

Intermittency in Resistive Ballooning Electromagnetic Turbulence

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Abstract

Turbulent fluctuations and the associated transport of heat and particles play a crucial role in magnetic fusion devices. Experimental observations reveal that even though turbulence is essentially electrostatic, magnetic fluctuations can have a strong impact on the plasma dynamics. Moreover, several analyses have revealed the non-Gaussian statistics in the edge fluctuations of a number of devices. In this study, we analyze the electromagnetic resistive ballooning turbulence, as modeled with the global code EMEDGE3d, and we focus on the statistical properties of the electromagnetic fluctuations.

Introduction

Plasma turbulence and the associated transport are limiting factors for the performance of fusion machines. Statistical descriptions of plasma turbulence can reveal basic characteristics of the turbulent dynamics over a broad range of experimental conditions. In this work, we focus on the statistical description of Resistive ballooning modes (RBM), a class of modes that plays a significant role in the confinement of the finite β edge plasma.

The analysis we perform is based on numerical results obtained with the three-dimensional global code EMEDGE3D that self-consistently computes the evolution of RBMs in toroidal geometry. The RBM turbulence is described by the following coupled equations for the normalized electrostatic potential ϕ , pressure p and magnetic field ψ [1]:

$$\partial_t \nabla_{\perp}^2 \phi + \{\phi, \nabla_{\perp}^2 \phi\} = -\frac{1}{\alpha} \nabla_{\parallel} \nabla_{\perp}^2 \psi - Gp + \nu \nabla_{\perp}^4 \phi \quad (1)$$

$$\partial_t p \{\phi, p\} = \delta_c G \phi + \chi_{\parallel} \nabla_{\parallel}^2 p + \chi_{\perp} \nabla_{\perp}^2 p + S(r) \quad (2)$$

$$\partial_t \psi = -\nabla_{\parallel} \phi + \frac{1}{\alpha} \nabla_{\perp}^2 \psi \quad (3)$$

The ballooning instability is driven on the low field side by the combination of the pressure gradient and the toroidal curvature of the magnetic field lines.

∇_{\parallel} and ∇_{\perp} correspond to the parallel and the perpendicular gradients along the magnetic field lines, respectively, G is the curvature operator, $G = \sin \theta \partial_r + \cos \theta / r \partial_{\theta}$, ν is the viscosity, χ_{\perp} and χ_{\parallel} are the perpendicular and the parallel diffusivity. Time is normalized to the resistive interchange time $\tau_{int} = \sqrt{R_0 L_p / 2} / C_s$, where C_s is the sound speed. The characteristic perpendicular and parallel length scales are the resistive ballooning length $\xi_{bal} = \sqrt{\rho \eta_{\parallel} / \tau_{int} L_s} / B$ and the magnetic shear length L_s , respectively. Here, ρ is the mass density, η_{\parallel} is the parallel resistivity, and α is given by $\alpha = \beta L_s^2 / (R_0 L_p)$. The thermal diffusivities $\chi_{\parallel, \perp}$ are normalized with respect to τ_{int} / ξ_{bal}^2 and the normalized viscosity with respect to $\tau_{int} / (\xi_{bal}^2 m_i n_0)$. Moreover, $S(r)$ represents a constant energy source, $\delta_c = 10 L_p / R_0$,

Magnetic flux surfaces are modeled by a set of concentric circular tori, with coordinates (r, θ, φ) that correspond to the minor radius, and the poloidal and toroidal angle, respectively. Assuming a monotonically increasing safety factor $q(r)$, the simulations cover the domain between $q = 2$ and $q = 3$ in the vicinity of a reference surface r_0 at the plasma edge. The values we have used for the parameters are $r_0 / \xi_{bal} = 500$, $R_0 / L_s = 1$, $\delta_c = 0.04$, and $\nu = \chi_{\parallel} = 2$. Throughout the analysis that follows, we analyze results obtained by DNS of RBM using different values for α , namely $\alpha = 0.01, 0.1, 0.2, 0.3$ (which correspond to $\beta = 0.02, 0.2, 0.4$ and 0.6 for $L_s = R_0 = 2.5$ m and $L_p = 0.4$ m), and we kept constant the rest of the parameters. Without losing generality, the analysis that follows is limited to data sets that belong to a given poloidal cross section.

Spectra of potential fluctuations

We have numerically calculated the temporal and the spatial spectra of the turbulent fluctuations for a chosen poloidal cross section. In Fig. (1), we present the averaged spectra for the potential fluctuations for four different values of the plasma $\alpha(\beta)$. The most striking difference

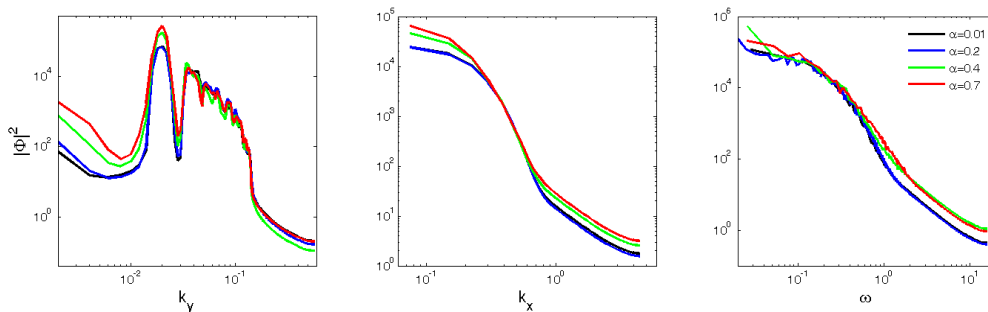


Figure 1: The averaged spectra of the potential fluctuation

between the different cases appears in the poloidal spectra, where it is evident that the increase of the plasma β value leads to a significant increase in the level of the potential fluctuations that correspond to large-scale poloidal flows.

