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Correlation of Parallel and Transverse Transport in Langmuir Turbulence

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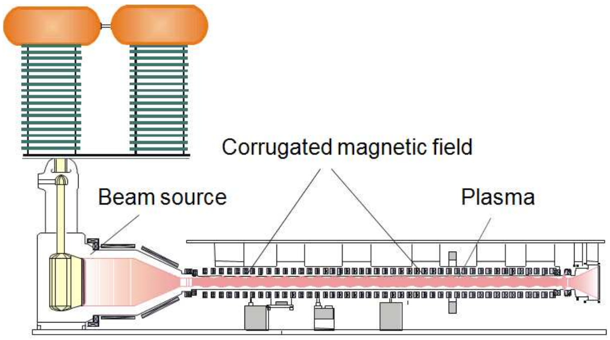
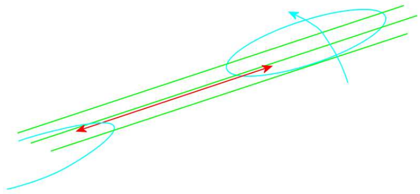
The Langmuir turbulence driven by relativistic electron beam is the main mechanism of plasma heating in the GOL-3 multiple-mirror trap. The typical beam parameters are: $1MeV$, $50kA$, $8\mu s$, $300kJ$, while the plasma parameters can be as high as $T_e \sim 3keV$, $T_i \sim 2keV$, $n \sim 10^{15}cm^{-3}$, at $B = 4.8/3.2T$, $L = 12m$, $L/\ell = 55$.



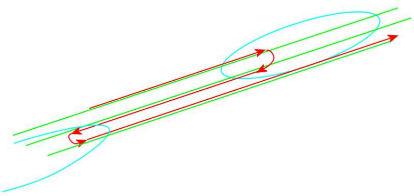
It was observed that besides heating, the turbulence reduced the parallel (to the magnetic field) heat transport by thermal electrons by a factor of $10^3 ... 10^5$, depending on the level of turbulence [1]. This is of course very favorable for the parallel heat confinement, which has always been the weak spot of all open traps. However, the additional collective scattering of electrons, provided by the turbulence, is sure to increase the transverse heat transport just as it reduces the parallel one. The aim of the present work is to estimate the cost of improving the parallel confinement in terms of enhancement of the transverse losses. The main difficulty here is that very little is known about the mechanism of interaction of such turbulence with thermal electrons. Indeed, the beam-generated waves have the phase velocity close to the speed of light, so that the thermal electrons can either be in resonance with only a far tail of the turbulent spectrum, or interact with some large-scale perturbations in a non-resonant way.

Three different models of collective scattering of thermal electrons are considered. These are: reflection from high-amplitude but rare perturbations, non-elastic scattering by low-frequency wavelets, and resonant interaction with Langmuir waves. It appears that in all cases the product of the parallel and the transverse heat conductivities is independent of the level of turbulence, so that $\sqrt{\chi_\parallel \chi_\perp} = \alpha D_{Bohm}$, where D_{Bohm} is the Bohm diffusion coefficient, $1 < \alpha < 10^3$, and α depends only on the scattering model. Note that for the Coulomb scattering $\sqrt{\chi_\parallel \chi_\perp} \approx 2\rho_e v_{Te} \approx 30 D_{Bohm}$, and it is also independent of the collision frequency. The result is that the degradation of the transverse confinement is significant but not too high.

Reflections. Any viable model of scattering should be able to explain the 10^3-times enhancement of parallel resistivity, which is observed in experiments, at reasonable levels of turbulent energy, $\bar{W}/nT < 0.4$. Assume drastic but rare scattering events, such as reflections in parallel velocity. They can occur if there are few but very strong cavern-like perturbations, which provide Miller-potential barriers for thermal electrons. In the absence of drifts and with stationary caverns all cold electrons would be trapped between such "caverns" and carry no current or heat flux along the field. The "caverns" may decay or move away, while the electrons may drift out of the local traps in the transverse direction. This leads to diffusion.



