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Comprehensive analytic study of a magnetised plasma-wall transition

F. Bint-e-Munir[1,2], S. Kuhn[1], D.D. Tskhakaya (sr)[1,3]

[1] *Association EURATOM-ÖAW, Institute for Theoretical Physics, University of Innsbruck, Innsbruck, Austria*

[2] *Department of Physics, Government College University, Lahore, Pakistan*

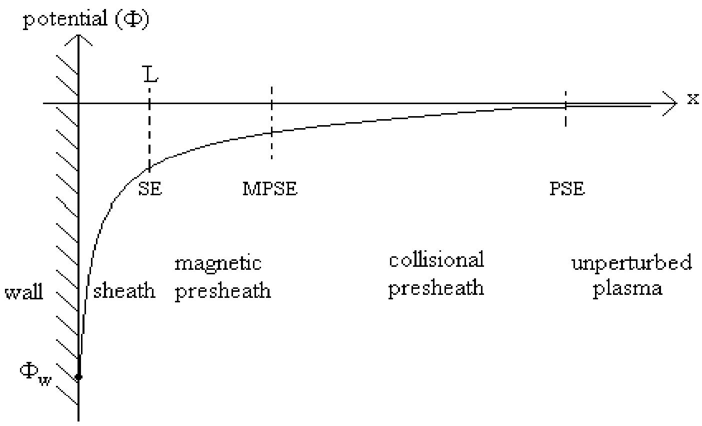
[3] *Institute of Physics, Georgian Academy of Sciences, Tbilisi, Georgia.*

Abstract

The matching of the collisional presheath (CPS) with the magnetic presheath (MPS), and of the MPS with the Debye sheath (DS) in the plasma-wall transition (PWT) layer is investigated. Evidence is found that in the hydrodynamic approximation the intermediate scales and the equations bridging the neighbouring PWT sublayers should have similar structures in both the non-magnetised and magnetised cases.

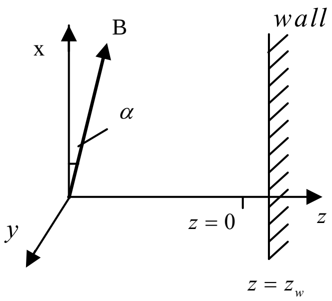
1. Introduction

In the presence of an oblique magnetic field, the plasma-wall transition (PWT) layer can be divided into three regions, namely: Debye sheath (DS), magnetic presheath (MPS) and collisional presheath (CPS) [1], with characteristic length scales λ_D (Debye length), ρ (ion gyroradius), and λ (relevant collisional length), respectively. For later use we define the ratios $\varepsilon_{Dm} \equiv \lambda_D / \rho$ and $\varepsilon_{mc} \equiv \rho / \lambda$. In the classical PWT problem without magnetic field, monotonicity of the electric potential requires fulfilment of the Bohm criterion at the CPS-DS interface. Chodura [2] was the first to investigate the collisionless MPS in the case of an oblique magnetic field. In the "asymptotic three-scale limit", $\lambda_D << \rho << \lambda$, which implies $\varepsilon_{Dm} \to 0$ and $\varepsilon_{mc} \to 0$, the DS is collisionless and non-neutral, the MPS is collisionless and quasi-neutral ($n_i = n_e$) [2], and the CPS is collisional and quasi-neutral.



The DS and MPS regions are separated by the "DS entrance", which is characterized by the marginal Bohm condition, $v_z = c_s = \left[k\left(T_e + \gamma T_i\right) / m_i \right]^{1/2}$ (where c_s is the ion-sound velocity, k is the Boltzmann constant, and γ is the local polytropic coefficient [3]), corresponding to a field singularity from the MPS side. The MPS and CPS regions are separated by a boundary surface defined as the "MPS entrance". Below we show that, quite in analogy with the unmagnetised PWT layer, the MPS entrance can be defined as a point were the electric field has a singularity from the CPS side. The condition imposed from the MPS side is similar to the Bohm condition, but the ion velocity must be directed along the magnetic field line, $v_{\|} = c_s$. This is known as the Bohm-Chodura condition. The dominant effect of the MPS is to deflect the ion orbits, so that the velocity component v_z, can fulfil the Bohm criterion at the DS entrance [1,4].

