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# Vertical Transport and Characteristic Modes of Astrophysical Plasma Disks *

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The plasma disk structures [1,2] that can surround compact objects such as black holes and that are dominated by the gravity of these objects can sustain a spectrum of typical collective modes. These are being identified and are found to be suitable for the interpretation of a variety of relevant experimental observations such as the winds emanating from disk structures around black holes or the so-called Quasi-Periodic Oscillations (QPO's) of X-ray emission from this kind of object. The driving factors of these modes are the differential rotation (radial gradient of the rotation frequency) and the vertical gradients of the plasma density and temperature.

The simplest configuration from which the modes that we shall analyze can emerge is a thin currentless disk that is threaded by a relatively weak vertical magnetic field $B_z$ and where the only component of the plasma flow velocity is toroidal. In particular, we assume that the central plasma pressure $p_0$ exceeds the magnetic pressure $B^2/8\pi$. The particle density profile, assuming an up-down symmetry, is represented by $n \simeq n_0\left(1-z^2/H_0^2\right)$, near the equatorial plane at the reference distance $R = R_0$ from the axis of symmetry, with $H_0^2 \ll R_0^2$. Different vertical temperature profiles, corresponding to different heating processes, are represented near the equatorial plane by related values of the parameter $\eta_T = -\left(d\ln T/dz^2\right)H_0^2$ where $2T \equiv p/n = T_e + T_i$. Considering Newtonian regimes, the radial equilibrium equation, to lowest order in the ratio $H_0^2/R_0^2$, reduces to $\Omega^2\left(R_0\right) = GM_*/R_0^3 \equiv \Omega_k^2\left(R_0\right)$ where $\Omega(R)$ is the rotation frequency, $\Omega_k\left(R_0\right)$ is the Keplerian frequency, $M_*$ is the mass of the central object and $\mathrm{v}_\phi = \Omega R$ is the toroidal velocity. The relevant vertical equilibrium equation is $0 = -\partial p/\partial z - z\Omega_k^2\rho$, where $\rho \equiv m_i n$ is the mass density, and we consider a variety of temperature profiles including the case $\eta_T = 0$.

**Axisymmetric Modes**

Normal mode perturbations, from the indicated initial state, are represented by $\hat{\mathrm{v}}_\phi = \tilde{\tilde{\mathrm{v}}}_\phi\left(R-R_0, z\right)\exp\left(\gamma_0 t - i\omega_0 t + i m_\phi \phi\right)$ where $\gamma_0$ is the mode growth rate, $\omega_0$, its frequency of oscillation and $m_\phi$, the toroidal mode number. The basic linearized equations for these perturbations include

$$\hat{\mathbf{E}} + \left(\hat{\mathbf{v}}\times\mathbf{B} + \mathbf{v}\times\hat{\mathbf{B}}\right)/c = 0\,, \tag{1}$$

considering that $-\partial\hat{\mathbf{B}}/\partial t = c\nabla\times\hat{\mathbf{E}}$, and the total momentum conservation equation

$$\mathbf{A}_m \equiv \rho\left(\frac{\partial}{\partial t}\hat{\mathbf{v}} + \hat{\mathbf{v}}\cdot\nabla\mathbf{v} + \mathbf{v}\cdot\nabla\hat{\mathbf{v}}\right) + \nabla\left(\hat{p} + \frac{\hat{\mathbf{B}}\cdot\mathbf{B}}{4\pi}\right) - \frac{1}{4\pi}\mathbf{B}\cdot\nabla\hat{\mathbf{B}} + z\hat{\rho}\Omega_k^2\mathbf{e}_z = 0. \qquad (2)$$

