

THE RECONSTRUCTION OF THE EFFECTIVE INTERACTION POTENTIAL IN DUSTY PLASMA

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The problem of the potential reconstruction for the system with known pair correlation function or structure factor is one of the fundamental tasks of the statistical physics (so called inverse task). There is an hypothesis that the behaviour of dusty particles can be considered the same as the behaviour of the macroscopic particles [1]. So the determination of the effective interaction potential is also important problem for the physics of complex plasmas. Below we will consider the 2D systems with pair-additive interaction.

The following results were obtained for the inverse task earlier. At first, the uniqueness theorem was proved [2]. This theorem states that the pair potential $\Phi(r)$ which gives rise to a given radial distribution function $g(r)$ is unique up to a constant. This constant is defined by the condition $\Phi(r) \rightarrow 0$, while $r \rightarrow \infty$. So we have only one potential for a given $g(r)$. At second, the reconstruction techniques were developed [3, 4]. These techniques were successfully applied to a number of real and model systems [5]. And, at third, the technique [3] was recently applied to obtain the interparticle potential in dusty plasma [6]. The potentials were extracted from the data of measurements [7]. It was shown that the potentials are dependent on the dusty particle density and they could have the attraction branch. There are two possible reasons of this attraction: the interaction between the particles themselves or the influence of the trap, which is always presented in experiments with the dusty plasma. In [7] the trap was a special ring mounted on the lower electrode. During the reconstruction of the potential in [6] the influence of the trap was not taken into account. So the analysis of the trap influence is the aim of this report.

The details of original reconstruction techniques are presented in [3, 4, 5, 6]. The interaction potential is obtained by iterations. At first some initial potential must be given. In [3] it was $\Phi_0(r) = -T \ln(g_{exact}(r))$ where $g_{exact}(r)$ is known (experimental) pair correlation function. (Here and below the temperature T is measured in the energy units). Then one can obtain $g_0(r)$ by Monte-Carlo or Molecular Dynamic simulations with this initial potential. It was shown in [3] that the potential on $n + 1$ iteration is:

$$\Phi_{n+1}(r) = \Phi_n(r) + T \ln(g_n(r)/g_{exact}(r)) \quad (1)$$

Every $g_n(r)$ is obtained by corresponding simulation with $\Phi_n(r)$. When $g_n(r)$ coincides with

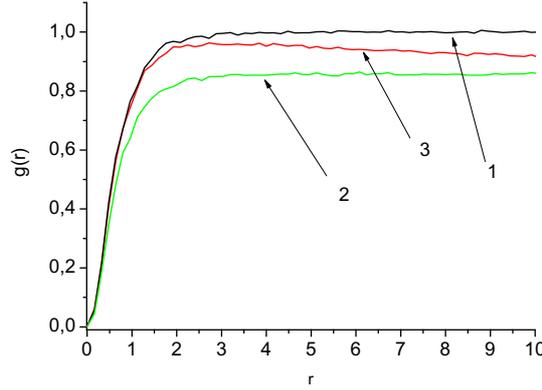


Figure 1: The pair correlation functions for the Yukawa system with and without trap. See text

$g_{exact}(r)$ the required potential is obtained. In [4] more complicated scheme than (1) was used. But the principle is the same: the coincidence of simulated and exact $g(r)$ gives the solution of the inverse task. The Monte-Carlo (MC) or Molecular Dynamic simulations use periodic boundary conditions. The presence of the trap imposes another type of boundary conditions. Let the radius of the trap is R . Then the potential energy of the dusty particles system is:

$$V(r) = \sum_{i,j} \Phi(\vec{r}_i - \vec{r}_j) + \sum_i U(\vec{r}_i) \quad (2)$$

Here Φ is the pair-wise interaction potential, which must be defined; U is the field of the trap which is supposed to be known. For example, we can set U as $U = 0$ at $r < R$ and $U = \infty$ at $r > R$ (infinite well trap). In general the potential U makes the system inhomogeneous, i. e. $g(\vec{r}) = g(\vec{r}_1 - \vec{r}_2) \rightarrow g(\vec{r}_1, \vec{r}_2)$. But as far as $U = 0$ inside the region of experiment we can consider the dusty particles as homogeneous system. In Fig. 1 the pair correlation functions obtained 2D simulations with potential $U/T = \Gamma \exp(-r)/r$ are presented. (Here the distance r is reduced to the screening length λ , $\Gamma = 1.0$, and dimensionless density $n\lambda^2 = 0.1$). The 1st curve is the result of standard Metropolis MC with periodic boundary conditions. The 2nd curve is the same as the first but without periodic boundary conditions. In this case $g(r)$ does not tend to unit as $r \rightarrow \infty$ because the particles can leave the system. The 3rd curve is the result of the simulation of the system with infinite well trap but without boundary conditions. One could see that the presence of the trap (the 3rd graph) partially compensates the absence of the periodic boundary conditions. So, the presence of the trap can change the resulting $g(r)$. The analysis made in [6] (without trap) showed that the reconstructed potential for the correlation functions measured in [7] could have the repulsive as well as the attractive branches. The fitting of the repulsive branch at the case of high densities gave rise to the Yukawa potential $\Phi(r)/T = Q^2 \exp(-r/\lambda)/(rT)$,