2. Model and basic equations



The problem is 1D, with the z axis perpendicular to the wall surface, placed at $z = 0$. The plasma occupies the region $z < 0$, and the electric potential $\Phi(z)$ decreases towards the wall monotonically. The magnetic field is assumed to be uniform and embedded in the xz plane, making a small angle α with the wall. The thermal motion of the ions is neglected $(T_i \to 0)$, and the electrons follow the Boltzmann distribution, $n_e = n_0 \exp(e\Phi/kT_e)$. The ions are assumed to undergo electron-impact ionisation (accounted for by the source term $\nu_i n_e$, where ν_i is the ionization frequency), and charge-exchange collisions (frequency ν_{cx}) with the neutrals. In the dimensionless variables

$$(v_z/c_s) \Rightarrow u, \quad (-e\Phi/kT_e) \Rightarrow \varphi, \quad (n_i/n_0) \Rightarrow n \text{ and } (c_s/\omega_c) \Rightarrow \rho \tag{2.1}$$

(where ω_c is the ion cyclotron frequency), the ion continuity, ion momentum and Poisson equations then read

$$(\partial nu/\partial z) = \lambda_i^{-1} \exp(-\varphi) \tag{2.2}$$

$$\{D^2 + \rho^{-2}\} Du = \{D^2 + Sin^2\alpha \cdot \rho^{-2}\}(\partial\varphi/\partial z) \tag{2.3}$$

$$\lambda_D^2 (\partial^2 \varphi/\partial z^2) = n - exp(-\varphi). \tag{2.4}$$

where $\lambda_i = c_s/\nu_i$ is the "ionisation length" and the three ion velocity component equations have been reduced to the the single Eq. (2.3), with

$$D = u(\partial/\partial z) + \lambda^{-1}, \quad \lambda^{-1} = \lambda_{cx}^{-1} + \lambda_i^{-1} e^{-\varphi}/n, \quad \lambda_{cx} = c_s/\nu_{cx}, \tag{2.5}$$

3. Matching of the MPS and the DS

Due to the asymptotic assumptions $\varepsilon_{Dm} \to 0$ and $\varepsilon_{mc} \to 0$, both the MPS and the DS can be considered collisionless and the system (2.2) – (2.4) can be reduced to the following single equation describing both regions:

$$\left\{\frac{1-u^2}{u}\right\}\left\{\frac{du}{dz}\right\}^2 = \frac{1}{\rho^2}\, f(u) + \lambda_D^4 \left\{u^2 \left(\frac{d\Phi}{dz}\right)^2 + \frac{1}{\rho^2}\, Sin^2\alpha \cdot \Phi\right\}, \tag{3.1}$$

where $\Phi = \frac{1}{2}\left(\frac{d\varphi}{dz}\right)^2 + \frac{d^2\varphi}{dz^2}$, and (3.2)

$$f(u) = Cos^2\alpha \left\{u_{\|}^2 + 2ln(u/u_{\|} Sin\alpha) - u^2\right\} - \left\{u_{\|} + u_{\|}^{-1} - Sin\,\alpha(u + u^{-1})\right\}^2. \tag{3.3}$$

with $u_{\|}$ the velocity component along the magnetic field. Let us first consider the MPS, where due to $\varepsilon_{Dm} \to 0$ the quasineutrality condition is fulfilled as well. From (3.1) it can then be shown that for $u \to 1$ (marginal Bohm condition) the electric field runs into the well-known sheath singularity, indicating the breakdown of quasi-neutrality and, hence, the DS entrance [5]. Close to the MPS-DS interface from the DS side, when $u - 1 + \delta u$ and $\varphi - \varphi_s \sim \delta u \ll 1$ (φ_s is the potential at the DS entrance), the Bohm criterion is fulfilled in the marginal form. Introducing the variable $\zeta = z/l$ by choosing the characteristic scale l we can make the contribution of the magnetic field having the same order as that of the charge separation. Close to the sheath edge for the renormalized potential $w = s \cdot (\varphi - \varphi_s)$ from (3.1) we obtain

$$\left(\frac{dw^2}{d\zeta}\right)^2 = f(1) + \left(\frac{d^3 w}{d\zeta^3}\right)^2. \tag{3.4}$$