Here, the initial magnetic field $B_z$ is considered to be varying over scale distances of the order of $R_0$. It is reasonable to assume that the collisional mean free path is short relative to $H_0$ the height of the disk and to the mode radial wavelengths. Thus, the thermal conductivity can be neglected and the adiabatic equation of state can be adopted. The perturbed linearized form of this is $\left[\gamma_0 - i\omega_0 + im_\phi\Omega(R)\right]\hat{p} + \hat{\mathrm{v}}_z(\partial p/\partial z) + \Gamma p\nabla\cdot\hat{\mathbf{v}} = 0$ where $\Gamma = 5/3$. In the case where $m_\phi \neq 0$, we choose to consider modes that co-rotate with the plasma at $R = R_0$. Therefore we take $\omega_0 = m_\phi\Omega(R_0) + \delta\omega_0$, where $\delta\omega_0 < \left|m_\phi\Omega(R_0)\right|$ and define $\gamma_t \equiv \gamma_0 + i\Omega'(R - R_0)$. Where $\Omega' \equiv d\Omega/dR$. Then Eq. (1) leads to $\hat{\mathrm{v}}_\phi = -\Omega'\hat{\xi}_R + \gamma_t\hat{\xi}_\phi$, $\hat{B}_R = B_z\,\partial/\partial z\,\hat{\xi}_R$, $\hat{B}_\phi = B_z\,\partial/\partial z\,\hat{\xi}_\phi$, $\hat{B}_z = B_z\,\partial/\partial z\,\hat{\xi}_z - B_z\left(\nabla\cdot\hat{\xi}\right)$, while $\hat{p} = -\hat{\xi}_z\,dp/dz - \Gamma p\nabla\cdot\hat{\xi}$. Likewise, the perturbed density is given by $\hat{\rho} = -\hat{\xi}_z(d\rho/dz) - \rho\nabla\cdot\hat{\xi}$ and, if we follow the arguments given in Ref. [3], we consider "isobaric" perturbations implying that $|\hat{p}/p| \ll |\hat{\rho}/\rho|$ and $\nabla\cdot\hat{\xi} \simeq -\hat{\xi}_z(dp/dz)/p$. We can verify that, for $m_\phi < R_0/H_0$, the toroidal pressure gradient can be neglected in the $\phi$-component of Eq. (2). Then this reduces to $\gamma_t\left(2\Omega_k\hat{\xi}_R\right) \simeq \mathrm{v}_A^2\,\partial^2\hat{\xi}_\phi/\partial z^2$ for $\gamma_t^2 \ll \mathrm{v}_A^2\,\partial^2/\partial z^2$. In fact we limit our analysis to the case where $\left|m_\phi(d\Omega/dR)(R - R_0)\right| < \gamma_0$. Following Ref. [3], we consider the $\partial\left(\mathbf{e}_\phi\cdot\nabla\times\mathbf{A}_m\right)/\partial z = 0$ equation, that is

$$\frac{\partial^2}{\partial z^2}\left[\rho\left(\gamma_t^2\hat{\xi}_R - 2\Omega_k\hat{\mathrm{v}}_\phi\right) - \frac{B_z^2}{4\pi}\frac{\partial^2}{\partial z^2}\hat{\xi}_R\right] - \frac{\partial}{\partial z}\left\{\frac{\partial}{\partial R}\left[\rho\gamma_t^2\hat{\xi}_z + z\Omega_k^2\hat{\rho} - \frac{B_z^2}{4\pi}\frac{\partial^2}{\partial R\partial z}\hat{\xi}_R\right]\right\} = 0, \qquad (3)$$

where $\partial\hat{\xi}_z/\partial z \simeq -\partial\hat{\xi}_R/\partial R$, given that, $\left|\nabla\cdot\hat{\xi}\right| \sim \left|\hat{\xi}_z/H_p\right|$ where $1/H_p^2 \equiv \left|d\ln p/dz^2\right|$.

The relevant axisymmetric modes, are represented by $\hat{\xi}_z = \tilde{\xi}_z^0 G_0(z)\exp[\gamma_0 t - i\delta\omega_0 t$ $+k_R(R - R_0)$ ] $\exp(\gamma_0 t)$ where $\hat{\xi}_z$ is the vertical displacement, $k_R \simeq k_0 \equiv (-2\Omega' R/\Omega)^{1/2}/\mathrm{v}_A$ $\equiv 1/H_c$, $\Omega' = d\Omega/dR$, $\Omega_k' = -3\Omega_k/(2R_0)$ and $\mathrm{v}_A \equiv B_z/(4\pi\rho)^{1/2}$ is the Alfvén velocity. In addition $G_0(z)$ is an even or odd function of $z$ that is localized over a distance

$H_c < \Delta_z \lesssim H_p$ represented, for instance, by $G_0 = \exp\left(-z^2/\Delta_z^2\right)$. The lowest order equation that gives $\gamma_0$ or $\delta\omega_0$ for this class of mode is obtained from Eq. (3 ) following the same steps given in Ref. [3],

$$\frac{\partial^2}{\partial z^2}\left\{\left(3\Omega_k^2\frac{z^2}{H_0^2}-2k_0\left|\delta k_R\right|\mathrm{v}_A^2\right)\tilde{\xi}_R^0\right\}+C_T\frac{\Omega_k^2}{H_0^2}ik_0\frac{\partial}{\partial z}\left(z^2\tilde{\xi}_z^0\right)-\mathrm{v}_A^2\frac{\partial^4}{\partial z^4}\hat{\xi}_R^0\simeq\frac{7}{3}\left(\gamma_0-i\delta\omega_0\right)^2k_0^2\hat{\xi}_R^0, \quad (4)$$

where $\delta k_R = k_R^2\mathrm{v}_A^2 - 3\Omega_k^2$ and $\tilde{\xi}_R^0 \simeq i\left(\partial\hat{\xi}_z^0/\partial z\right)/k_0$ . Then Eq. (4) can be integrated and reduced to a quartic equation for $\tilde{\xi}_z^0$ that can be solved analytically. As a significant example we note that when $C_T \equiv (4/5)(3\eta_T/2-1) > 0$, the growth rate corresponding to the lowest eigenfunction, for which $\Delta_z \simeq (H_0/k_0)^{1/2}$, is $\gamma_0 \simeq (\Omega_k\,\mathrm{v}_A/H_0)^{1/2}\left[\left(6\sqrt{3}/(35)\right)(\eta_T-2/3)\right]^{1/2}$.

