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A Kinetic Model for the Diffusion Coefficients of Magnetically Confined Plasmas in the Low-collisional Regime

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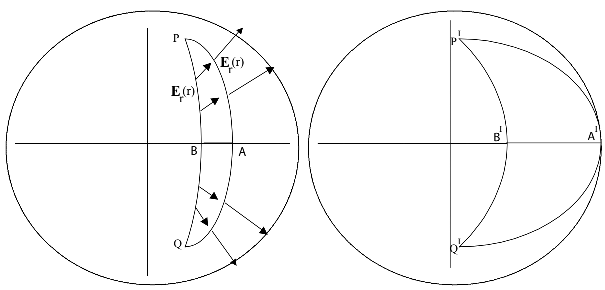
Introduction

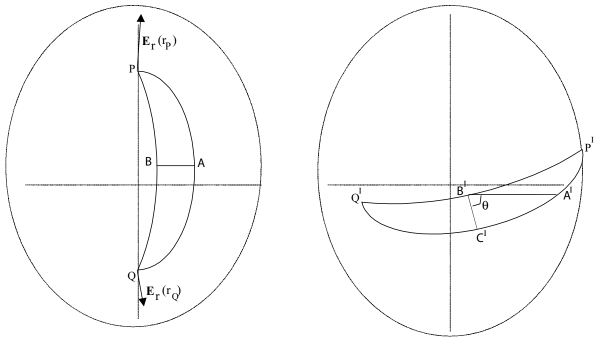
In order to interpret the expressions of the transport coefficients obtained by the TFT in terms of collisional mechanisms, a heuristic model has been introduced in ref. [1]. In this paper, the effects of the radial electric drift and the toroidal geometry of the tokamak, on the deformation of the guiding centre orbits of the trapped particles are studied in detail. Through a non-perturbative calculation, we have estimated the maximum displacement of the guiding centre orbit away from its initial magnetic surface. Explicit calculations for a JET-plasma are performed. In the core of the plasma, our result coincides with the expression obtained by the Berk, Galeev, Sagdeev [2] and Balescu [3] theory (BGSB). However, in the edge of the plasma we found that the toroidal geometry of the tokamak can significantly enhance the value of the electron (radial) diffusion coefficient and the intensity of the electronic bootstrap current. The values for the ion diffusion coefficient and the bootstrap current remain, however, substantially unaltered.

The Model

We have three possible mechanisms able to increase significantly the magnitude of the diffusion coefficient: **1)** The *orbit-squeezing* effect: the banana orbit deforms towards the direction



of the thermodynamic forces as the latter increase in magnitude. In this case, the diffusion coefficient can increase. **2)** The *banana orbit rotation* effect: the banana orbit tends to rotate in the direction of the thermodynamic forces as the latter increase in magnitude. This effect is expected to be visible when the *up-down* symmetry of the Tokamak plasma is broken.



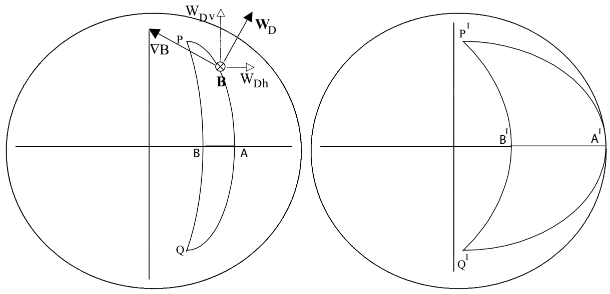
Let $\mathbf{E}_r(r)$ indicate the radial electric drift. In the local triad $(\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\zeta)$, the magnetic field in the standard hight aspect ratio, low β, circular tokamak equilibrium model, here referred to as the *standard model*, reads

$$\mathbf{B} = B_0 G(r)\ \mathbf{e}_\theta + \frac{B_0}{1+(r/R)\cos\theta}\ \mathbf{e}_\zeta \qquad \text{with} \quad G(r) \equiv \frac{r}{Rq(r)} \tag{1}$$

