

Continuous spectrum of shear Alfvén waves in the presence of a magnetic island*

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Introduction

Shear-Alfvén waves (SAW) are e-m waves propagating in plasmas with the characteristic Alfvén velocity v_A . SAW in a nonuniform equilibrium experience an energy absorption mechanism near resonant surfaces, called *continuum damping* [1,2]. The resonance frequency in a tokamak plasma has a continuous spectrum with gaps [3]. Two types of global shear-Alfvén instabilities exist in tokamak plasmas: Energetic Particle continuum Modes (EPM) [4], with frequency determined by fast particle characteristic motions, and discrete Alfvén Eigenmodes (AE), with frequency inside the continuum gaps [5]. The former can become unstable provided the drive exceeds a threshold determined by the continuum damping absorption; the latter, on the other hand, has a generally lower instability threshold, being practically unaffected by continuum damping [6]. For this reason, the importance of understanding the continuous spectrum topology is clear if one faces the stability problem of a tokamak and its potential impact on reaching the ignition condition.

The SAW continuous spectrum in a given equilibrium can be modified by the interaction with low-frequency MHD fluctuations, such as magnetic islands. Magnetic islands are formed when the original sheared equilibrium field lines break due to non ideal effects (such as finite resistivity) and reconnect in different magnetic topology [7]. In this work, we study the non-linear interaction of the Alfvén continuous spectrum with low-frequency MHD modes within fluid theory, considering finite- β regime and taking into account the toroidicity effects associated with the geodesic curvature, responsible in tokamaks of the opening of the Beta-induced Alfvén Eigenmodes (BAE) gap in the low frequency part of the continuous spectrum [8, 9].

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To Prof. Pegoraro and Prof. Chen: would you like to add something here?

SAW continuous spectrum topology in a tokamak equilibrium with magnetic islands

The fluid equation for continuum SA modes in tokamak with quasi-static magnetic islands ($\omega_{isl}/\omega \ll 1$) and finite- β regime can be written in the form:

$$\frac{d}{d\psi} [(\omega^2 - \omega_{BAE}^2) + v_A^2 \nabla_{\parallel}^2] \frac{d}{d\psi} \varphi = 0 \quad (1)$$

where φ is the electrostatic perturbed potential, ω_{BAE} is the Beta-induced Alfvén Eigenmodes frequency and ∇_{\parallel} is the gradient calculated along the sum of the equilibrium and island magnetic fields, with flux surfaces labeled by ψ . We

describe the axisymmetric equilibrium with magnetic shear with a coordinate system (q, u, ζ) . Here $q = (r_0 B_{tor}) / (R_0 B_{pol}) = \epsilon B_{tor} / B_{pol}$ is the safety factor (used in tokamak geometry as radial coordinate, where B_{tor} is constant and B_{pol} is a function of the minor radius r), ζ is the direction of the original sheared equilibrium field (without magnetic islands) at the rational surface $q = q_0$, and u is the helical variable, perpendicular to both q and ζ . Since we are only interested in the SAW continuous spectrum modified by the finite size magnetic island at the rational surface $q = q_0$, we may adopt a slab approximation for describing nonlinear interaction near $q = q_0$. The modes coupled by the presence of the islands are the modes with its same helicity, namely $m = q_0 n$, where m and n are respectively the poloidal and toroidal mode numbers in tokamak geometry. We define the flux function $\psi = \Delta q^2 / 2 + M(\cos(u) + 1)$, with $M = (q_0 s)(B_{isl,0} / B_{pol})$, s being the magnetic shear. The ψ value goes from zero, at the O -point of the island, to $+\infty$, passing through $\psi_{sx} = 2M$ at the separatrix. Since we are looking for the continuous spectrum frequencies, i.e. those for which the solution φ is singular in ψ , the calculation of the continuum reduces to an eigenvalue problem with solution $\omega^2 = \omega^2(\psi)$.

We solve the eigenvalue problem numerically, with a shooting method both inside and outside the separatrix, and benchmark the solutions analytically far from the island and near the O -point. The beta-induced Alfvén Eigenmode continuum accumulation point (BAE -CAP) is found to be

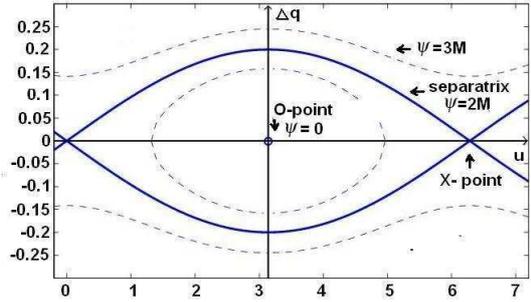


Figure 1: Coordinate system of an equilibrium with a magnetic island.

