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Nonlinear waves in collisional dusty plasma

B.P. Pandey[1,2], S. V. Vladimirov[2], A. Samarian[2]

[1] *Department of Physics, Macquarie University, Sydney, NSW 2109, Australia*

[2] *School of Physics, The University of Sydney, NSW 2006, Australia*

The Alfvén waves are possible source of turbulence in the interstellar medium [1]. These waves are responsible for the transport of angular momentum and energy in the accretion discs [2]. When the amplitude of the waves is large, nonlinear effect plays an important role in the propagation of these waves. It is known that in a homogenous, uniform background medium, the interplay between dispersion and nonlinearity can give rise to solitary wave structures. When the dispersion is kept finite and the wave amplitude is small (but finite), the evolution of the perturbed magnetic field is described by the nonlinear Schroedinger equation (NLSE). However, when both dispersion and nonlinearity are comparable, the propagation of the wave is governed by the derivative nonlinear Schroedinger equation (DNLS). Such waves have been studied for last several decades in space plasmas [3].

Most of the space plasma such as those in cometary tails, interstellar molecular clouds and planetary nebulae is dusty [4, 5]. It contains charged grains and, the coupling of the grains to the magnetic field determines the wave propagation in the planetary and interstellar medium [6, 7, 8]. The plasma–grain collision can causes not only the damping of the high frequency waves but also assist the excitation and propagation of the low frequency fluctuations in the medium. For example, if the plasma-dust collision frequency is higher than the dynamical frequency then collision will be responsible for dragging the dust along the plasma fluctuations. In such a scenario collision will always cause the propagation of the waves in the medium without any damping.

The present work investigates the nonlinear wave properties of collisional dusty plasma consisting of electrons, ions and charged grains. We show that when the electrons and ions are magnetized, the relative drift between the charged grains and the plasma particles (electrons and ions) gives rise to the Hall diffusion in the medium. This Hall diffusion causes the wave dispersion. It is shown that the collisional dusty medium is inherently dispersive in nature and the balance between the dispersion and nonlinearity leads to DNLS equation.

We shall assume that the dusty plasma consists of the inertialess electrons, ions and the inertia of the dusty fluid is due to the presence of the charged grains. We shall define mass density of the bulk fluid as $\rho = \rho_e + \rho_i + \rho_d \approx \rho_d$. Then the bulk velocity $\mathbf{v} = (\rho_i\,\mathbf{v}_i + \rho_e\,\mathbf{v}_e + \rho_d\,\mathbf{v}_d)/\rho \approx \mathbf{v}_d$.

The continuity equation (summing up the electron, ion and dust continuity equations) we get

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \tag{1}$$

The momentum equation can be derived by adding the electron, ion and dust momentum equations

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P + \frac{\mathbf{J} \times \mathbf{B}}{c}. \tag{2}$$

Here $P = P_e + P_i + P_d$ is the total plasma Pressure and $\mathbf{J} = -e n_e \mathbf{v}_e + e n_i \mathbf{v}_i$ is the current density in the dust frame. It is interesting to note that the collision terms have disappeared from the above momentum Eq. (2).

In order to derive an induction equation, we note that the electron and ion momentum can be inverted to yield

$$\mathbf{v}_{\mathbf{i}\perp} = \frac{\frac{c}{B}\beta_i \left(\mathbf{E}_\perp - \frac{\nabla_\perp P_i}{e n_i}\right) + \beta_i^2 \frac{c\mathbf{E}\times\mathbf{B}}{B^2} - \beta_i \frac{\nabla P_i \times \hat{\mathbf{B}}}{\rho_i \nu_{id}}}{(1+\beta_i^2)},$$

$$\mathbf{v}_{\mathbf{e}\perp} = \frac{-\frac{c}{B}\beta_e \left(\mathbf{E}_\perp + \frac{\nabla_\perp P_e}{e n_e}\right) + \beta_e^2 \frac{c\mathbf{E}\times\mathbf{B}}{B^2} + \beta_e \frac{\nabla P_e \times \hat{\mathbf{B}}}{\rho_e \nu_{ed}}}{(1+\beta_e^2)}, \tag{3}$$

where $\hat{\mathbf{B}} = \mathbf{B}/B$, and,

$$\mathbf{v}_{\mathbf{i}\parallel} = \beta_i \frac{c\mathbf{E}_\parallel}{B} - \frac{\nabla_\parallel P_i}{\rho_i \nu_{id}}, \quad \mathbf{v}_{\mathbf{e}\parallel} = -\beta_e \frac{c\mathbf{E}_\parallel}{B} - \frac{\nabla_\parallel P_e}{\rho_e \nu_{ed}}. \tag{4}$$