where (r, θ, ζ) are the toroidal coordinates. We easily find the expression for the grad-B drift velocity

$$W_D \sim \frac{B_0^2}{R}\Big[\Big(1-\frac{r}{R}\cos\theta - RG(r)G'(r)\cos\theta\Big)\ \mathbf{e}_v - RG(r)G'(r)\sin\theta\ \mathbf{e}_h + G(r)\cos\theta\,\mathbf{e}_\zeta\Big] + O(\eta^2) \tag{2}$$

where prime denotes the derivative with respect to the minor radius. We can then assist to the following effect **3)** The *geometry-induced banana orbit deformation* effect: when $RG(r)G'(r)$ is



not negligible and changes sign, the banana orbit of the particles can be significantly deformed. This constitutes a case where toroidicity can appreciably enhance the value of the radial diffusion coefficient.

The Expressions of the Transport Coefficients for a JET Plasma

In the low-collisional regime, we propose the following model for a plasma consisting of electrons and a single species of ions, in the presence of an external axisymmetric confining magnetic field

$$-\frac{\Omega_0}{\varepsilon}\int_0^r d\rho G(\rho) + \Big(1+\frac{r}{R}\cos\theta\Big)\Big(U(r,\theta,\mathscr{E},M) - \varepsilon\frac{c}{B_0}\frac{E_{r0}(r)}{G(r)}\Big) = const. \tag{3}$$

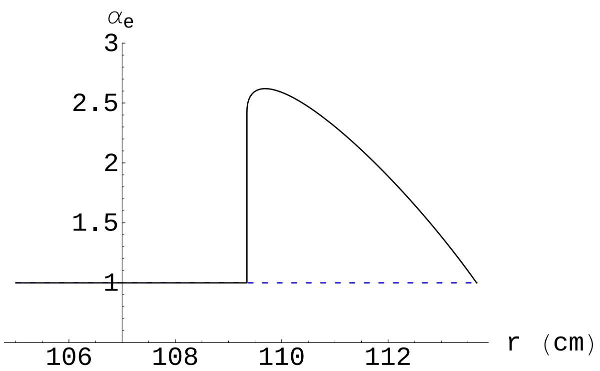
where Ω_0 is the Larmor frequency, ε denotes the drift parameter, $U(r,\theta,\mathscr{E},M)$ is the modified parallel velocity of the charged particle and c is the speed of light. Notice that, for one particle, Eq. (3) corresponds to the toroidal invariant L. Our model consists in *replacing in Eq. (3) the drift parameter, ε, with the inhomogeneous expression* $\varepsilon(r) = \rho_{L_\iota}(r)/(RG(r))$ $(\iota = e,i)$. Under this assumption, it is possible to show that the expression for the radial diffusion coefficient of charged particles in the banana regime reads

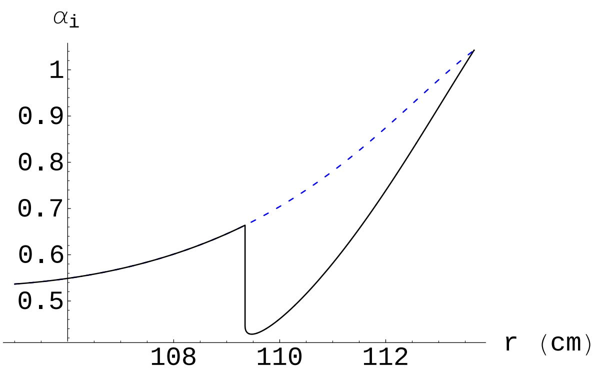
$$\tilde{D}_{\iota B} \approx \alpha_\iota^2 \sqrt{2}\pi^2 \varepsilon^{1/2} \frac{r^2}{\tau_\iota} \qquad (\iota = i,e)$$
$$\alpha_\iota = -\frac{G(r)\Omega_{L_\iota}}{U_\iota(r,0)}(r'-r) + \frac{1}{S_\iota(r')}\frac{G(r)}{G(r')} - \frac{c}{U_\iota(r,0)B_0}\frac{G(r)}{G(r')}\frac{S_\iota(r)}{S_\iota(r')}\left[\frac{E_r(r)}{G(r)} - \frac{E_r(r')}{G(r')}\right]$$
$$F(r') = F(r) \qquad F(r) \equiv \frac{G(r)}{\sqrt{T}(r)}\int_0^r G(\rho)d\rho \tag{4}$$