Structure functions

One of the most efficient tools to study the scaling and self-similarity properties of turbulence is the structure function [2]. The structure function of order q for a field u in a fully developed turbulent state is defined as [2] $S_q(u; r) = \langle |u(x+r) - u(x)|^q \rangle$ where r represents a spatial separation and $\langle \dots \rangle$ is the ensemble average. The structure function $S_q(u; \tau)$ for the temporal scales is defined similarly. We have properly calculated the temporal and the spatial structure functions for the spatio-temporal fluctuations of the potential, pressure, magnetic field and vorticity, for various values of α . When the signal u is self-similar over some range of spatial (temporal)

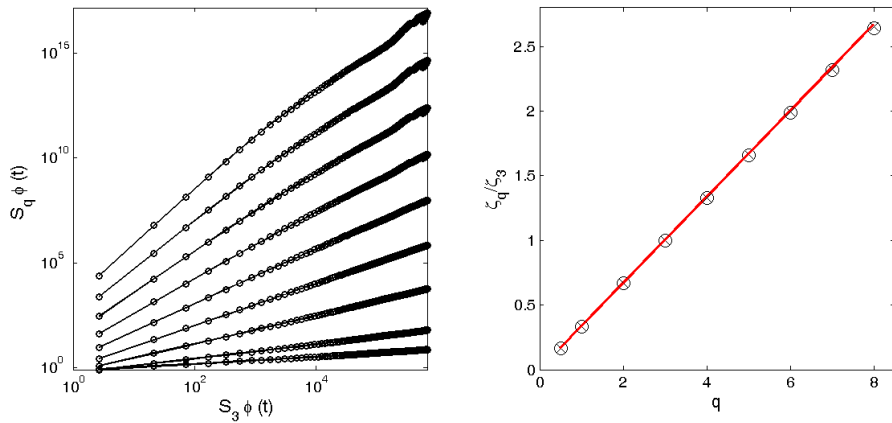


Figure 2: The structure functions S_q ($q=0.5,1,2..8$) versus S_3 for the potential fluctuations (Left panel). The ESS relative scaling exponents of the potential ϕ (Right panel).

scales, the structure function shows a power-law dependence on the scale-size, i.e. the q -th order structure function is expected to scale as $S_q \propto r^{\zeta_q}$, where ζ_q is the scaling exponent. This is theoretically the case in the inertial range of turbulence - the range of wave numbers where the dominant process is the energy transfer and not the energy injection or dissipation. However, in plasmas the different scaling regimes are not widely separated, which causes difficulties in the accurate estimate of the scaling exponents. The notion though of Extended-Self-Similarity (ESS) [3] provides a successful method that allows to extend the structure function analysis into the dissipation and the large scale region. With the ESS method, the dependence of the ratio ζ_q / ζ_3 on q can numerically be determined. For the case of RBM turbulence, we find a $q/3$ linear scaling for ζ_q / ζ_3 , which is in agreement with the structure function exponent for the potential as it is experimentally found in edge turbulence [4], indicating the mono-fractal scaling for the fluctuations.

The scaling between kurtosis and skewness

In a recent Letter, Labit et al. [5] found a unique parabolic relation, $K = (1.502 \pm 0.015)S^2 - (0.226 \pm 0.019)$, between the skewness S and kurtosis K of about ten thousand observed density fluctuation signals that are associated with drift-interchange turbulence over a broad range of experimental conditions. Using the results of our RBM turbulence simulations, we calculated the skewness and the kurtosis of $\sim 3 \times 10^5$ time series. The K - S scatter plot is shown in Fig. (3), and it has clear a parabolic character. A least-squares fit with a quadratic polynomial yields $K = (1.476 \pm 0.006)S^2 - (0.496 \pm 0.002)$. The uncertainty in the estimated coefficients is determined as the 95% confidence limits. Parabolic relations of this kind have been found in different physical systems. In a recent report [6], Krommes discussed the remarkable similarity in the K - S relation of Torpex density fluctuations with the K - S relation of sea-surface temperature fluctuations, and suggested a generalized non-linear Langevin theory that includes linear wave propagation for its explanation.

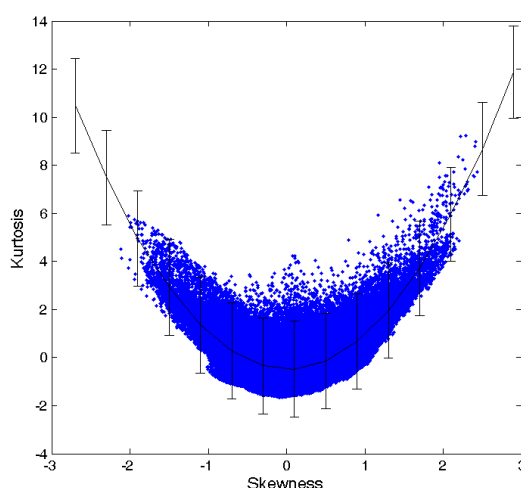


Figure 3: Kurtosis vs. skewness computed for $\sim 3 \times 10^5$ time-series and the fitted quadratic polynomial.

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