*35th EPS Conference on Plasma Phys. Hersonissos, 9 - 13 June 2008 ECA Vol.**32D**, P-2.059 (2008)*

Flow Shear Effects on Resistive MHD Instabilities in Tokamaks

D.P. Brennan[1], S.E. Kruger[2], R.J. La Haye[3]

[1] *University of Tulsa, Tulsa, OK, USA*

[2] *Tech-X Corporation, Boulder, CO, USA*

[3] *General Atomics, San Diego, CA, USA*

MHD theory has been used with great success over recent years in describing the physics of many observed instabilities in tokamak experiments relevant to fusion. However, typically these analyses have not considered the presence of equilibrium flow shear, despite the facts that flow shear is known to be important in these experiments[1, 2].

The consideration of flow shear can significantly affect the overall diagnosis of experimental observations by strongly affecting the equilibrium, stability and transport in the theoretical descriptions. Several studies have been published in the literature which analytically and numerically address the effect of flow shear on resistive MHD modes in simplified geometries. Recently, it has become possible to realistically and accurately address the nonlinear effects of flow shear in arbitrary axisymmetric toroidal configurations at finite β[3, 4, 5, 2].

A series of nonlinear simulations of coupled tearing/interchange modes driven by an internal kink mode in the presence of flow shear are presented and analyzed for the dependence of nonlinear drive on flow. The equilibria used as initial conditions are based on the DIII-D tokamak, as shown in Fig.1. The effects of sub-sonic flow on the driven nonlinear phase are detailed in the analyses, where both the inner layer physics and the coupling between surfaces can be affected. Flow shear provides free energy to the instability in the inner layer in the experimental regime[6], while differential flow between neighboring rational surfaces can have a damping influence by decoupling their outer region solutions.

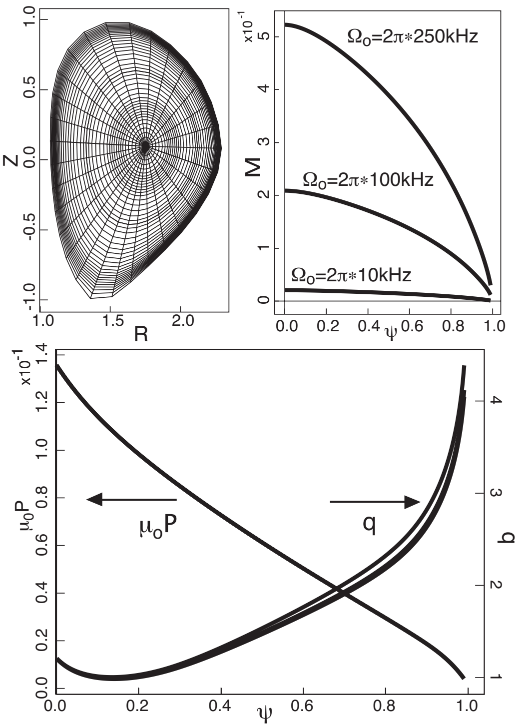

Figure 1: The shape, pressure, q and flow profiles of the equilibria used in this study.

Equilibrium

The inclusion of flow shear effects in this context can be separated into three parts, the effects on the equilibrium, the effects on the inner layer solutions, and the effects on the coupling between resonant modes.

Including the flow in the equilibrium solutions is crucial in initial value computations, but does not significantly affect the outer region solutions for the instabilities. Accurate force balance must be met for the initial value simulations to be valid. The change in the force balance is proportional to the square of the Mach number. The Mach number $M = \Omega R_0 \sqrt{\rho_i/(2P_0)}$ is varied from 0 to ~ 0.5, giving a maximum of 25% percent change in the equilibrium due to flow, as shown in Fig. 1. The flow inclusive pressure is $P = P_0 e^{((R/R_0)^2-1)M^2}$. The profiles of flow imposed on the system are linear in the poloidal flux $\Omega(\psi) = \frac{d\Omega}{d\psi}(1-\psi)$. The mass density ρ_i is constant.

Fluid elements on neighboring rational surfaces can rotate past each other several times during the growth of the mode. The growth of the $1/1$ drives the system to saturation in $\Delta t\ 10^3 \tau_A$. The significant flow differential $\Delta\Omega \sim \Omega_0 \Delta\psi \sim 3\times 10^4 Hz$ for $\Omega_0 \sim 1\times 10^5 Hz$. Were the modes uncoupled they would experience ~ 10 rotations past each other in a growth time.