In the region where $r' \neq r$, we can then write

$$\frac{\alpha_e(r) - 1/S_e(r)}{\alpha_i(r) - 1/S_i(r)} \sim \sqrt{\frac{m_i}{m_e}} \tag{5}$$

The following figures report on the behaviour of electron and ion parameters α_e, α_i against the minor radius for a JET plasma. The dashed curve corresponds to the inverse of the squeezing orbit factor. Eq. (5) suggests that electrons diffuse much faster than ions. The maximum value





of α_e is of about 2.7 whereas the maximum value of α_i is of about 1.1. If we also take into account that, under these conditions, the electron collision time τ_e may reduce its initial value by a factor 3 whereas the ion collision time remains substantially unaltered [1], we can finally estimate the maximum values of the diffusion coefficients

$$\tilde{D}_{eB} \approx 21\, D_{eB}\Big|_{BGSB} \qquad \text{and} \qquad \tilde{D}_{iB} \approx D_{iB}\Big|_{BGSB} \tag{6}$$

In conclusion, according to our model, the values of the diffusion coefficients exceed the values computed by the BGSB theory by a factor α^2/τ_ι, which may be of order 10 for electrons and ranges from 0.5 to 1 for ions.

Conclusions

In the low-collisional regime, in particular in the banana regime, our analysis can be summarized as follows. As the thermodynamic forces increase their magnitudes, after some collisions between trapped particles, a radial gradient of the electric field is generated inside the plasma. As a consequence, the guiding centre orbits of the trapped particle tend to deform towards the direction of the thermodynamic forces or, when the *up-down* symmetry of the Tokamak plasma is broken, to rotate towards the direction of the thermodynamic forces. However, experience shows that in JET, far from the periphery, the intensity of the radial gradient of the electric drift remains quite weak and then its influence on the radial displacement of the guiding orbit of the trapped particles is negligible at first approximation. On the contrary, the toroidal geometry of the reactor may deform the guiding centre orbits enhancing significantly the value of the diffusion coefficient. We showed that the toroidal geometry may

a) enhance the value of the electron (radial) diffusion coefficient;

b) increase the intensity of the electron bootstrap current.

The effect of the radial electric field is very important close to the periphery of JET. Indeed, in proximity to the divertor, the radial gradient of the electric field may reach very large values and, in this case, the squeezing orbit effect and the banana rotation effect (for a dissymmetric up-down tokamak plasma) should be more visible. A model has been introduced to evaluate the maximum displacement of the guiding centre away from its initial magnetic surface. We showed that, due to the difference of masses, an asymmetry appears between the ion and electron diffusion coefficients: the geometry of the tokamak corrects the value of the electron parameter α_e with a factor, which may be of order 2.7, leaving almost unaltered the ion one. For a value of $\alpha_e \sim 2.7$ and taking into account that the electron collision time may reduce its value by a factor of 3, we can finally estimate that the maximum value of the electron diffusion coefficient may be 21 times greater than the value computed by the BGSB theory. The intensity of the electron bootstrap current may also increase by a factor 2.7.

References

[1] G. Sonnino, *Physicalia* 29, no 4, 161 (2008).

[2] H.L. Berk and A.A. Galeev, *Physics of Fluids*, **10**, 441 (1967). A.A. Galeev and R.Z. Sagdeev, *Sov. Phys. JETP*, **26**, 233 (1968)

[3] R. Balescu 1988, *Transport Processes in Plasmas. Vol 2. Neoclassical Transport*, *Elsevier Science Publishers B.V. North-Holland*