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# A MODEL FOR ION ACCELERATION IN A Z-PINCH DURING AN m=0 INSTABILITY

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## I. Introduction

Fusion reactions in a plasma focus or in a z-pinch are mainly due to a beam-target process resulting from ions accelerated during the discharge. Quite different models for ion acceleration have been proposed, based on various effects, ranging from the E-field induced by the symmetric B-field compression, to the fields due to plasma instabilities. It is likely that more than one acceleration mechanism is in effect during a discharge. As a matter of fact, in some experiments it has been observed that more than one neutron peaks are produced, indicating different processes of ion acceleration. In a model based on the development of the m=0 instability proposed by Haines [1], the existence of an asymmetry along the z-axis was pointed out, which causes ions near the symmetry axis to be preferentially accelerated. This arises from the average axial drift of off-axis ions due to the radial electric field $Zn_ieE_r = \partial p_i/\partial r$. This axial flow has to be balanced by an equal and opposite flow of ions on-axis that have singular orbits.

To account for this effect it is necessary to include the Hall term in Ohm's law, as this is the one that breaks the symmetry. However, the assumption of no longitudinal plasma flow made in [1] is inconsistent with momentum equations. We consider here a model based on Haines' idea, that includes a finite axial velocity. Our model solves the Hall-MHD equations for a cylindrical plasma column, initially in equilibrium, after a pinching perturbation is applied.

## II. Model for the m=0 instability

In order to determine the ion orbits and obtain the acceleration they experience, we first need to model the evolution of the m=0 instability of the plasma column. In order to do that we use a Hall-MHD model in order to account for the asymmetry along the axial direction resulting from the singular orbit ions; the Hall and the electron pressure gradient terms have mixed parities. We use a cylindrical plasma column with a uniform axial current $I$, radius $a$ and an azimuthal magnetic field $\mathbf{B} = B_\theta\hat{\theta}(r)$. The corresponding equations for the density $\rho$, radial velocity $v_r$ and axial velocity $v_z$, are,

$$\frac{\partial \rho}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) - \frac{\partial}{\partial z}(\rho v_z) \tag{1}$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} - \frac{J_z B_\theta}{c} \tag{2}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{J_r B_\theta}{c} \tag{3}$$

where $J_r = (Ir/\pi a^3)\partial a/\partial z$, and $J_z = (I/\pi a^2)$. As the plasma evolves, the associated electric fields are obtained from Ohm's law as,

$$E_r = \frac{v_z B_\theta}{c} + \eta J_r - \frac{J_z B_\theta}{nec} - \frac{1}{ne}\frac{\partial p_e}{\partial r} \quad (4)$$

$$E_z = -\frac{v_r B_\theta}{c} + \eta J_z + \frac{J_r B_\theta}{nec} - \frac{1}{ne}\frac{\partial p_e}{\partial z} \quad (5)$$

(6)

where $\eta$ is the resistivity. The system is closed using a politropic equation for the pressure: $p = k\rho^\gamma$ with $\gamma$ a free parameter. In this model the B-field is frozen to the electrons and thus there is a difference between the guiding-center and the center-of-mass velocities.

The evolution of the column radius $a(t)$ follows from Faraday's law. In addition, we keep a finite $v_z$ as opposed to Haines consideration who used $v_z = 0$ [1], which is inconsisten with the formation of a neck in the column. The existence of an axial flow is essential. Therefore, the equation for $a$ is,

$$\frac{\partial a}{\partial t} = \frac{a^3}{2}\frac{\partial}{\partial z}\left(\frac{v_z}{a^2}\right) + \frac{\eta c^2}{4\pi}\frac{\partial^2 a}{\partial z^2} - \frac{3\eta c^2}{4\pi a}\left(\frac{\partial a}{\partial z}\right)^2 - \frac{Ia^3}{2\pi e}\frac{\partial}{\partial z}\frac{1}{\rho a^4} + \frac{a}{2r}\frac{\partial}{\partial r}(rv_r) - \frac{mI}{2\pi era^2}\frac{\partial a}{\partial z}\frac{\partial}{\partial r}\left(\frac{r^2}{\rho}\right) \quad (7)$$

This equations are solved by in a domain that includes the plasma and the vacuum region, starting from $r = 0$. The electric fields can have values even outside the plasma, when the neck starts forming, but the particles cannot cross there.

