*35th EPS Conference on Plasma Phys. Hersonissos, 9 - 13 June 2008 ECA Vol.**32D**, P-2.089 (2008)*

Nonlinear regime of Bragg backscattering leading to probing wave trapping and time delay jumps in fast frequency sweep reflectometry.

E.Z. Gusakov[1], S. Heuraux[2], A. Yu. Popov[1]

[1] *Ioffe Institute, St.Petersburg, Russia*

[2] *LPMIA, UMR-CNRS 7040, Vandoeuvre CEDEX, France*

Microwave reflectometry is widely used technique providing information on plasma density profile and drift wave turbulence in magnetic fusion devices. The RADAR modification of this diagnostics utilizing frequency or amplitude modulation of the probing wave allows separation of profile and turbulence information. In spite of been a robust method of density distribution measurements [1], in some regimes the fast frequency sweep reflectometry demonstrate huge time delay jumps which are difficult to explain based upon a model of the microwave propagation along a smooth density profile to the cut off and back.

In the present paper an alternative model based on numerical and analytical study of Bragg back scattering (BBS) nonlinear regime is proposed to explain the effect of time delay jumps.

1. Physical model

Considering an incident ordinary wave propagating perpendicular to the external magnetic field, we treat it assuming slab plasma geometry in the framework of the Helmholtz equation

$$\left\{\partial^2/\partial x^2 + k_0^2\left(1-n(x)/n_c\right)\right\}E(x)=0\ ,\ \left(\ n(x)=n_0(x)+\delta n(x)\right) \qquad (1)$$

where $n(x)$ is the density profile, $\delta n(x)=\sum_{j=1}^{m}\delta n_j(x)=\sum_{j=1}^{m}\delta_j n_c \exp\left[-\left(x-x_j\right)^2/L^2\right]\cos\left(\kappa_j x\right)$, is modeling a superposition of quasi-coherent turbulent fluctuations localized in a position of the corresponding Bragg resonance (BR) where the BBS condition $\kappa_j = 2k\left(x_j\right)=2k_0\sqrt{1-n_0\left(x_j\right)/n_c}$ is fulfilled, $k_0=\omega/c$, δ_j is a relative fluctuation amplitude, L is the width of the fluctuation envelope and $n_c = n_0(x_c) = m_e\omega^2/\left(4\pi e^2\right)$. We solve (1) numerically and investigate its solutions analytically using methods developed in the three wave interaction theory. In the later case assuming weak plasma inhomogeneity $\kappa_j x_c >> 1$ we seek a solution to (1) in the form of the incident and reflected WKB waves

$$E(x)=a_i(x)\Big/\sqrt{k(x)}\exp\left(i\int^x k(x')dx'\right)+a_r(x)\Big/\sqrt{k(x)}\exp\left(-i\int^x k(x')dx'\right) \qquad (2)$$

The amplitudes $a_i(x)$ and $a_r(x)$ are varying slowly due to BBS in the vicinity of the BR points obeying the following reduced differential equations

$$ia_i' + \sqrt{Z}a_r \exp(i\Phi) = 0, \quad -ia_r' + \sqrt{Z}a_i \exp(-i\Phi) = 0 \tag{3}$$

where $Z(x) = k_0^4 l_j^2 / \kappa^2 \cdot \delta n_j^2(x)/n_c^2$ and $\Phi(\xi)$ is a phase mismatch caused by plasma inhomogeneity. In vicinity of the BR it takes the form $\Phi(\xi) = \xi^2$, $\xi = (x - x_j)/l_j$, where $l_j = |dk(x_j)/dx|^{-1/2}$ is a scale determining the linear BBS region size. The system (3) was analyzed in the theory of three-wave parametric decay instability [2-4]. In the case $L \to \infty$ its solutions are expressed in terms of the parabolic cylinder functions.

