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# TEST PARTICLE TRANSPORT IN TURBULENT ELECTROMAGNETIC FIELDS

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The transport of charged particles in a magnetohydrodynamic (MHD) plasma is influenced by two main effects. First we have the interaction between the particles and the plasma constituents trough collisions and second we have the influence of the electromagnetic field on the particles by means of the Lorentz force. Assuming the transport is dominated by the particles interaction with the electromagnetic field, we can neglect their collisions with the background plasma. In this case, the geometry of the electromagnetic field and the type of structures present will have a crucial role in the behavior of the particle transport. We are in the case of anomalous transport, defined as the process in which the transport coefficients depend on variables unrelated to collisions, that characterize the degree of disorder in the medium. For a turbulent state of the plasma, the magnetic field that acts on the particles can be considered as the superposition of a constant external field and of a fluctuating component. Since the fluctuating part of the magnetic field is generated by the turbulent movement of the plasma, we will refer to this as turbulent magnetic field. Because of the interaction with the plasma flow, the turbulent magnetic field will generate coherent structures from any random initial condition given a sufficient long time. As was shown in [1], the proper coherent field structures will have a different effect on the particle acceleration compared with the structures obtained by random phase correlation. Because of this, the best way of generating the proper electromagnetic field structures is represented by solving the MHD equations. The incompressible MHD equations solved are:

$$\partial_t \mathbf{u} = -\nabla p - \mathbf{u}\cdot\nabla\mathbf{u} + \mathbf{b}\cdot\nabla\mathbf{b} + \nu\nabla^2\mathbf{u} + \mathbf{B}_0\cdot\nabla\mathbf{b} + \mathbf{f} \quad (1)$$

$$\partial_t \mathbf{b} = -\mathbf{u}\cdot\nabla\mathbf{b} + \mathbf{b}\cdot\nabla\mathbf{u} + \eta\nabla^2\mathbf{b} + \mathbf{B}_0\cdot\nabla\mathbf{u} \quad (2)$$

where $\mathbf{u} = \mathbf{u}(\mathbf{x},t)$ is the zero mean velocity field of the fluid. The magnetic field $\mathbf{B}$ is split into two parts, an external magnetic field $\mathbf{B}_0$ considered constant and stationary and a zero mean turbulent part denoted by $\mathbf{b} = \mathbf{b}(\mathbf{x},t)$. In the present study, the magnetic field is expressed in Alfvèn velocity units ($\mathbf{B} \to \mathbf{B}/\sqrt{\rho\mu_0}$, with $\rho$ the fluid mass density and $\mu_0$ the magnetic permeability). The fluid viscosity $\nu$ and the magnetic diffusivity $\eta$ are taken to be equal, so that the magnetic

Prandtl number ($\mathrm{Pr} = \nu/\eta$) is unity. When desired, we force the velocity equation by means of an known, external force $\mathbf{f} = \mathbf{f}(\mathbf{x},t)$. The equations (1) and (2) are joined by the incompressibility condition for the fluid ($\nabla \cdot \mathbf{u} = 0$) and the magnetic field zero-divergence ($\nabla \cdot \mathbf{b}$=0). Because of the incompressible condition, the total pressure (hydrodynamic + magnetic) $p = p(\mathbf{x},t)$, is not an independent variable and depends on $\mathbf{u}$ and $\mathbf{b}$.

In the present study we will concentrate on the particle transport due to the magnetic field. The nonrelativistic equations of motion for a charged particles in the presence of the magnetic field are:

$$\frac{d\mathbf{r}(t)}{dt} = \mathbf{v}(t) \qquad ; \qquad \frac{d\mathbf{v}(t)}{dt} = \alpha\left[\mathbf{v}(t) \times \mathbf{b}(\mathbf{r},t) + \mathbf{v}(t) \times \mathbf{B}_0\right] \tag{3}$$