The numerical scheme followed is based on a leapfrog trapezoidal algorithm, with a predictor-corrector evolution. The boundary conditions in the $z$ direction are periodic for all variables except $v_z$, for which we set its derivative to be zero. In the radial direction we imposed Newmann boundary conditions at $r = 0$ (zero derivative), while for the vacuum region we assumed free boundary conditions. The perturbation is applied as a small sinusoidal radial velocity: $v_r(r, z, t = 0) = V_{r0} r \cos(kz)$. The free parameters that we can vary are the total current $I$ and the pressure $P$. The initial density profile was parabolic, satisfying the equilibrium equations.

In figure (1) we show the results for the plasma column density for $I = 150kA$ and $P = 1.8 \times 10^7 din/cm^2$. This produces the maximum compression rate at the neck at a time of 38 ns. The values of the politropic index were varied and the best compression was obtained for $\gamma = 2$ corresponding to a 2D adiabatic evolution, as expected. The same evolution of the m=0 instability is shown for the velocity field, in Figure (2). It is clear that the axial velocity is present near $r = 0$. Finally, in Figure (3) we show the electric field vectors, where one can see that they are maximum near the edge at the neck.

## III. Ion orbits

Next we take the two-fluid equations developed in [3] for the slow dynamics, in which the flow velocity and time scales are of the order of the diamagnetic velocity and

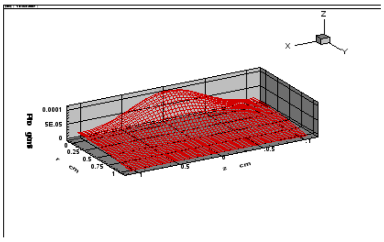
Figure 1: Evolution of the plasma column density.

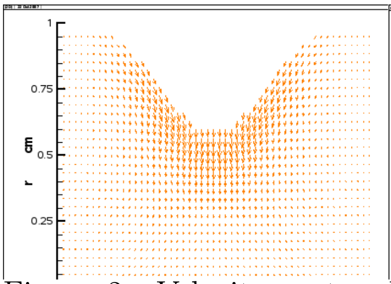

Figure 2: Velocity vectors at the of instability.

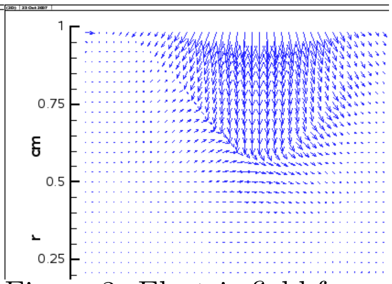

Figure 3: Electric field form Ohm's law with Hall term

frequency, respectively. Here, the effect of anisotropic pressures for ions and electrons is retained. However, the electron inertial effects are not present, so a completely collisionless reconnection could not be considered. The small, but finite collisionality is what will drive reconnection. When these equations are normalized such that $t \to t/\tau_A$, $x \to x/d_i$, $\phi \to \phi/(d_i^2 B_0/\tau_A)$, $\psi \to \psi/(d_i B_0)$, $n \to n/n_0$, $p \to 8\pi p/(\rho_s B_0/d_i)^2$ and adapted to our cartesian geometry, the reduced model for the seven fields can be cast as,

$$\frac{\partial n}{\partial t} = [n, \phi] \tag{8}$$

$$\frac{\partial p_{e\|}}{\partial t} = [p_{e\|}, \phi] + 2\nu_e(p_{e\|} - p_{e\perp}) \tag{9}$$

$$\frac{\partial p_{i\|}}{\partial t} = [p_{i\|}, \phi] + 2\nu_i(p_{i\|} - p_{i\perp}) \tag{10}$$

$$\frac{\partial p_{e\perp}}{\partial t} = [p_{e\perp}, \phi] - \nu_e(p_{e\|} - p_{e\perp}) + \frac{\rho_s^2}{3n^2}(p_{e\|} - p_{e\perp})[n, p_{e\perp}] \tag{11}$$

$$\frac{\partial p_{i\perp}}{\partial t} = [p_{i\perp}, \phi] - \nu_i(p_{i\|} - p_{i\perp}) - \frac{\rho_s^2}{3n^2}(p_{i\|} - p_{i\perp})[n, p_{i\perp}] \tag{12}$$

$$\frac{\partial \psi}{\partial t} = [\psi, \phi] - \frac{\rho_s^2}{n}[\psi, p_{e\|}] - \frac{\rho_s \epsilon \nu_e}{n}(p_{e\|} + p_{e\perp}) \tag{13}$$

$$\frac{\partial U}{\partial t} = [U, \phi] + \frac{1}{2n}[n, |\nabla\phi|^2] + \frac{1}{n}[\psi, \nabla^2\psi] + \frac{\rho_s^2}{n}[\nabla\phi; \nabla p_{i\perp}] - \rho_s^2 \frac{\partial}{\partial x}(p_{i\|} + p_{i\perp} + p_{e\|} + p_{e\perp}) \tag{14}$$