Competing Influences

The effects of flow can be somewhat separated into flow shear and differential flow effects. The combination of shear and differential flow can produce complicated results, and this is the case for realistic profiles. Even for moderate toroidal flow shear, the inner layer solutions can be affected, and the relative rotation between rational surfaces can significantly affect the coupling. The solution to the inner layer equations, even for a single uncoupled mode, can be either destabilizing or stabilizing, depending on conditions, as has been widely reported in the literature. One well known example is Chen and Morrison[6]. This study follows the basic approach of FKR with added flow. The results showed that in the typical constant ψ regime resistive modes were more unstable with flow, with $\gamma \sim \alpha^{2/5}\Delta'^{4/5}S^{-3/5}\hat{\gamma}$.

Two highly relevant studies were conducted by Kruger[3] and Chandra et al. [4].

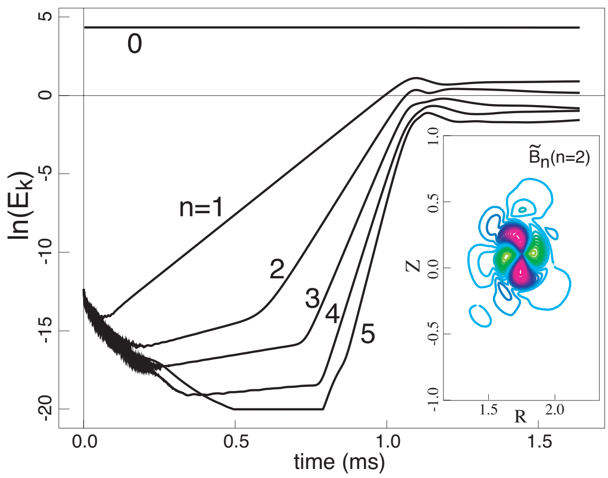

Figure 2: The log of the kinetic energies in a NIMROD computation of a $1/1$ driving various harmonic modes.

Both studies employ reduced MHD initial value computations of coupled modes in toroidal geometry. These studies focus on the interaction of two resonant modes, address only the tearing parity, and have several results in common, in particular the damping effect that flow has on coupling drive between modes.

The coupling between surfaces can be understood in terms of a simple model. Linearly, for two surfaces, and ignoring the interchange component, we can solve the quadratic dispersion relation[3] for the linear mode based on the 2x2 determinant $|D' - D(Q)| = 0$, where Q becomes complex in the inner layer $Q \equiv Q_r + iQ_i$ where $Q_r = \mathbf{k} \cdot \mathbf{V}_\phi$ (ignoring poloidal precession from ω_* effects) represents the toroidal angular frequency at the rational surface, and Q_i represents the mode growth rate. Moving to the reference frame of the flow we can write a quadratic for the complex growth rate as:

$$2\omega = \Omega_1 + \Omega_2 + i(\gamma_{11} + \gamma_{22}) \pm [\Delta\Omega + i(\gamma_{22} - \gamma_{11})][1 - \frac{4\gamma_{12}\gamma_{21}}{(\Delta\Omega + i(\gamma_{22} - \gamma_{11})^2}]^{\frac{1}{2}} \tag{1}$$

where γ_{ij} is the growth rate in the rest frame of the surface i, and $\Delta\Omega = \Omega_1 - \Omega_2$ is the differential flow. Taking $\Delta\Omega >> \gamma_{ij}$ and looking at surface 2

$$\omega \approx \Omega_2 - \frac{\gamma_{21}\gamma_{12}}{\Delta\Omega} + i\gamma_{22} \tag{2}$$

Assuming surface 2 is linearly stable, the growth of mode 2 can be estimated as

$$\frac{\psi_2}{\psi_1} \approx \frac{\gamma_{21}}{\Delta\Omega} \tag{3}$$

The competing influences of inner layer drive and flow damping gives us some intuition for the initial value analysis.