2. The strong reflection due to the BBS effect.

Based on reduced equations (3) we start with consideration of the nonlinear BBS phenomena in the vicinity of the single BR. For this purpose we suppose that the only fluctuation exists on the density profile (m=1) and assume no cut off in plasma (see Fig 1a). Under these assumptions we may write the field of the wave incident from the plasma edge ($x = 0$) onto the Bragg resonance region ($x = x_1$) and transmitted through it, as

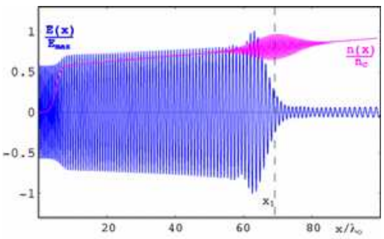

Fig.1a. Normalized density (violet) and electric field (blue) distributions in case of single perturbation. (Density gradient length 390λ₀)

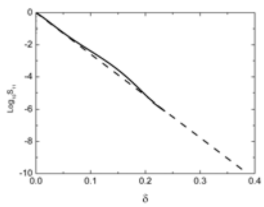

Fig.1b. Dependence of transmission on the fluctuation amplitude. Analytical formula (dashed line), simulation (solid line).

$$E(x) = a_i^{in} k(x)^{-1/2} \exp\left(i\int_0^x k(x')dx'\right) + S_{ir} a_i^{in} k(x)^{-1/2} \exp\left(-i\int_{x_1}^x k(x')dx'\right), \qquad x_1 - x >> l_1 \tag{4}$$

$$E(x) = S_{ii} a_i^{in} k(x)^{-1/2} \exp\left(i\int_{x_1}^x k(x')dx'\right), \qquad x - x_1 >> l_1$$

The matrix S_{ij} elements here stand correspondingly for reflection (S_{ir}) and transmission (S_{ii}) coefficient. Analyzing the asymptotic behavior of system (3) solutions at $\xi \to -\infty$ and $\xi \to \infty$ we determine them, according to [4], in the form

$$S_{ii/rr} = \exp\left(-\frac{\pi Z_*}{2}\right), \; S_{ir/ri} = \mp 2\sqrt{\frac{\pi}{Z_*}} \frac{\exp(-\pi Z_*/4)}{\Gamma(\mp i Z_*/2)} \tag{5}$$

where $Z_* = Z/\sqrt{1 + 2(l_j/L)^2 Z}$

An important feature of the transmission coefficient is its exponential dependence on parameter Z_* and thus on the fluctuation amplitude. In the case $Z > 1$ this dependence leads to suppression of transmission and to next to 100% reflection ($S_{ii}\big|_{Z>>1} \to 0$, $S_{ir/ri}\big|_{Z>>1} \approx \exp(\pm i\varphi_\Gamma), \varphi_\Gamma = Z_*/2(\ln(Z_*/2) - 1) - \pi/4$). Then a picture of the wave propagation resembles total reflection of the incident wave from the BR region with an additional phase φ_Γ imposed by scattering (see blue curve in fig.1a). The analytical dependence of $\log_{10}(S_{ii})$ on the fluctuation amplitude δ agrees well with the computed transmission coefficient, as it is seen in fig.1b.

3. The microwave trapping between the BR and cut-off.

In this section we consider the case of the influence of a single quasi-coherent mode on microwave propagation at plasma parameters for which the cut-off is in the plasma volume, however far from the BR so that the inequality $l_1 << \min\{L, x_c - x_1\}$ is valid (see fig.2a). In

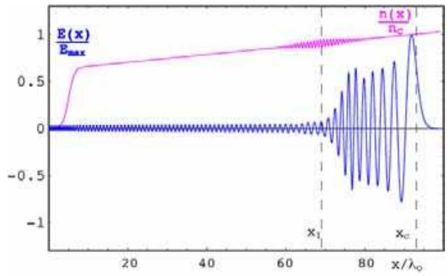

Fig.2a. Normalized plasma density (violet) and electric field (blue) distributions in case of single perturbation and cut off.

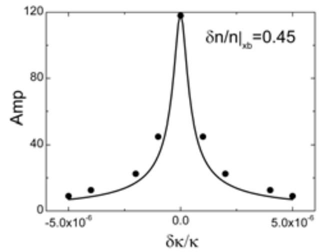

Fig.2b Amplification factor versus $\delta\kappa$. Solid curve formula, scattered circles-simulation.

