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A contribution to the equilibrium and stability of axisymmetric plasmas with field aligned flow

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In the ordinary tokamak discharges the current density is usually peaked at the plasma center and poloidal field is finite everywhere in the plasma column except the magnetic axis. However, stable tokamak plasmas with nearly zero poloidal magnetic field and therefore nearly zero toroidal current density in the appreciable central region (a "current hole") were observed in the JET [1] and JT-60U [2] tokamaks and were sustained for several seconds. In the JET discharges it was noticed that the core current density is clamped at zero, indicating the existence of a physical mechanism which prevents it to becoming negative. This issue is related to further experimental and theoretical studies, e.g. [3, 4, 5] and Refs. cited therein, according to which a possible route to current reversal in the plasma core is the formation of configurations with non-nested magnetic surfaces. In particular, experimental results of the HT-7 tokamak [3] indicated the existence of two oppositely flowing currents in the high-field-side and low-field-side during current reversal. Equilibria of this kind with zero average toroidal current were constructed recently in Ref. [5]. Another issue which plays a potential role is plasma flow; for example, in the presence of toroidal flow the existence of tokamak equilibria with central current reversal and nested magnetic surfaces was claimed in Ref. [6].

The aim of the present contribution is twofold: (i) to construct analytic equilibrium solutions with parallel incompressible flow and either peaked or reversed current density in connection with nested or non-nested magnetic surfaces and (ii) to apply a recent sufficient condition [7] to a particular solution [Eq. (4) below] as a first-step stability consideration.

The equilibrium of an axisymmetric plasma with incompressible flow parallel to the magnetic field satisfies the generalized Grad-Shafranov equation [8, 9]

$$\Delta^\star u = -R^2\frac{dP_s}{du} - \frac{1}{2}\frac{d}{du}\left(\frac{X^2}{1-M^2}\right) \tag{1}$$

Here, $(z,\ R,\ \phi)$ are cylindrical coordinates, $u(R,z)$ is the poloidal magnetic flux function, P_s represents the pressure when the flow vanishes, X relates to the toroidal magnetic field, M is the Alfvén Mach function and $\Delta^\star = R^2\nabla\cdot(\nabla/R^2)$. Eq. (1) should be solved under appropriate boundary conditions after assigning the free functions $P_s(u)$, $X(u)$ and $M(u)$; i.e., for tokamak and reversed filed pinch equilibria u can be taken constant on a fixed boundary while for equilibria of compact toroids u must be additionally constant on the axis of symmetry.

We will construct an analytic solution of Eq. (1) by using the linearizing ansatz

$$\begin{aligned} M^2 &= \frac{M_a^2 u^2 \left(X_0^2 + u_a^2 \Lambda_1\right)}{u_a^2 \left(X_0^2 + u^2 \Lambda_1\right)}, \\ X &= \left(1 - M^2\right)^{1/2} \left(X_0^2 + \Lambda_1 u^2\right)^{1/2}, \\ P_s &= P_{sa} \frac{u^2}{2}, \end{aligned} \quad (2)$$

where M_a, X_0, P_{sa}, Λ_1, and u_a are constants. Then, on separation of variables for up-down symmetric configurations of arbitrary aspect ratio Eq. (1) admits the solution

$$u(R,z) = \alpha \left[F_0(\eta,\rho) + \gamma G_0(\eta,\rho)\right] \cos\left(\lambda \frac{z}{R_0}\right), \quad (3)$$

where α, γ, λ and R_0 are constants, $\rho = \sqrt{P_{sa}} R^2/(2R_0)$, $\eta = (\lambda^2 - \Lambda_1)/(4\sqrt{P_{sa}})$, and F_0 (G_0) the Coulomb wave function of the first (second) kind. For vanishing flow (3) reduces to the Hernegger-Maschke solution [10] and for $\eta = 0$ it assumes the simpler form

$$u = \alpha \left(\sin\rho + \gamma \cos\rho\right) \cos\left(\lambda \frac{z}{R_0}\right). \quad (4)$$