Here $\beta_j = \omega_{cj}/\nu_{jd}$ is the plasma Hall parameter and is a measure of magnetization of the electrons and ions. For example, when plasma cyclotron frequency ω_{cj} dominates the plasma-dust collision frequency, i.e. $\beta_e \gg 1$, and $\beta_i \gg 1$, the Hall term ($\sim \mathbf{E}\times\mathbf{B}$) will dominate in Eq. (3). Physically, in the dust frame, $\beta_j \gg 1$ implies that the plasma drift due to collision is very small compared to the transverse gyration of the plasma particles across the magnetic field. This results locally in the plasma particles going away or coming close to a stationary observer in the dust frame resulting in a time-dependent Hall electric field. This Hall field is generated over plasma-cyclotron time scale and depending upon the sign of the grain charge, the Hall scale can become arbitrary large [7]. In $\beta_j \gg 1$ limit, one may assume that the relative drift between electrons and ions are small, i.e. $\mathbf{v}_e \approx \mathbf{v}_i$ and the electron velocity $\mathbf{v}_e$ can be written as $\mathbf{v}_e = -\mathbf{J}/Zen_d$. While writing this expression for $\mathbf{v}_e$, plasma quasineutrality condition $n_e = n_i + Zn_d$ have been used. After taking curl of the electron momentum equation $\mathbf{E} + \mathbf{v}_e \times \mathbf{B}/c \approx 0$ and making use of Maxwell's equation, the induction equation can be written as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [(\mathbf{v}_d \times \mathbf{B}) - (\mathbf{J}\times\mathbf{B}/Zen_d)]. \tag{5}$$

In a dusty plasma, or for that matter in any multicomponent plasma, since $n_e \neq n_i$, even when relative drift between the plasma particles are absent, i.e.,$\mathbf{v}_e = \mathbf{v}_i$, owing to the presence of a third, charged component, the Hall effect will always be present.

We investigate the dusty plasma dynamics in the $\beta_j \gg 1$ limit with the help of Eqs. (1), (2) and (5) along with an equation of state $P/\rho^\gamma = \text{const}$. Assuming a uniform background field in the z-direction and the perturbations is assumed one-dimensional with the variation along z-direction. Then above set of Eqs. (1), (2) and (5) in the stretched variable $\xi = \varepsilon\,(z - V_A\,t)\,,\, \tau = \varepsilon^2 t$ can be written as

$$D\rho + \frac{\partial\,(\rho\,v_z)}{\partial\xi} = 0\,,$$

$$\rho\,\left(D + v_z\frac{\partial}{\partial\xi}\right)v_z = -\frac{\partial}{\partial\xi}\left(p + \frac{|\mathbf{B}_\perp|^2}{8\pi}\right)$$

$$\rho\,\left(D + v_z\frac{\partial}{\partial\xi}\right)\mathbf{v}_\perp = \frac{B_z}{4\pi}\frac{\partial\mathbf{B}_\perp}{\partial\xi}$$

$$D\mathbf{B}_\perp = B_z\frac{\partial\mathbf{v}_\perp}{\partial\xi} - \frac{\partial}{\partial\xi}\,(v_z\mathbf{B}_\perp) + \varepsilon\,\sigma\,V_A\,\delta_d\,\hat{\mathbf{z}}\times\frac{\partial}{\partial\xi}\left(\frac{\rho_0}{\rho}\frac{\partial\mathbf{B}_\perp}{\partial\xi}\right). \tag{6}$$

Here $D = \varepsilon\,\partial/\partial\tau - V_A\,\partial/\partial\xi$, $V_A^2 = B_z^2/4\,\pi\,\rho_0$, $\delta_d = V_A/\,\omega_{cd}$ is the dust skin depth, $\sigma = \pm 1$ and $\mathbf{v}_\perp$ and $\mathbf{B}_\perp$ are the trasvesrse (to $\hat{z}$) components of velocity and magnetic field respectively.

