Zonal flow dynamics and GAM oscillations in tokamaks with resonant magnetic field perturbations

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Introduction
Resonant magnetic perturbations (RMP) are known to have a strong impact on turbulent dynamics and zonal flows in the tokamak plasma edge [1], and the Dynamic Ergodic Divertor experiment at TEXTOR [2] offers the opportunity to induce deliberately large islands by generation of magnetic perturbations with well defined structure. In these experiments it has been proved that small scale turbulence, blobby density transport and GAM oscillations (Geodesic Acoustic Modes) are strongly affected simultaneously by the RMP [3, 4, 5, 6]. The GAM oscillations are basically stable plasma oscillations where energy is transferred between the zonal flows and the poloidal $m = \pm 1$ density sidebands. The GAMs are always present in experimental signals detected in the edge plasma region, e.g. by means of reflectometry, and in fluid drift turbulence simulations based on a four field model [7] usually a certain signature in the frequency spectrum of turbulent fluctuations close to the GAM frequency can be found. In this work we focus on scenarios with pronounced GAM activity, which means for the model used that neoclassical ordering is valid and parallel ion flow and Alfvén dynamics of poloidal sidebands are small. A series of simulations performed with the 3D ATTEMPT code is presented for TEXTOR like parameters. It is shown that the effect of RMPs on GAMs is twofold: on the one hand GAM oscillations are enhanced by RMP, but on the other hand the resonances introduce a structure in the zonal flow by damping the GAMs at the resonant surfaces.

Fluid model and GAM oscillations
Basic features of turbulent plasma dynamics can be represented by a three-field model describing the non-linear evolution of the electric potential $\phi$, the density $n$ and the parallel magnetic potential $A$.

$$\frac{\partial n}{\partial t} = -v_E \cdot \nabla n - \mathcal{K}(\phi - n) + \nabla_{\|} J$$

(1)

$$\frac{\partial w}{\partial t} = -v_E \cdot \nabla w + \mathcal{K}(n) + \nabla_{\|} J$$

(2)

$$\hat{\beta} \frac{\partial A}{\partial t} + \hat{\mu} \frac{\partial J}{\partial t} = -\hat{\mu} v_E \cdot \nabla J + \nabla_{\|} (n - \phi) - \hat{C} J$$

(3)

These are the scaled equation of continuity, the quasineutrality condition and Ohm’s law, respectively, with vorticity $w$ and current density $J$ defined by $w=-\nabla_{\perp}^{2} \phi$ and $J=-(\nabla_{\perp}^{2} A)$. The definitions of the operators $v_E \cdot \nabla$, $\nabla_{\|}$, $\nabla_{\perp}^{2}$ and $K$ and the parameters $\hat{\beta}$, $\hat{\mu}$, $\hat{\epsilon}$ and $\hat{C}$, can be found in [7].

Assuming now that dissipative effects, parallel ion flow and Alfvénic dynamics of sidebands are small and that neoclassical ordering is allowed (skip terms of order $\rho_s^2/L_{\perp}^2$, where $\rho_s = c_s/\omega_{ci}$ is the drift scale with $\omega_{ci} = eB/m_i$, and $L_{\perp}$ is the gradient scale length of the density),
The plasma dynamics can be reduced to

$$\frac{\partial^2 \langle n \sin \theta \rangle}{\partial t^2} = -\omega_{\text{GAM}}^2 \langle n \sin \theta \rangle \quad ; \quad \frac{\partial^2 V_{00}}{\partial t^2} = -\omega_{\text{GAM}}^2 V_{00}$$

(4)

where $\langle \cdots \rangle$ denotes a flux surface average. Thus a simple oscillation results with frequency $\omega_{\text{GAM}} = \sqrt{2} c_s / R_0$ in physical units. These are the GAM oscillations originating from toroidal curvature (compressibility of drift velocities), which couples the zonal flow $V_{00} = \partial \langle \phi \rangle / \partial x$ and the density side band $\langle n \sin \theta \rangle$. Experimental observations demonstrate that the presence of a resonant magnetic perturbation field (RMP) changes the GAM signature significantly [6]. On the one hand the switch on of RMP first amplifies the GAM oscillations signature. With increasing RMP the GAM oscillations disappear. To study a basic interplay between the GAM oscillations, driven by coupling between density and electric potential due to the curvature terms $\mathcal{K}(\phi)$ and $\mathcal{K}(n)$ in Eqs.1 and 2, and the RMP, appearing in the parallel derivatives of the model equations, we consider a simplified model by neglecting the $E \times B$-advection and additional subscale dissipation. The implementation of the RMP is done as described in [7], i.e. splitting the parallel derivative into three parts

$$\nabla_\parallel f = \frac{B}{B} \cdot \nabla f = \frac{B_0}{B} \cdot \nabla f + \frac{B_s}{B} \cdot \nabla f + \frac{\tilde{B}}{B} \cdot \nabla f$$

(5)

The first term on the rhs represents the parallel derivative along the unperturbed equilibrium field, the second the parallel derivative along the static RMP, and the last one stems from the magnetic flutter, i.e. the magnetic field due to plasma response currents.

