Modelling of beam-driven high frequency Alfvén eigenmodes in MAST

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Introduction: Alfvén eigenmodes (AEs) are studied due to the potential redistribution of fast ions that they can cause and also for the purpose of diagnosing plasma parameters [1]. Modes driven by both the radial gradient of the fast ion pressure, e.g. Toroidal Alfvén Eigenmodes (TAEs) [2], and by energy space gradients, e.g. Compressional Alfvén Eigenmodes (CAEs) [3], have been previously studied on NSTX [4] and observed on MAST [5], driven by super Alfvénic ions produced by neutral beam injection (NBI) [6, 7]. For instabilities with frequency $\omega$ comparable to the ion cyclotron frequency $\Omega_i$, the relevant fast particle resonance responsible for the mode excitation is the Doppler shifted cyclotron resonance [8],

$$\omega - k_{\parallel}v_{\parallel,\text{res}} - l\Omega_b = 0$$ (1)

where $k_{\parallel} = k \cdot B_0/|B_0|$, $l$ is an integer, $l = +1$ corresponds to the Doppler resonance, $l = -1$ corresponds to the anomalous Doppler resonance and $v_{\parallel,\text{res}}$ is the parallel velocity of the resonant beam particles. Note on MAST the NBI and thermal plasma ions are both deuterium so that $\Omega_b = \Omega_D$. New MAST data [9], obtained using high power super Alfvénic NBI, shows significant AE activity in the ion cyclotron frequency range with the intermediate frequency modes having a large compressional component [10]. The aim of the present paper is to perform kinetic modelling of the linear drive and damping for global AEs. A 1-D model [11, 12] for computing AEs is used with the NBI distribution function computed using the TRANSP code [13]. The 1-D eigenmode model is simpler than the models from Refs. [14, 4], but allows extensive parameter space scans of toroidal mode number and $k_{\parallel}$.

Experimental data: MAST is a small aspect ratio spherical tokamak (ST) with major and minor radii of $R_0 = 0.86$m and $a = 0.6$m respectively. In a recent series of MAST discharges, with co-$I_p$ NBI with $E_{\text{NBI}}^\text{max} \approx 65$keV and $v_{\parallel,\text{b}}^\text{max}/v_A \approx 2.5$, we observed persistent AEs which span a frequency range between $\Omega_D(R_0)/4$ and just above $\Omega_D(R_0) \approx 2.3 \times 10^7$rad$^{-1}$ ($f_{gD} \approx 3.6$MHz) (where $\Omega_D(R_0)$ is the cyclotron frequency at the geometric axis). Figures 1 - 3 show a typical example of such data.
Figure 2 shows toroidal mode numbers \((n)\) of the AEs, which are only negative and decrease in frequency as \(|n|\) increases \((n < 0\) implies counter \(I_p\)). The fine frequency splitting between successive mode numbers can be explained by toroidal plasma rotation \(f_{\text{rot}}^\text{max} \approx 19\text{kHz},\) driven by NBI. However, this rotation cannot explain large frequency separations e.g. \(\Delta f_{n=-5\cdots-6} \approx 150\text{kHz}\). It is important to note that AEs with a maximum frequency of 3.8MHz, exceeding the cyclotron frequency \(f_{cD}(R_0) = 3.77\text{MHz}\), were observed in pulse #18886 (not shown).

**Modelling the NBI distribution function:** Using TRANSP [13] with \(10^5\) macro particles the deuterium NBI distribution function at \(t = 0.32s\) has been calculated, which shows that most of the NBI produced ions are deposited in the core. Figure 4 shows that a high density bump on tail in NBI energy exists, due to the balance of charge exchange losses at lower energy and beam sources at high energy, out to a normalised radius of \(r/a = 0.425\), where \(n_0 = 4 \times 10^{19} \text{m}^{-3}\), \(T = 950\text{eV}\), \(n_b = 5 \times 10^{17} \text{m}^{-3}\).
A shifted Gaussian distribution in velocity space can represent the NBI distribution function in the relevant phase space region,