this case the amplitude of the electric field propagating from the BR to the cut-off is determined both by transmission through BR and by reflection due to BBS of the wave reflected in the cut off and as a result is given by expression

$$a_i^{out} = a_i^{in} S_{ii} \Big/ \left(1 - S_{ri} \exp(-i2\varphi_0)\right) \qquad (6)$$

with $\varphi_0 = \int_{x_1}^{x_c} k(x')dx' + \delta\varphi(\delta n) - i\pi/4$ consisting of the regular WKB phase and a smaller part $\delta\varphi(\delta n)$ related to the dispersion relation modification due to BBS important at $Z > 1$. According to (6) the transmission is generally exponentially small, unless the specific resonance condition $2\varphi_0 + \varphi_\Gamma(\delta n_0) = 2\pi n$ is met by the phase φ_0 minimising the absolute value of the denominator in (6). In this resonant case at $Z > 1$ the transmitted wave is exponentially large $a_i^{out} \sim \exp(\pi Z_*/2) a_i^{in}$, which is explained by formation of a "weakly coupled cavity" between the cut off and strongly reflecting BR. The wave trapping by the cavity is clearly seen in numerical modelling shown in fig.2a. The "cavity" quality factor is determined by exponentially small tunnelling through the BR region. Therefore at high $Z >> 1$ the width of the "resonance curve" is very narrow, as it is seen in fig.2b where the dependence of the amplification factor $amp = a_i^{out} / a_i^{in}$ on fluctuation wave number $\delta\kappa$ obtained numerically and analytically is shown. The wave trapping effect can manifest itself in the experiment by fast reflected wave phase jumps and enhanced scattering produced in the region of high field localisation.

4. The microwave trapping by two quasi-coherent modes.

A similar wave trapping effect is possible also far from the cut off if a couple of strong fluctuations satisfying condition $Z_1 > 1, Z_2 > 1$ exist in plasma. Being good reflectors, these fluctuations situated in the vicinity of corresponding BR are leading to the effective cavity formation. The electric field structure in such a cavity is shown in fig.3a. The analytical treatment provides in this case the resonance phase condition $2\varphi_0 + \varphi_\Gamma(\delta_1) - \varphi_\Gamma(\delta_2) = \pi(2n+1)$ and the maximal amplification expression $a_i^{out} \sim a_i^{in} \exp(\pi Z_{*1}/2) / \left(1 + \exp\left[\pi(Z_{*1} - Z_{*2})\right]\right)$ valid for $Z_{*1} < Z_{*2}$. The perfect fit of this expression to the modeling results is shown in the fig .3b.

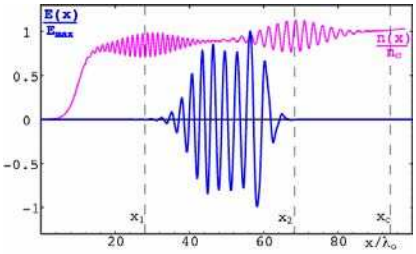

Fig.3a. Normalized plasma density (violet) and electric field (blue) distributions in a double case perturbation.

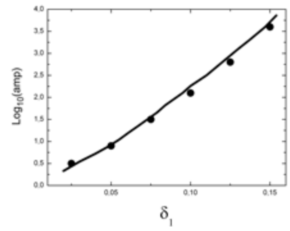

Fig.3b Amplification factor versus fluctuation amplitude δ_1. *Solid curve-formula, circles-simulation.*

5. Conclusions.

As a result of 1D analyses it is shown that reflection of the probing microwave may occur in the vicinity of the BR point (far from the cut off) at a high enough density fluctuation level leading to large jump of the reflected wave phase and corresponding time delay. The criterion for transition to this nonlinear regime of BBS in the case of linear density profile takes a form

$$\delta n^2 / n_c^2 > \kappa / k_0^2 x_c \qquad (7)$$

The possibility of probing wave trapping due to BBS is demonstrated. The analyzed nonlinear BBS phenomena may occur in the ITER scale machine at the level of turbulent density fluctuations of less than 1% routinely observed in the present day tokamaks.

Financial support of RFBR grants 06-02-17212, и *07-02-92162-CNRS is acknowledged*

References.

1. F. Clairet *et.al.*, *Rev. Scient. Instrum.* 73, 1481 (2003)
2. Piliya A. D. *Proc. 10th Conf. Phenomena in Ionized Gases (Oxford)* p 320, (1971)
3. M. Rosenbluth, *Phys. Rev. Lett.* 29, 564 (1972);
4. Piliya A. D. *Zh. Eksp. Teor. Fiz. (Sov. Phys.—JETP)* 64 1237, (1973)