Thus, significant progress can be made in understanding mode onset and evolution with equilibrium flow in a nonlinear single fluid resistive MHD treatment. The NIMROD code solves linear and nonlinear MHD equations as initial-value computations with a mesh of finite elements for the poloidal (R-Z) plane and finite Fourier series for the toroidal direction. The single fluid form of the NIMROD equations is

$$\begin{aligned}
\rho\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right) &= \mathbf{J}\times\mathbf{B} - \nabla p - \nabla\cdot\Pi \\
\mathbf{E} &= -\mathbf{V}\times\mathbf{B} + \eta\mathbf{J} \\
n\frac{\partial T}{\partial t} + n\mathbf{V}\cdot\nabla T + (\Gamma-1)nT\nabla\cdot\mathbf{V} &= -(\Gamma-1)\nabla\cdot\mathbf{q} + (\Gamma-1)Q \\
\mathbf{q} &= -(\kappa_{\|} - \kappa_{\perp})\nabla_{\|}T - \kappa_{\perp}\nabla T
\end{aligned} \tag{4}$$

along with Maxwell's and continuity equations, where V is the fluid velocity, ρ the mass density, J the current density, B the magnetic field, p the pressure, Π the stress tensor (including a numerical $-\rho\nu\nabla^2 V$), E the electric field, η the electric resistivity, T the temperature, Γ the ratio of specific heats, q the heat flux, and κ the thermal diffusivity. The single fluid model ignores drift waves, but captures the essential physics of nonlinear coupling.

The log of the kinetic energy from an initial value computation of the nonlinear 1/1 driven instability is shown in Fig. 2, note the large $n = 0$ component (flow). Growth rates increase with flow in general, which stems from the inner layer influence. The nonlinear perturbed magnetic field normal to the flux surfaces is shown in the insert to Fig. 2. Taking a Fourier decomposition of these fields in the poloidal direction, we can determine the phase of the now separate poloidal components $m = 2$ and $m = 3$ of the $n = 2$ mode by $\phi = tan^{-1}(Im(B_r)/Re(B_r))$ at each surface. In Fig. 3(a) this phase information is plotted for for the duration of a computation with $\Omega_0 = 1 \times 10^4 s^{-1}$. In Fig. 3(b) the ratio of the amplitudes of the 3/2 and 2/2 B_r components is shown, which is essentially a measure of the reconnected flux. At low flow this ratio increases with flow due to inner layer drive, while high differential flow damps the drive to the 3/2 mode; the influences are mixed. The torque on the mode $sin(\Delta\phi)$, with $\pi/2$ a maximum. An increase in torque is clearly visible at the transition to nonlinear drive $\sim 0.7ms$, as is an abrupt change in $\Delta\phi$ as nonlinear saturation is approached.

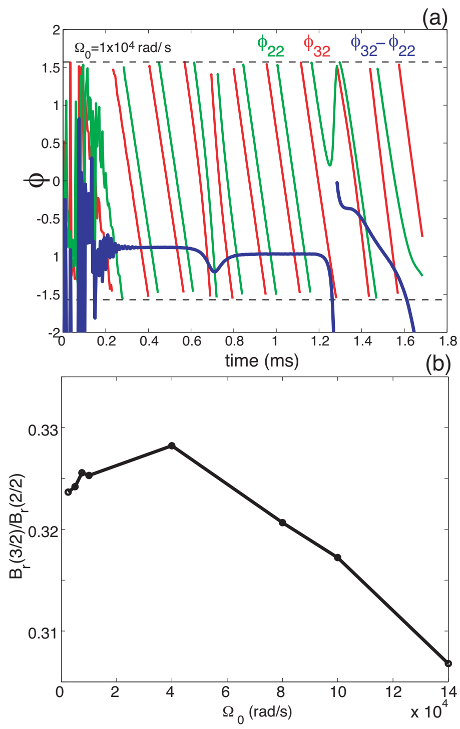

Figure 3: The poloidal phases of the 3/2 and 2/2 components and the ratio of their amplitudes.

Supported by US DOE Grant DE-FG02-08ER54950

References

[1] M.S. Chu, L. Chen, L.-J. Zheng, C. Ren and A. Bondeson, Nucl. Fusion **39** 2107 (1999).

[2] D.P. Brennan, S.E. Kruger, T.A. Gianakon *et al.*, Nucl. Fusion **45**, 1178 (2005).

[3] S.E. Kruger, PhD Thesis, The University of Wisconsin - Madison (1999).

[4] Chandra, Nucl. Fusion **45**, 524 (2005).

[5] C.R. Sovinec, A.H. Glasser, T.A. Gianakon *et al.*, J. Comp. Phys. **195**, 355 (2004).

[6] X.L. Chen and P.J. Morrison, Phys. Fluids B **2**, 495 (1990).