We will consider further (4) for a tokamak of rectangular boundary cross section of height $2b$ and width $2a$. The quantities a and b will be chosen as free parameters together with the radial position of the geometric center (R_0), the vacuum toroidal magnetic field at the geometric center ($B_{\phi 0} = X_0/R_0$), the safety factor on one of the magnetic axes (q_a) and the Mach function on the respective magnetic axis (M_a). To avoid potential current driven instabilities the value of q_a will be chosen

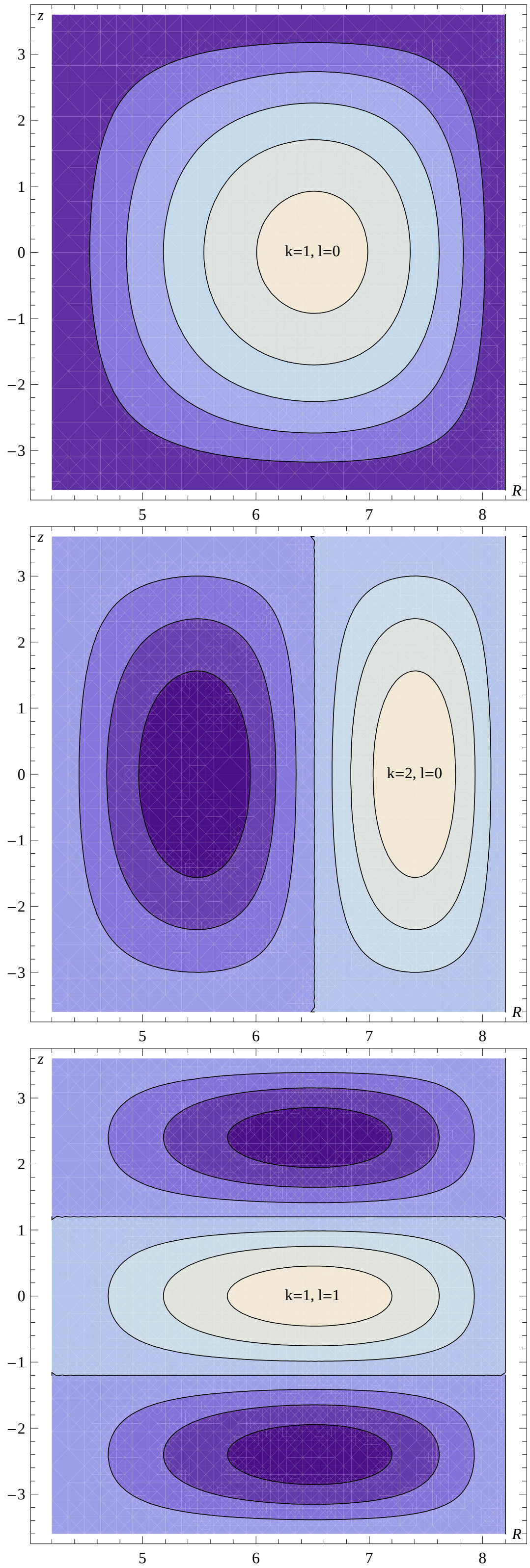

Figure 1: *A variety of configurations determined by the equilibrium solution 4.*

so that the safety factor on all magnetic axes be larger than unity. The boundary condition $u(R = R_0 \pm a) = 0$ and $u(z = z_0 \pm b) = 0$ leads to discrete values of P_{sa}, Λ_1 and γ. Simply or multiply toroidal configurations can be constructed for different values of a couple of integers k and l. In particular, for the configurations shown in Fig. 1 we have chosen $M_a^2 = 0.1$ and ITER relevant values for the other free parameters: $a = 2$m, $b = 3.5$m, $R_0 = 6.2$m, $B_{\phi 0} = 5$Tesla and $q_a = 1.1$. As expected current reversal on axis is possible for multiply toroidal configurations. In particular, the doubly toroidal configuration of Fig. 1 has nearly zero average toroidal current as shown in Fig. 2. Also, the positions of the magnetic axes have been analytically determined together with the Shafranov shift of any of them:

$$\Delta R = R_0 \left\{ \left[1 + \frac{a^2}{R_0^2} + \frac{2a}{R_0 k}(1-k) \right]^{1/2} - 1 \right\}. \quad (5)$$