We apply reductive perturbation method and assume that the perturbations to physical quantities vanish at infinity, i.e. $\rho \to \rho_0\,,\, p \to p_0\,,\, v_z \to 0$ and $\mathbf{B}_\perp \to \mathbf{B}_{\perp\mathbf{0}}$. We shall use inverse of plasma $\beta = c_s^2/V_A^2$ where $c_s^2 = \gamma\,p_0/\rho_0$ is the acoustic speed) as the smallness parameter, i.e. $\varepsilon = \beta^{-1}$ for perturbative expansion of the physical quantities. Introducing scaled sound speed $\hat{c}_s = \varepsilon\,c_s$ and the scaled equilibrium pressure $\hat{p}_0 = \varepsilon\,p_0$, the scaled pressure is expanded as $p = \varepsilon^{-1}\,\hat{p}_0 + p_1 + \varepsilon\,p_2 + \ldots$. We note that p_1, p_2 are scaled pressure here. Expanding all other quantities as $f = f_1 + \varepsilon\,f_2 + \ldots$, from Eqs. (6) collecting terms of the order ε^0 and ε and imposing boundary conditions at infinity, we will arrive at the following derivative nonlinear Schrodinger equation (DNLS)

$$\frac{\partial\mathbf{B}_{\perp 1}}{\partial\tau}\alpha\,\frac{\partial}{\partial\xi}\mathbf{B}_{\perp 1}\,\left(|\mathbf{B}_{\perp 1}|^2 - |\mathbf{B}_{\perp 0}|^2\right) + \sigma D\,\hat{\mathbf{z}}\times\frac{\partial^2\mathbf{B}_{\perp 1}}{\partial\xi^2} = 0\,. \tag{7}$$

where $\alpha = V_A/(16\,\pi\,\rho_0\,\hat{c}_s^2)$, and, $D = V_A\,\delta_d/2$. By writing above equation in the component form in the x and y direction, and combining them into a single complex variable $b = B_x + i\,B_y$, one gets the complex scalar version of DNLS

$$\frac{\partial b}{\partial\tau} - \alpha\,\frac{\partial}{\partial\xi}\left[b\,\left(|b|^2 - |b_0|^2\right)\right] + i\,\sigma\,D\,\frac{\partial^2 b}{\partial\xi^2} = 0\,. \tag{8}$$

The DNLS equation (8) describes the evolution of weakly nonlinear, weakly dispersive waves propagating either exactly parallel to the ambient magnetic field $\mathbf{B} = B\,\hat{z}$ (corresponding to the boundary condition $b_0 \to 0$ as $\xi \to \infty$) or slightly oblique propagation ($b_0 \neq 0$ at infinity). Dispersion in the DNLS Eq. (8) is caused by the dust inertia and nonlinear term arises due to coupling between the transverse magnetic field and plasma pressure perturbations. We note that the derivation of the above DNLS equation is very similar to the MHD case although we have considered a set of dissipative multi-component dusty plasma equations. In the low-frequency limit (i.e. frequencies much lower than the plasma-dust collision frequencies), the dust-plasma collision causes the dust and the plasma fluids to stick together as a single fluid. Exactly parallel ($b_0 = 0$) , circularly polarized, finite amplitude Alfvén waves, $b = B_0\, e^{i\,(\omega\,\tau - k\,\xi)}$ satisfies following nonlinear dispersion relation

$$\omega = \alpha\, B_0^2 k + D\,\sigma\, k^2\,. \tag{9}$$

We note that when $\alpha = 0$, Eq. (9) is a linear dispersion relation describing the whistler mode. When $\alpha \neq 0$, for the lefthand polarized waves ($\sigma = -1$), $k > 0$ is unstable to parallel modulation while the righthand polarized branch ($\sigma = 1$), $k < 0$, is stable to the parallel modulation [3].

For exactly parallel, circularly polarized, finite amplitude Alfvén waves, Eq. (8) admits localized envelop soliton solutions [10]

$$|b|^2 = \frac{2V}{\alpha}\left[\sqrt{2}\cosh\left(\frac{\xi V\,\tau}{L}\right) - 1\right]^{-1}, \tag{10}$$

where V represents the velocity in the co-moving frame, $L = D/V \equiv \left(\frac{V_A}{2V}\right)\,\delta_d$ is the width of the solition and $2V/\alpha$ is the maximum amplitude of the soliton. Here we have assumed $\sigma = 1$.

To summarize, we have shown that in the low frequency limit, the nonlinear propagation of the waves in the magnetized, collisional dusty medium can be described by the DNLS. The soliton solution of DNLS could be probably invoked to explain the structures in the space plasmas.

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