Particle path, $\eta \ll 1$



Particle path, $\eta \sim 1$

Introduce the volume fraction occupied by "caverns":

$$N(J) \equiv \pi\lambda_\perp^2\lambda_\parallel n_\varphi \ll 1, \tag{1}$$

the characteristic time of interaction, $t_e = m_e v_\parallel \lambda_\parallel / e\hat{\varphi}$, and the transverse drift displacement during interaction

$$\delta \sim v_E t_e = c\frac{\hat{\varphi}}{\lambda_\perp B}\frac{m_e v_\parallel}{e\hat{\varphi}}\lambda_\parallel = \rho_e \frac{\lambda_\parallel}{\lambda_\perp}. \tag{2}$$

Note that for each reversal of the parallel velocity there is a step of transverse diffusion, which is independent of the perturbation amplitude but depends on its geometry. The dimensionless ratio

$$\eta = \frac{\delta}{\lambda_\perp} \sim \frac{\rho_e\lambda_\parallel}{\lambda_\perp^2} \tag{3}$$

determines how large is the transverse displacement of the test electron compared to the cavern size. (The "cavern" parameters for GOL-3 are: $\lambda_\parallel \sim c/\omega_p \sim 1mm$, $\lambda_\perp \sim \rho_i \sim 0.1mm$, and hence $\eta \sim 1$. [2]) The estimate for the parallel conductivity is

$$\chi_\parallel = \frac{\lambda_\parallel v_{Te}}{N}\left[1 + \left(\frac{\lambda_\parallel}{N v_{Te}\tau_\varphi} + \eta^2\right)^{-1}\right]^{-1}, \tag{4}$$

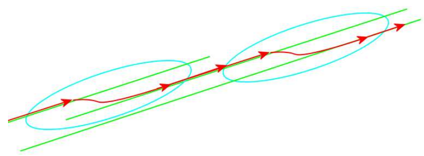
where τ_φ is the life-time of a cavern. Meanwhile, the transverse heat diffusion depends only on time between interactions:

$$\chi_\perp = \frac{\delta_{eff}^2}{l_\varphi / v_\parallel} = N\rho_e^2 \frac{\lambda_\parallel v_{Te}}{\lambda_\perp^2 (1+\eta)^2}. \tag{5}$$

In the most typical case $\eta \lesssim 1$, $\tau_\varphi \gg N\lambda_\parallel / v_{Te}$ the product of the two diffusion coefficients does not depend on N or $\hat{\varphi}$,

$$\sqrt{\chi_\perp\chi_\parallel} \approx \rho_e v_{Te} \frac{\lambda_\parallel}{\lambda_\perp (1+\eta)} = \lambda_\perp v_{Te} \frac{\eta}{(1+\eta)}. \tag{6}$$

Inelastic collisions. It is very hard to observe rare events such as "caverns", and it is possible that they do not exist at all. Still, there will be wave packets of Langmuir waves around anyway. Their amplitude will be moderate and a typical particle will pass them. However, there will be more of them, $N \lesssim 1$. If the parallel interactions were elastic, there would be no significant suppression of the parallel diffusion, while the transverse heat diffusion would be present. However, interaction with the Miller force is inelastic, since it depends on the frequency and is subject to the Doppler shift.



Inelastic collisions

The time of interaction with each wavelet $\tau_k = \lambda_\parallel / v_\parallel$, determines the mean transverse displacement $\delta r = cE_\perp / B\tau_k$. The change of the parallel velocity is proportional to the degree of inelasticity, $\iota = \delta\varepsilon / e\hat{\varphi} \sim v/c \ll 1$. Then, $\delta v_\parallel = \iota e\hat{\varphi}/mv_\parallel$.

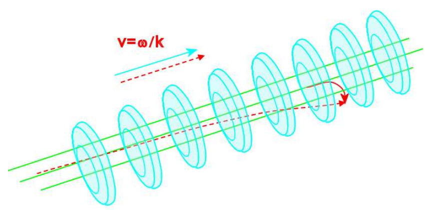
As a result,

$$\chi_\perp = \frac{N\delta r^2}{\tau_k} = N\frac{\lambda_\parallel}{v_{Te}}\left(\frac{c\hat{\varphi}}{\lambda_\perp B}\right)^2, \qquad D_v = N\iota^2\frac{v_\parallel}{\lambda_\parallel}\left(\frac{e\hat{\varphi}}{mv_\parallel}\right)^2, \tag{7}$$

$$\chi_\parallel = \frac{v_\parallel^4}{D_v} = \frac{\lambda_\parallel v_{Te}^3}{N\iota^2}\left(\frac{e\hat{\varphi}}{mv_{Te}}\right)^{-2}, \qquad \sqrt{\chi_\parallel\chi_\perp} = \rho_e v_{Te}\frac{\lambda_\parallel}{\iota\lambda_\perp}. \tag{8}$$

In this case the transverse transport is relatively larger (by a factor of $\sim c/v_{Te}$) for the same value of suppression of the parallel heat diffusion.