Here we assumed simmetry along the perpendicular $z$ direction and used the simplest approximation for the collisional friction: $F_e^{coll} \sim \nu_e p_e$, $g_\alpha^{coll} \sim \nu_\alpha(p_{\alpha\|} - p_{\alpha\perp})$. The dimensionless constants are $\epsilon = (m_e/2m_i)^{1/2}$, $\nu_\alpha \to \nu_\alpha \tau_A$ and the function $U$ is,

$$U = \frac{1}{n}\left(\nabla \cdot (n\nabla\phi) + \rho_s^2 \nabla^2 p_{i\perp}\right) \tag{15}$$

The characteristic length is the ion skin depth, so the reconnection region must be smaller than this. It is clear that the driving term for reconnection in Eq.(13) is quite small being proportional to $\epsilon$.

Based on Eqs.(8-14) we created a code for the simulation of the reconnection Taylor problem, as described in the previous section. An approximation was made, that

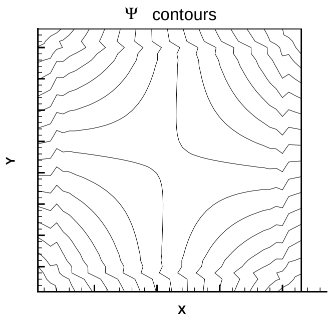


Figure 4: Magnetic field lines at X-point for $t = 6\tau_A$

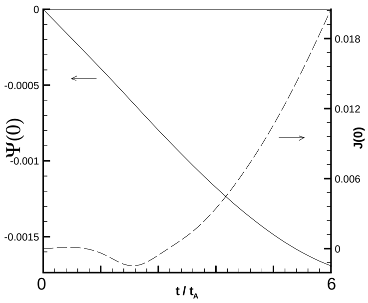


Figure 5: Evolution of $\psi(0)$ and central current

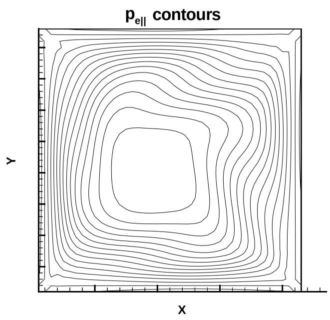


Figure 6: Pressure for high collisionality, $\nu = 0.9$

assumes the density in Eq.(15) is constant, in order to speed up the process of inverting the operator to obtain $\phi$. It was checked *a posteriori* that the assumption is indeed satisfied, when $n$ is kept constant at the boundaries. For the pressures, although they have constant boundary values, there is a continuous increment towards the central X-point, as it is shown in Figure (3) for $p_{e\|}$; similar behaviors are observed for the other pressures when $p_{\alpha\|}/p_{\alpha\perp} = 1.1$.

An interesting result, not observed in the zero-pressure case, is that the forcing flow at the border gives rise to an MHD wave that wiggles the magnetic field lines. Figure (4) shows how the originally smooth field lines are twisted as a result of the wave. The reconnected flux can be seen in Fig.(5) and is quite small, for the value $\nu = 0.09$ used. It has opposite sign to that for the collisionless case, since the currents are also negative initially. For higher collisionality, $\nu = 0.9$, the distorting effects are quite large and the reconnected flux does not vary so much (see Fig.(6)).

### IV. Conclusions

The collisional contributions to the reconnection rate in a quasi-collisionless plasma were analyzed using two different approaches. When pressure variations are ignored, the collisionality increases the instantaneous reconnection rate but the asymptotic values of $\psi(o)$ do not change, while the current at the X-point is reduced substantially. With a model that includes anisotropic pressure perturbations the presence of MHD waves is observed and the reconnection rate is very low.

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## References

[1] M.G. Haines, Nuclear Instrum. Methods **207**, 179 (1983)