where the $\mathbf{r} = \mathbf{r}(t)$ represents the position of the particle and $\mathbf{v} = \mathbf{v}(t)$ its velocity. The parameter $\alpha = \frac{q}{m}\sqrt{\rho\mu_0}$ represents the coupling between the particle (of charge $q$ and mass $m$) and the fields generated by the plasma with density $\rho$. By adimensiolizing the particle motion equation (3), we find the standard interpretation of the coupling parameter as the ratio of the MHD time scale to the characteristic time scale of the particle, given by the inverse of the Larmor frequency. Selecting the initial veocity of the particle to be equal to the refrence Alfvèn velocity of the magnetic field ($v_A = \left\langle B^2 \right\rangle^{1/2}$), we obtaine in our case the interpretation of $\alpha$ as the invers of the particle's Larmor radius. This allows us to directly couple the test particle to a turbulent scale and compare the influence of the perceived magnetic field disorder on the particle transport. We will look at the time evolution of the mean squared displacement (MSD), defined for a direction $i$ as: $\left\langle \delta r_i(t)^2 \right\rangle = \left\langle [r_i(t) - r_i(0)]^2 \right\rangle$. The angle brackets denote averaging over the test particle ansamble. By looking at the anomalous scaling law $\left\langle \delta r(t)^2 \right\rangle = At^{\mu}$ and extracting $\mu$ we find the diffusion regimes as indicated by Figure 1. We would like to emphasize that even for the case of normal diffusion ($\mu = 1$) we still talk about anomalous transport.

Numerically, employing a pseudo-spectral code, we solve the MHD equations (1)-(2) in a box of length $2\pi$ with periodic boundary conditions. Starting from random initial conditions prescribed by means of a spectrum, we use the force $\mathbf{f}$ to inject a constant level of energy in the velocity equation. We evolve the MHD equations until we reach a statistically stationary state, for which the energy injected by the force is equal to the energy lost do to the dissipative effects. This will ensure that the phases are properly corre-

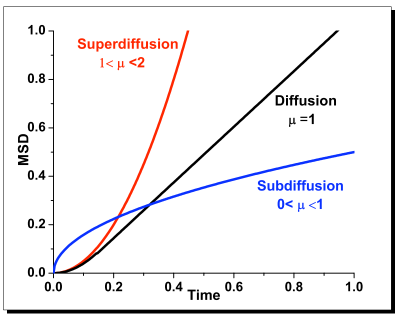


Figure 1: Schematic representation of diffusion regimes as defined by the exponent $\mu$.

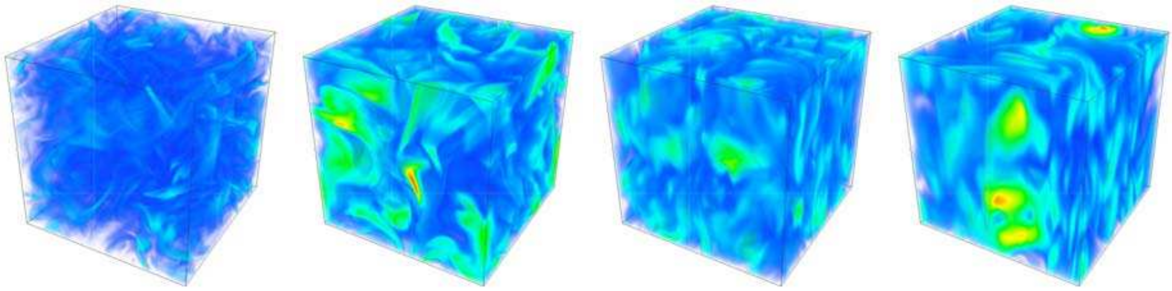

Figure 2: Turbulent magnetic energy for different values of the external magnetic field $\mathbf{B}_0$. Left to right cases $M = \{0, 1, 10, 100\}$. From blue to red we have an increase in energy level.

lated and the coherent field structures are present. All trough our simulations we check that the smallest turbulent scale (the Kolmogorov length $l_K$) is properly solve by our simulation. Using 128 modes in each direction we reach a value for the Taylor micro-scale Reynolds number ($R_\lambda$) of 120. Once the stationary regime has been achieved, we impose an external magnetic field that has an energy multiple ($M = \{0, 1, 10, 100\}$) of the turbulent magnetic energy ($B_0^2 = M\left\langle b^2 \right\rangle$) and wait for a new stationary regime to be achieved, Figure 2. For the four MHD cases, we inject a number of 10.000 mono-energetic test particles with isotropic distributed velocity direction. The particles are distributed randomly in the simulation box. We evolve the particles by a forth order Runge-Kutta method with an adaptive time step, which differs from the MHD time step. At each particle time step, to find the value of the fields acting on a particle at position $\mathbf{r}$ we use a linear or spline interpolation (depending on the problem solved) of the surrounding grid nodes values. Taking into account the level of turbulence, we chose for the coupling parameter of our test particles the values $\alpha = \{5, 10, 50, 100, 200\}$.