Interaction with resonant waves. Assume that the particle moves in the field of a resonant electrostatic perturbation, $E_\parallel$. Then, due to finite transverse coherence, there is also the transverse field component, $E_\perp \sim E_\parallel \lambda_\parallel / \lambda_\perp$. If this is the case, then on each length interval δl there will be the change of the parallel velocity and the transverse particle drift simultaneously: $\delta v_\parallel = eE_\parallel \delta l / mv_\parallel$,



Resonant interaction

$$\delta r = c\frac{E_\perp}{B}\frac{\delta l}{v_\parallel} = \frac{E_\perp}{E_\parallel}\frac{\delta v_\parallel}{\omega_{ce}} = \frac{\lambda_\parallel \rho_e}{\lambda_\perp}\frac{\delta v_\parallel}{v_\parallel} = \eta\lambda_\perp\frac{\delta v_\parallel}{v_\parallel}. \tag{9}$$

Decorrelation with the wave affects both processes simultaneously, so that the diffusion coefficients are also proportional to each other:

$$\chi_\perp = \frac{\delta r^2}{\tau} = \left(\frac{\eta\lambda_\perp}{v_\parallel}\right)^2 D_v. \tag{10}$$

Diffusion in the parallel velocity reverses its sign in $\tau \sim v_\|^2/D_v$, so that the mean free path is $l_\| = \tau v_\| = v_\|^3/D_v$, and the parallel heat conductivity looks like

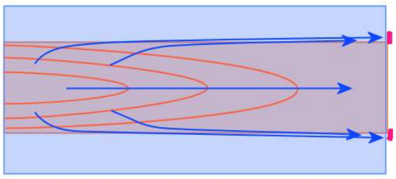
$$\chi_\| = \frac{v_\|^4}{D_v} = \frac{(\eta\lambda_\perp v_\|)^2}{\chi_\perp}, \qquad \sqrt{\chi_\|\chi_\perp} \approx \lambda_\perp v_{Te}\eta. \tag{11}$$

As before, *the product of diffusion coefficients is independent of the level of turbulence.* The difference with the previous models is in the definition of $\lambda_\|$. In the resonant case it is just the parallel wavelength. For the Langmuir waves $\lambda_\| = r_d$, so that η and $\sqrt{\chi_\|\chi_\perp}$ will be one or two orders of magnitude lower than in the first model!

$$\sqrt{\chi_\|\chi_\perp} = \lambda_\perp v_{Te}\frac{\rho_e r_d}{\lambda_\perp^2} \sim 10^{-2}\lambda_\perp v_{Te}. \tag{12}$$

This means that the resonant scattering is generally better for the transverse confinement.

Heat flux in the GOL-3 device. The current density of the beam, and hence the level of turbulence depend on the radius. As the result, the edge layer will exhibit the classical type of heat conductivity, while the ratio of the transverse to the parallel heat diffusion in the central area will be much higher. If $\chi_\| \sim 10^{-3}\chi_{\|Cl}$, then $\chi_\perp \sim \alpha^2\chi_{\perp Cl}\cdot(10^3/30^2)$! The model of resonant interaction leads to lowest possible enhancement of the transverse diffusion, namely, at this level of turbulence it will not be much higher than the collisional diffusion, since $\alpha \sim 1$.



Heat flux in the beam shadow.

The enhanced radial heat flux is converted into the parallel heat flux at the edge of the beam shadow. It means that the total resulting heat flux in thermal electrons goes to the end wall as a hollow ring surrounding the beam imprint. The heat flux to the tube wall remains almost unchanged.

Conclusion. Estimates based on dynamics of test particles show that for different models of turbulence $\sqrt{\chi_\|\chi_\perp} = \alpha D_{Bohm}$, where $1 < \alpha < 10^3$.

Turbulent suppression of the parallel heat flux by a factor of up to 10^3 is compatible with moderately good transverse confinement.

References

[1] V.T.Astrelin, A.V. Burdakov, V.V.Postupaev, *Plasma Phys. Reports*, 24(5), 414 (1998).

[2] A.V. Burdakov, private communication.