The intermediate scale is found as $l = (\rho \cdot \lambda_D^4)^{1/5}$ and $s = (\rho/\lambda_D)^{2/5}$ Proceeding from (3.4) we suppose that for matching one can use the equation

$$\frac{d^2 w}{dz^2} = w^2 + \sqrt{f(1)}\,(\zeta - \zeta_s), \tag{3.5}$$

Under appropriate conditions (3.5) also correctly describes each region separately.

4. Matching of the collisional and magnetic presheaths

Below we show the way to bridge these distinguished regions in the smooth form. The CPS and the MPS both are quasineutral and therefore $n = \exp(\ \varphi)$. Introducing $\eta = z/\lambda$ as a scale variable, from (2.2) – (2.4) we find

$$\{Sin^2\alpha - u^2\}\frac{du}{d\eta} = u^2 + \frac{\lambda}{\lambda_i} Sin^2\alpha. \tag{4.1}$$

Obviously from the CPS side at $u = \sin\alpha$ (or at $u_{\|} = 1$), du/dz and the electric field have the singularity. The corresponding spatial point we define as the MPS entrance. From (3.1), (3.3) it follows that close to the CPS – MPS interface from the MPS side, when $u = u_{\|}\sin\alpha + w\,(w \ll 1)$ and the Bohm – Chodura criterion is fulfilled in the marginal form we have

$$\left(\frac{dw}{d\zeta}\right)^2 = \left(1 - \frac{1}{u_{\|}^2}\right)\frac{w^2}{1 - u_{\|}^2 Sin\alpha}. \tag{4.2}$$

As we see the behaviour of plasma characteristics at the different sides of the interface is quate similar to that observed at MPS - DS matching. Therefore one can follow the procedure used there. Introducing the new variable $\chi = z/l$ by choosing the characteristic length l from (2.2)-(2.4) we find close to the CPS-MPS interface the equation, which describes the smooth CPS-MPS transition

$$\frac{d^2\overline{w}}{d\chi^2} = \frac{1}{Sin\alpha \cdot Cos^2\alpha}\overline{w}^2 + \left(1 + \frac{\lambda}{\lambda_i}\right)\frac{1}{Cos^2\alpha}(\chi - \chi_m), \qquad (4.3)$$

Where $\overline{w} = \beta \cdot w$ and χ_m is the point of the MPS entrance. For the intermediate scale we find $l = (\rho^4 \cdot \lambda)^{1/5}$ and $\beta = (\lambda/\rho)^{2/5}$.

It is interesting to note that , as given in [4], the Painleve equation represents the Posson's equation, while in our analysis of CPS – MPS transition, we have used quasineutrality condition (instead of the Posson's equation) from the very beginning of the analysis.

For the intermediate scale in the classical analysis of the transition without magnetic field it was found $l_r = (\lambda_D^4 \cdot \lambda)^{1/5}$ [4]. It looks like that in the hydrodynamic approximation the intermediate scale has the generasl form $= \lambda_1^{4/5} \cdot \lambda_2^{1/5}$, where λ_1 is smaller and λ_2 is the larger characteristic scale-lengths of matching regions respectively.

Apparently the both transitions CPS-MPS and MPS-DS are described by the Painleve equation (see (3.5) and (4.3)) quite similar to the case of the unmagnetized PWT layer [1]. Hence one can say that the Painleve equation plays somewhat an universal role in the matching procedure in the hydrodynamic approximation.

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