To the best knowledge of the authors, no study of test particle transport has taken into account the evolution of the MHD fields. The fields are found as solutions of the MHD equations (proper structures exist) but then they are frozen in regard for the particle evolution ([1], [2]). By allowing the MHD field to evolve we destroy the small to intermediate scale structures of the field. This aspect is of great importance since trapped particles will release themselves much faster changing the behavior of the transport form subdiffusion to normal diffusion, Figure 3.

| Frozen Field ($\mathbf{B}_0 = 0$) | | | | | Evolving Field ($\mathbf{B}_0 = 0$) | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 5 | 10 | 50 | 100 | $\alpha$ | 5 | 10 | 50 | 100 |
| $\mu$ | 1.03 | 0.99 | 0.87 | 0.85 | $\mu$ | 1.01 | 1.00 | 1.02 | 1.07 |
| $A$ | 1.02 | 1.15 | 1.16 | 1.18 | $A$ | 1.09 | 1.36 | 1.80 | 1.94 |

Table 1: Anomalous exponents and coefficients for the frozen fields compared to time evolving fields in the case of isotropic turbulence ($\mathbf{B}_0 = 0$).

A large value for $\alpha$ means a small Larmor radius for the test particle. The smaller the Larmor radius, the better the particle will follow the field line of the magnetic field. For this reason, selecting a Larmor radius smaller then the smallest turbulent scale for the particle (around $\alpha = 200$) will give similar results as solving field line equations. In this case the particle will folow a field line and be trapped by the chaotic structure of the field. In this situation we will find subdiffusive behavior even for the evolving isotropic turbulence ($\mu = 0.45$). If the test particle Larmor radius is larger then the smallest turbulent scale, the particle will orbit around different magnetic field lines. By the nature of turbulence, these field lines will diverge and the particle will be relesed. The successive trapping and release of particles will give in the end a normal diffusion regime.

For anisotropic turbulence, an external magnetic field will organize the turbulence, creating a preferred direction along its direction. Structures will elongate in the direction of the external magnetic field, large structures will appear and the flow will tend towards two-dimensional turbulence, Figure 3. This in turn will increase the trapping of particles in the perpendicular direction while suppressing it in the parallel direction. Because of this structures, the transport will be tend to be subdiffusive in the direction perpendicular to the $\mathbf{B}_0$ and superdifusive in the parallel direction, Figure 4. For time evolving fields we see the tendency of the particles to reach normal diffusion.

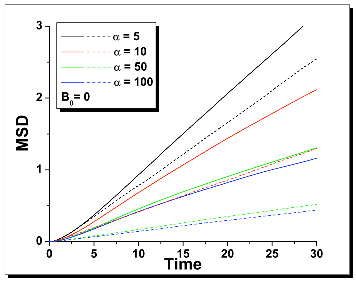

Figure 3: MSD for the frozen fields (full line) compared to time evolving simulation (dash line) in the case of isotropic turbulence ($\mathbf{B}_0 = 0$).

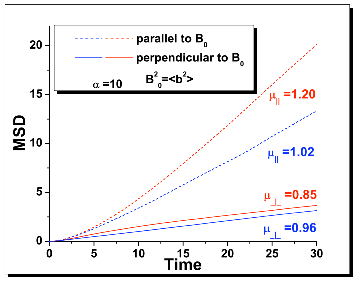

Figure 4: MSD for the frozen fields (red) compared to time evolving simulation (blue) in the case of anisotropic isotropic turbulence ($B_0^2 = \langle b^2 \rangle$).

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