When $C_T < 0$, the higher eigenfunctions remain unstable [3] and $\gamma_0$ is given by a different expression while, the lowest eigenfunction, is purely oscillatory (i.e. $\delta\omega_0 \neq 0$ and $\gamma_0 = 0$).

**Tri-dimensional Spirals**

Unstable non-axisymmetric modes with relatively low $m_\phi$'s, can be represented typically by

$$\hat{\xi}_z \simeq \tilde{\xi}_z^0 F_0(R-R_0)\,G_0(z)\sin\left\{k_R(R-R_0)-m_\phi\left[\Omega_k(R_0)t-\phi\right]\right\}\exp(\gamma_0 t) \quad (5)$$

where $F_0 = \exp\left[-7(R-R_0)^2/\left(6\Delta_R^2\right)\right]$, $k_R^2 \simeq k_0^2$, $\Delta_R^2 \equiv -\gamma_0/\left(m_\phi k_R\Omega'\right)$ and $\operatorname{sgn} k_R = \operatorname{sgn} m_\phi$.

Clearly, these are tri-dimensional trailing spirals that are localized radially. The excitation of the lowest harmonics of these structures $\left(m_\phi = 2,3\right)$ lends itself to the formulations of a theoretical model for high frequency QPO's as proposed in Ref. [4].

Purely oscillatory spirals that can be found for $C_T < 0$ are instead represented by

$$\hat{\xi}_z \simeq \tilde{\xi}_z^0 G_0(z)\sin\left[\delta\omega_0 t-\frac{\sigma}{2}(R-R_0)^2\right]\sin\left[m_\phi(\phi-\Omega_0 t)+k_R(R-R_0)\right] \quad (6)$$

where $\sigma = 7m_\phi k_R\Omega'/(3\delta\omega_0)$. These spirals can be superposed to produce mode packets propagating away from $R = R_0$ for $\operatorname{sgn} k_R = \operatorname{sgn} m_\phi$ (trailing spiral configuration) and lend themselves as a means to transport angular momentum away from $R = R_0$. The effective diffusion coefficient associated with the propagation of the relevant mode packets is $Deff = \partial\delta\omega_0/\partial\sigma$ and is of significant magnitude. We may argue that at the surface $R = R_0$, in the vicinity of a black hole where the higher eigenfunctions (in z) have their highest growth rates, these modes can sustain the excitation of the mode packets mentioned above. Therefore $R = R_0$ could be considered as the surface toward which the accreting matter would flow.

**Particle Outflows and Inflows**

The considered mode can produce particle density transport, in the vertical direction, that is of contrary sign to that of the temperature transport and modify the density and temperature profiles in such a way as to lead $\eta_T$ toward 2/3. Thus, if $\eta_T > 2/3$ a particle inflow toward the equatorial plane is induced. This process is similar to that proposed for the theoretical explanation of the observed particle inflow in magnetically confined toroidal plasmas that is associated [5] with the outflow of thermal energy related to the ratio of the gradients of the radial electron temperature and the particle density.

When $\eta_T < 2/3$, including the case where $\eta_T = 0$ or where the surface of the disk can be hotter than the interior, the particle transport is away, from the equatorial plane. These arguments are based on the quasilinear analysis that gives the vertical particle flux produced by unstable modes as

$$\Gamma_{pz} = \left\langle\left\langle \hat{n}\hat{\mathrm{v}}_z \right\rangle\right\rangle \simeq -\frac{4}{5}\gamma_0 \left\langle\left\langle \left|\hat{\xi}_z\right|^2 \right\rangle\right\rangle \times \left[\frac{\partial}{\partial z}n - \frac{3}{2}\frac{n}{T}\frac{\partial}{\partial z}T\right],$$

where $\langle\langle\ \rangle\rangle$ indicates an average over a radial distance $\Delta R$ such that $1/k_R < \Delta R < R_0$. The corresponding temperature flux is $\left\langle\left\langle \hat{T}\hat{\mathrm{v}}_z \right\rangle\right\rangle \simeq -\left\langle\left\langle \hat{n}\hat{\mathrm{v}}_z \right\rangle\right\rangle T/n$. The outflows produced by these modes can be considered as candidates to explain the origins of the winds that have been observed to emanate from disk structures such as those at the core of AGN's. *Sponsored in part by the U.S. Department of Energy.

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