GAMs like dynamics due to nonlinear interaction of multiple NTMs in tokamaks

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Neoclassical tearing modes (NTMs) are one of the most serious concerns for operation of advanced fusion devices like ITER \cite{1}. NTMs are driven by the perturbed bootstrap currents and can limit the normalized plasma $\beta_N$ of a tokamak. There have been a large number of studies in the recent past \cite{2-7} investigating the dynamics of NTMs. Most of them have been single helicity studies and the topic of mode coupling among NTMs of different helicities has not received adequate attention. However several recent experiments on ASDEX and JET show evidence of mode coupling effects influencing the evolution of NTMs \cite{2, 3}. There have been some theoretical studies aimed at understanding the physics of nonlinear coupling of different NTMs but the results so far are not very conclusive \cite{3-6}. These past studies suggest that the interaction can arise due to harmonic coupling of the waves \cite{3, 6} or from stochastic coupling due to overlapping of the modes \cite{5}. The latter can be a dominant mechanism only when the resonant surfaces are very close to each other which is not always the case.

In the present work we have addressed this issue by carrying out numerical simulation studies of multiple NTMs using a fully toroidal code NEAR which solves a set of generalized reduced MHD equations \cite{7, 8}. The model equations are,

\begin{align}
\frac{\partial \psi}{\partial t} - \nabla \cdot \phi &= \eta J_{||} - \frac{1}{ne} \mathbf{b}_0 \cdot \nabla \cdot \pi_e \\
\frac{dU}{dt} &= B_0 \nabla \cdot \left( \frac{J_{||}}{B_0} \right) + \nabla \cdot \frac{\mathbf{b}_0 \times \nabla p}{B_0^2} \\
\frac{dp}{dt} + \Gamma p \mathbf{V} \cdot \nabla q &= -(\Gamma - 1) \nabla \cdot \mathbf{q} \\
\frac{dV_{||}}{dt} &= -\nabla_{||} p
\end{align}

where, $U = \nabla \cdot \left( \frac{\nabla \phi}{B_0^2} \right)$, $J_{||} = \nabla_{\perp}^2 \psi$, $q = -\chi_{\perp} \nabla \cdot p - \chi_{||} \nabla_{||} p$, $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla$, $\mathbf{V} = \frac{\mathbf{b}_0 \times \nabla \phi}{B_0^2} + \mathbf{V}_{||}$ and other notations are standard. We have taken $(m,n) = (2,1)$ and $(3,1)$ as perturbed modes ($m$ is poloidal and $n$ is toroidal mode number) to ensure well separated resonant surfaces. We have used a circular equilibrium obtained from TOQ code \cite{9} with $R/a \sim 10$, $S (=\tau_R/\tau_A)$ is $10^5$. 

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Figure 1: Evolutions of magnetic energies with time for coupled and single NTMs (on the left) and comparison of magnetic and flow energies (on the right) for $\Delta' < 0$, finite $\mu_e$.

Figure 2: Evolutions of magnetic energies with time for coupled TMs (on the left) and comparison of magnetic and flow energies (on the right) for $\Delta' > 0$, $\mu_e = 0$.

At first we have started with toroidal $\beta_0 = 0.009$ and an equilibrium which is stable for the (2,1) and (3,1) classical tearing modes ($\Delta' < 0$). These modes can be destabilized by the neoclassical driving term which is proportional to viscosity $\mu_e$. The left panel of Fig.1 shows that coupled NTMs lead to oscillations in the energy while there are no such oscillations for a single (3,1) NTM evolution. The right panel of Fig.1 shows that coupled NTMs also generate large perpendicular flows. We find that the nature of the oscillations depends on the plasma $\beta_p$ values.

Next, we have taken an equilibrium which is slightly unstable ($\Delta' > 0$) for classical tearing modes so that we can switch off or on the neoclassical term in the code and compare the results. Fig.2 shows that there are no oscillations of energy for the pure classical tearing modes with $\mu_e = 0$. The flow energy also remains small compared to the magnetic energy. Upon switching on the neoclassical term for the same equilibrium oscillations of energy appear as shown in Fig.3. This also leads to higher energies for the $n=0$ modes and the generation of higher perpendicular flows. Unlike the pure classical tearing modes, flows are not restricted to their respective
Figure 3: Evolutions of magnetic energies with time for coupled TMs (on the left) and comparison of magnetic and flow energies (on the right) for $\Delta' > 0$, finite $\mu_e$

Figure 4: $\Phi$ contours of coupled tearing modes with $\Delta' > 0$, $\mu_e = 0$ (on the left) and with $\Delta' > 0$, finite $\mu_e$ (on the right) at saturation

Figure 5: Evolutions of magnetic energies with time for coupled TMs (on the left) and evolutions for different $\beta_p$ (on the right) for $\Delta' > 0$, finite $\mu_e$ in absence of $V_\parallel$ nonlinear terms
resonant surfaces as can be seen in Fig.4. Small scale flows appear inside the resonance layer and an expanded flow pattern exists outside the resonant surfaces providing coupling between the two modes. It is to be noted that none of these phenomena appear for linear runs and these are basically nonlinear effects. Lastly, to identify the role of parallel dynamics, we have made some runs by switching off the nonlinear $V_\parallel$ terms. The left panel of Fig.5 shows that a suppression of large oscillations occurs during the Rutherford phase leading to the conclusion that the convective derivative term in the $V_\parallel$ equation is the major contributor towards these large oscillations. However there are still small regular oscillations to be seen at saturation which seem to strongly depend on the value of the plasma $\beta_p$ values (see right panel of Fig.5). At lower $\beta_p$ these oscillations disappear. It is also observed that the energy of the $\phi^{(1,0)}$ mode starts increasing at first before that of any other mode and is almost instantaneously followed by the growth of the $p^{(2,1)}$, $p^{(0,0)}$ and $p^{(1,0)}$ modes. Eventually other components respond and lead to oscillations. The nature of these oscillations are very similar to GAMs i.e. geodesic acoustic modes [10, 11]. Just like GAMs, the oscillations are accompanied or triggered by higher $\phi^{(1,0)}$, $p^{(0,0)}$ and $p^{(1,0)}$ modes. The increase of oscillation frequency with plasma $\beta$ as shown in Fig.5 also conforms with the typical characteristics of GAMs. The characteristic frequency of the oscillations, calculated from the power spectrum of energy evolution of Fig.1 is found to be in the range of values typical for GAMs.

In conclusion, the presence of finite neoclassical electron viscosity in a high $\beta$ plasma can lead to GAMs-like oscillations that mediate the coupling between multiple tearing modes in the nonlinear regime even when their resonant surfaces are well separated.

References