where $Q^2/(\lambda T) = 7.1$ and $\lambda = 0.5$ mm . This form of the potential is in qualitative agreement with the data of the measurements [8]. (It is the only known to author experimental data on the dusty particle potential). At low densities the repulsive branch was not of Yukawa type. Now we will try to take into account the trap, supposing that the potential between the dusty particles is of Yukawa type with $\lambda = 0.5$ mm.

There are two sets of experiments in [7] that were earlier analyzed in [6]. The first set is obtained for a relatively small density of dusty particles $n_d = 150$ cm². The second set is for a relatively high density $n_d = 486$ cm². The error of the measurements was estimated as 10 %. For the case of small densities the MC simulations with the trap gave rise to the experimental $g(r)$ at $\Gamma = Q^2/(\lambda T) = 1.1$. This value is close to the theoretical estimates made in [7] i. e. $\Gamma = 1.2$. Corresponding correlation functions are presented in Fig. 2. The circles are the measured $g(r)$ and the line is the result of our simulation with the trap. For the case of high density the MC simulation with trap could not give rise to the measured $g(r)$. No one of calculated $g(r)$ can reproduce the 1st maximum of experimental correlation function. The best value of coupling parameter is $\Gamma = 1.66$. Corresponding $g(r)$ are presented in Fig. 3. The notation is the same as in Fig. 2.

So, the present simulations partially confirm the conclusions made earlier in [6]. I. e. that the system of dusty particles can not be described by purely repulsive potential. The new conclusion is that the effective attraction between the dusty particles, predicted by many researches (see [1]), cannot be explained by the influence of the trap only. The shadowing effects and the drag forces should be taken into account to obtain correct effective pair-wise potential between the the dusty particles.

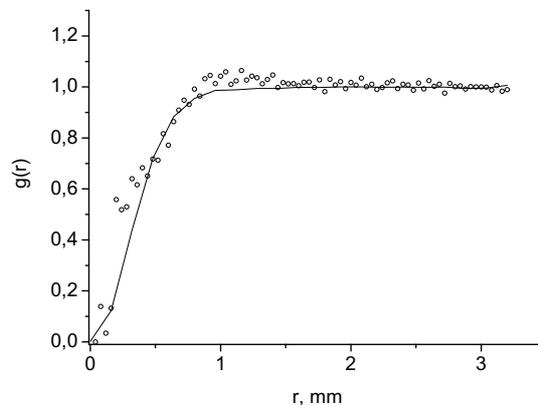


Figure 2: The simulation with the trap at small density.

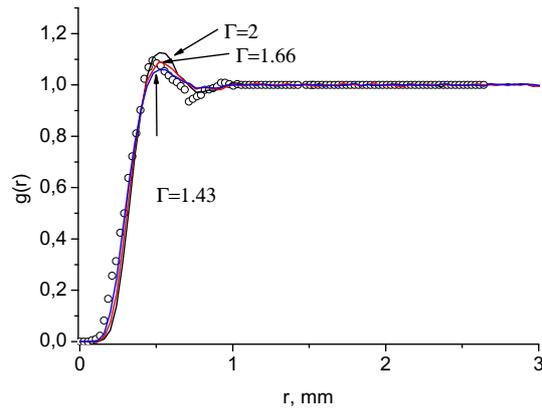


Figure 3: The simulation with the trap at high density.

References

- [1] V.E. Fortov, A. V. Ivlev, S. A. Khrapak, A. G. Khrapak, G. E. Morfill, Phys. Rep., **421**, 1 (2005)
- [2] R. L. Henderson, Phys Letters A, **49**, 197 (1974)
- [3] W. Schommers, Phys. Rev. A, **28**, 3599 (1983)
- [4] L. Reatto, D. Levesque, J. J. Weis, Phys. Rev. A, **33**, 3451 (1986)
- [5] D. K. Belashchenko, Physics Uspekhi, **42**, 297 (1999)
- [6] E. M. Apfelbaum, Physics of Plasmas, **14**, 123703 (2007)
- [7] V.E. Fortov, A. V. Gavrikov, O. F. Petrov, I. A. Shakhova, V. S. Vorob'ev, Phys. Plasmas, **14**, 040705 (2007)
- [8] U. Konopka, G. E. Morfill, L. Ratke, Phys Rev. Lett., **84**, 891 (2000)