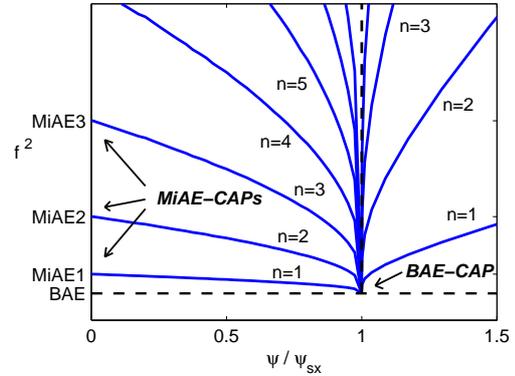


Figure 2: Continuous spectrum branches for several n -modes.

shifted in space from the rational surface of the island ($q = q_0$) to the separatrix flux surface position (labeled $\psi = \psi_{sx}$ in Figure 2), following the behavior of the magnetic field helicity pointed out in [10, 11, 12]. The BAE-CAP frequency, f_{BAE} (we use the notation for frequency: $f = \omega/2\pi$), is not modified by the presence of the island. Outside the island the continuous spectrum frequency branches of the several n modes grow reaching asymptotically the behavior $f_{--} = (f_{BAE}^2 + f_A^2(2\psi - 2M))^{1/2}$, which physically corresponds to the absence of the island. The non-uniformity of the magnetic field intensity along the field lines generates a splitting between the frequency branches of the modes with even and odd eigenfunctions. The most remarkable feature is the presence of new continuum accumulation points at the *O-point* of the island ($\psi = 0$), which depend on the toroidal mode number n , and, thereby, give rise to gaps in the continuous spectrum and regions free of continuum damping. This fact could make the existence of new magnetic-island induced Alfvén Eigenmodes (MiAE) possible, excited via wave-particle resonances, provided that the island size is sufficiently wide with respect to the mode radial localization. The MiAE-CAP frequencies are given by:

$$f_{MiAE-CAP} = f_{BAE} \sqrt{1 + q_0 s \frac{B_{isl,0}}{B_{pol}} \frac{f_A^2}{f_{BAE}^2} n^2}, \quad (2)$$

where q_0 , s , and B_{pol} are respectively the values of the safety factor, shear and poloidal magnetic field calculated at the rational surface of the island, $B_{isl,0}$ is the magnetic island amplitude and f_A is the Alfvén frequency: $f_A = v_A/(2\pi qR)$. For small amplitude magnetic islands the scaling is linear with the amplitude, and the approximate value is:

$$f_{MiAE-CAP} \simeq f_{BAE} + \frac{q_0 s}{2} \frac{B_{isl,0}}{B_{pol}} \frac{f_A^2}{f_{BAE}^2} n^2. \quad (3)$$

The regime of validity of the linear approximation is given by:

$$\frac{B_{isl,0}}{B_{pol}} n^2 \ll \frac{1}{q_0 s} \frac{f_{BAE}^2}{f_A^2} \sim \frac{\beta}{q_0 s},$$

which can be broken for high mode numbers ($n > \sqrt{\beta/M}$) even for finite- β plasmas, where $M = q_0 s B_{isl,0}/B_{pol}$. In the case of low- β plasmas, it is worthwhile to note that the order of magnitude of the island-induced frequency shift can be comparable with the BAE-CAP frequency itself.

Conclusions and discussion

In this work, the continuous spectrum of Shear Alfvén waves has been calculated for finite- β tokamak equilibrium in the presence of a finite-size magnetic island. The Beta-induced Alfvén Eigenmodes continuum accumulation point (*BAE-CAP*) is found to be shifted in space from the

rational surface of the island to the separatrix flux surface position, while the frequency f_{BAE} remains the same. New continuum accumulation points are found at the O -point of the island and, consequently, new gaps in the continuous spectrum. This result has strong potential implications in explaining stability properties of tokamak plasmas in presence of magnetic islands. In fact, new magnetic-island induced Alfvén Eigenmodes could be excited inside a magnetic island if the thermal or energetic component of the plasma provides sufficient free energy for driving the mode. Here, we want to emphasize that our present theoretical approach is not limited to finite magnetic shear. For $s = 0$, i.e. the typical condition under which reversed shear Alfvén Eigenmodes can be excited [13, 14], we can easily generalize Eq. (2) by simply considering that the $BAE - CAP$ is shifted by $k_{\parallel}^2 v_A^2$ [15, 16]. Due to the dependence of the MiAE-CAP frequency on mode numbers and the magnetic island size, the possibility of using Eqs. (2) and (3) as novel magnetic island diagnostic is evident. Comparisons with available experimental data are in progress.

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