**Figure 1:** Poincaré-plots of the perturbed vacuum magnetic fields. Left: RMP 1, right: RMP 2.

**Numerical results**

The following set of parameters have been used for the computational setup: electron temperature $T_e=100$ eV, particle density $n_0=1.0 \times 10^{19}$ m$^{-3}$, ion mass $m_i=2 m_p$, minor radius $a=0.5$ m, major radius $R_0 = 1.75$ m, density gradient length $L_\perp=20$ cm, magnetic shear $\hat{s}=1$. The computational domain covers the radial region $2 \leq q \leq 4$ and to focus on the large scale perturbations, the computational grid was chosen with $N_x \times N_y \times N_z = 8 \times 128 \times 16$, covering the complete toroidal annulus. Numerical simulations have been performed with the parameters described above varying strength of magnetic perturbation fields. These were chosen to be a sum of three modes $(5/2, 6/2$ and $7/2)$ corresponding to a magnetic potential

$$A_s = - \sum_{m=5,7} (-1)^m A_0 \exp [m (r - a) / a] \cos (m \theta - 2 \varphi)$$

(6)
where $A_0=-10^{-6}$ Tm for the lower perturbation field RMP 1 and $A_0=-20^{-6}$ Tm for the higher perturbation strength. The perturbed vacuum magnetic fields are illustrated by Poincaré-plots shown in Fig.1. The simulations have been started with a random density perturbation and have been followed over a period of 2000 $L_\perp/c_s$. The results in Fig.2, left column, show a clear pronounced peak at the GAM frequency $\omega_{\text{GAM}}$ in the frequency spectrum of the zonal flow $V_{00}$ over almost the entire radial domain $2 \leq q \leq 4$.

![Figure 2: Spectra of zonal flow and variances of zonal flow, density sideband and Alfvénic trigger for the cases without RMP (top), with RMP1 (mid) and RMP2 (bottom)](image)

The same signature can be observed in the frequency spectra of the density side band $\langle n \sin \theta \rangle$. The corresponding plots are very similar - except an additional strong contribution at lower frequencies up to $\omega_{\text{GAM}}/3$ - and not shown here. For the case of RMP fields it can be seen, that GAM oscillations are present, but they do not occur uniformly in the computational domain. Gaps with lower amplitude appear at the resonant surfaces. The variances of the time traces of the zonal flows in Fig.2, 2nd column, are obviously strongly correlated to the GAM oscillations, whereas the variances of the density side band (3rd column) do not reflect the GAM pattern of the frequency spectra so clearly. It is apparant that the resonant surfaces at $q=5/2, 3, 7/2$ form some kind of border for the GAM activity and that the outer region is strongly affected by damping of the GAMs with increasing perturbation field strength. Also shown in Fig.2 are the variances of the Alfvénic trigger term $\langle J \partial A / \partial y \rangle$. This term appears in
an extension of Eq.4, if the same approximations are used as described above, but the Alfvénic coupling of vorticity and current density are kept, i.e.

\[
\frac{\partial}{\partial t} \langle n \sin \theta \rangle = \frac{\omega_B}{2} \frac{\partial \langle \phi \rangle}{\partial x}, \quad \frac{\partial}{\partial t} \frac{\partial \langle \phi \rangle}{\partial x} = -\omega_B \langle n \sin \theta \rangle + \hat{\beta} \left\langle J \frac{\partial A}{\partial y} \right\rangle
\]  

(7)

where the total magnetic potential \( A \) contains the RMP field and the selfconsistent magnetic flutter. The last term modifies the GAM dynamics by a triggering due to currents flowing in the plasma as a response to the magnetic perturbation. Analysis of these modified equations for the GAM oscillations in a local approximation shows that - dependent on amplitude and frequency of the Alfvénic trigger - the GAMs can be amplified or damped, due to a redistribution of energy between the GAM oscillations and Alfvén waves. This simplified model also reproduces the different character of response of zonal flow and density sideband oscillations, i.e. the density oscillations show a much stronger component in the frequency of the Alfvénic trigger (the frequency band up to \( \omega_{\text{GAM}}/3 \) mentioned above) than the zonal flows which are much more contolled by GAM oscillations.

Summary and comments

The simulated spectra and radial profiles of the zonal flow \( \langle \partial \phi / \partial r \rangle \) and density side band \( \langle n \sin \theta \rangle \) show a clear signature of GAM oscillations. This signature is due to the computational setup, focussing on large scale plasma perturbations and avoiding strong GAM damping caused by small scale dissipation. By switching on the RMP fields the GAM oscillations change their character and it is observed that on the one hand the GAM amplitude is increased. But on the other hand the GAM amplitude of the zonal flows shows a clear structure in radial direction, strongly correlated with the location of the resonant surfaces, where induced plasma currents interact with the GAMs in such a way that oscillations are less pronounced. With increasing perturbation field the GAM amplitude is reduced again in the regions where the RMP is strongest. This result fits well into the experimentally observed trend found by reflectometry [6]. The correlation between RMP and GAMs is most prominent in the structure of the zonal flows, whereas the density side bands exhibit oscillations in the entire radial domain, but at frequencies different from \( \omega_{\text{GAM}} \) and linked to the oscillations in currents induced by the magnetic perturbation.

References