\[
f_b = \frac{n_p e^{-(v_\parallel-v_{\parallel b})^2/2\Delta v_\parallel^2} e^{-(v_\perp-v_{\perp b})^2/2\Delta v_\perp^2}}{2\pi^2 v_{\perp b} \Delta v_\perp \Delta v_\parallel}
\]

with following NBI parameters (which are fitted to the TRANSP result) \( v_{\perp b} = 6 \times 10^6 \text{ms}^{-1}, v_{\parallel b} = -1.47 \times 10^6 \text{ms}^{-1}, \Delta v_\perp = 1 \times 10^5 \text{ms}^{-1} \) and \( \Delta v_\parallel = 1.03 \times 10^5 \text{ms}^{-1} \). We will assume \( v_{\perp b}^2 \gg \Delta v_\perp^2 \) in all calculations that follow.

**Global kinetic analysis:** In order to calculate the global linear kinetic drive, \( \gamma^{gl} \), associated with the NBI distribution function in Eq. (2) and Maxwellian thermal plasma species, we perform the following weighted integral of the local kinetic drive and damping, \( \gamma^{loc} \), with the wave electric field \( (E(r)) \) over the region of the eigenmode localisation [3]

\[
\gamma^{gl} = \frac{\int_0^a r dr |E(r)|^2}{\int_0^a |E(r)|^2}.
\]

where, for small \( k_\perp \), the NBI contribution to \( \gamma^{loc} \) takes the following form

\[
\gamma^{loc} = -\sqrt{\pi e^{-x^2}} \frac{\omega_{\parallel b}^2}{\omega} \left[ \frac{1}{k_\parallel \Delta v_\parallel} - \frac{v_{\parallel b}}{\Delta v_\parallel} + x_\parallel \left( -\frac{1}{\omega} + \frac{3}{2} \frac{\Delta v_\perp^2}{\Delta v_\parallel^2} + \frac{v_{\perp b}^2}{\Delta v_\perp^2} \right) \right] \text{sgn}(k_\parallel),
\]

\[
x_\parallel = \frac{\omega - k_\parallel v_{\parallel b} + \Omega_D}{k_\parallel \Delta v_\parallel},
\]

We use a 1-D ‘hollow cylinder’ model [12] for calculating a discrete spectrum of AEs in the frequency range compatible with the experimental observations and use the Doppler resonance condition from Eq. 1 to identify the resonant particle velocity for a given eigenmode. As a result of the eigenmode analysis, we obtain \( E(r) \) and the global drive given by Eq 3. For the NBI distribution function given by Eq. 2 the value of the drive is sensitive with respect to \( v_{\parallel b} \) and \( v_{\perp b} \). In order to assess this sensitivity we vary \( v_{\parallel b} \) and \( v_{\perp b} \) within the TRANSP error bars (of about 10%) for a mode with \( n = -9, k_\parallel = 6.8 \text{m}^{-1} \) and \( f \approx 1 \text{MHz} \) (Fig. 5).

The maximum growth rate calculated within the TRANSP error bars is between \( \gamma^{gl}/\omega \approx 0.1\% \) and 1% which is then consistent with experimentally measured linear growth rate of \( \gamma^{gl}/\omega = 0.5\% \).
Absence of $n > 0$ modes: The observed $n < 0$ modes were driven via the Doppler resonance, $l = +1$ in Eq. 1, while the $n > 0$ modes can only be driven via the anomalous Doppler resonance, $l = -1$. Although the NBI on MAST was super Alfvénic, it was still below the critical beam speed required for exciting the right hand polarised compressional Alfvén eigenmodes (CAEs),

$$v_{\|b} > \frac{3\sqrt{3}}{2}v_A.$$  \hspace{1cm} (5)

The validity of Eq. 5 can be tested experimentally by lowering the magnetic field.

Conclusions: It is now be understood that high frequency AEs on MAST are driven via the Doppler shifted cyclotron resonance condition by a bump on tail free energy source associated with the NBI ions. The absence of $n > 0$ AEs can now be understood to be a result of the existence of a critical beam velocity, which was not exceeded experimentally.

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