The linear stability of the equilibria described by (4) will now be considered by applying a recent sufficient condition [7]. This condition states that a general steady state of a plasma of constant density and incompressible flow parallel to **B** is stable to small three-dimensional perturbations if the flow is sub-Alfvénic ($M^2 < 1$) and $A \geq 0$, where A is given by Eq. (20) of Ref. [7]. For an axisymmetric equilibrium A is put in the form

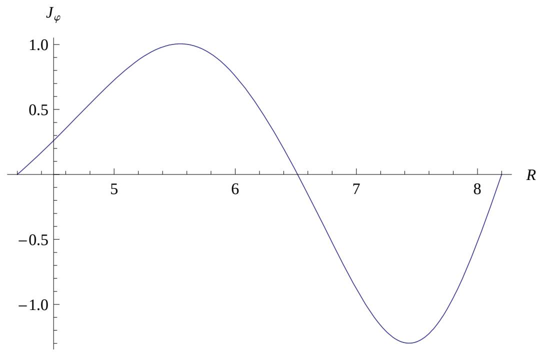

Figure 2: *The toroidal current density profile on the mid-plane $z = 0$ normalized with respect to the inner magnetic axis for the doubly toroidal equilibrium of Fig. 1.*

$$A = -g^2 \left\{ (\mathbf{J} \times \nabla u) \cdot (\mathbf{B} \cdot \nabla) \nabla u + \frac{1}{2}\frac{dM^2}{du} \left(1 - M^2\right)^{-1} |\nabla u|^2 \left[\left(1 - M^2\right)^{-1/2} \nabla u \cdot \frac{\nabla B^2}{2} + g \left(1 - M^2\right)^{-1} |\nabla u|^2 \right] \right\} \quad (6)$$

where

$$g = \left(1 - M^2\right)^{-1/2} \left(\frac{dP_s}{du} - \frac{dM^2}{du}\frac{B^2}{2} \right). \quad (7)$$

For elongations (b/a) varying from 0.25 to 4, q_a from 1.1 to 10 and M_a^2 from 0 to 0.5 it turns out that the condition $A > 0$ is never satisfied irrespective of nested magnetic surfaces. As an example the variation of A in the mid-plane $z = 0$ for the singly toroidal configuration of Fig. 1 is given in Fig. 3. However, since the stability condition is sufficient, this result does not necessarily imply that the equilibrium (4) is unstable.

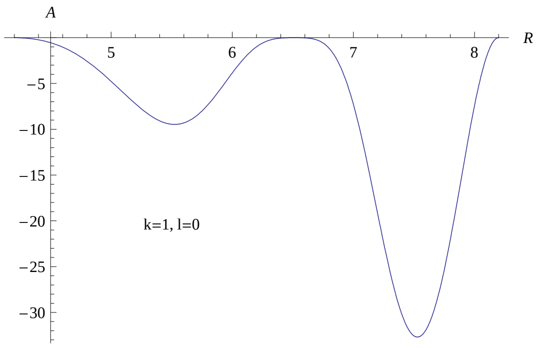

Figure 3: *Radial variation of A in the mid-plane $z = 0$ normalized with respect to the absolute value at the geometric center for the singly toroidal configuration of Fig. 1.*

In summary, we have constructed analytic equilibria in terms of Coulomb wave functions with incompressible flow parallel to the magnetic field and pressure gradient, toroidal current density and flow vanishing on the plasma boundary. The possibility of regular configurations (with nested magnetic surfaces) and reversed current density configurations (with non nested magnetic surfaces) has been demonstrated by means of a particular sinusoidal solution [Eq. (4)]. Also, as a first step study of linear stability, a recent sufficient condition was applied to the sinusoidal solution. For wide parametric regions it turns out that the condition is not satisfied irrespective of current reversal and Mach numbers. Since the condition is sufficient, however, not fulfillment does not necessarily imply that the equilibrium is unstable. Finally, it is noted that extension of the present study for the generic Coulomb wave function solution (3) and realistic boundary shapes is under way.

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