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Understanding non-diffusive transport in gyro-kinetic simulations of electrostatic ITG turbulence in tokamak geometry

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It is widely accepted that the rate at which particles, energy or any passive quantity are transported by turbulence can be significantly lowered in the presence of a perpendicular sheared flow. This is a situation of particularly relevance in tokamak plasmas, in which both externally-driven and self-consistently generated (by turbulence itself) sheared poloidal mean flows seem to be central to the formation of radial transport barriers. The reduction of turbulent fluxes along the direction perpendicular to the flow is ultimately related to the fact that the flow can somehow either reduce the amplitude of the fluctuations of the advecting and/or advected fields, or alter the cross-phase between them (or both) [1]. The way in which the flow achieves this suppression is still not well understood. However, it is customary to quantify it in practice by using reduced perpendicular diffusivities/conductivities. We have examined here whether this practice is appropriate and found that it fails to capture correctly the dynamics of transport across sheared flows [2]. These results may have implications not only for the theoretical understanding of the suppression, but for modeling purposes as well.

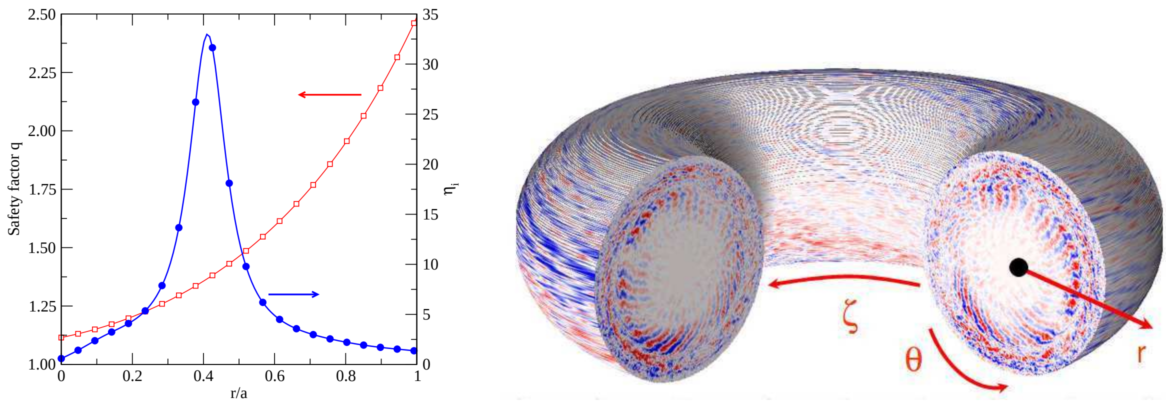

Fig 1.-: (left) η_i and safety factor (q) radial profiles used in the simulations; (right) geometry, coordinate definitions and snapshot of temperature fluctuation contours.

We have carried out simulations of electrostatic ITG turbulence in toroidal geometry with the global, δf-gyrokinetic PIC code UCAN [3]. UCAN evolves the non-Maxwellian part of the ion distribution function of a collisionless deuterium plasma confined by a toroidal magnetic field, coupled to the Poisson equation for the electrostatic potential ϕ. Externally-driven mean flows can be easily included by adding an external electric potential. The geometry and the safety factor and η_i (kept fixed in the simulations) profiles used are shown in Fig. 1. Other physical parameters are $a=0.56$ m, $R_0=2.5$ m, $B_0=1.87$ T, $n_0=3.1x10^{19}$ m^{-3}, $T_e=T_i=0.7$ KeV. These values have been used in the past to simulate discharges of the DIII-D tokamak in a different context. The radial resolution used is such that ~*200* ion Larmor radii are included; the temporal resolution is *0.15* μs, much smaller than the local decorrelation time (*3-10* μs). Since UCAN is a PIC code which pushes kinetic ions along their gyro-averaged orbits, we use these orbits to compute the ion radial propagator: $P(r, t \mid r_0, t_0)$. This propagator represents the probability of finding one ion at some radius r and time t, if it was located at some another radius r_0 at a time $t_0 \leq t$. Its shape can be used to characterize transport dynamics. We use it here to diagnose the nature of radial transport in the presence of sheared poloidal mean flows. To understand how the method works, we review some results from stochastic transport theory. Diffusion is a macroscopic reflection of Brownian motion. This can be shown by assuming that the microscopic motion is governed by a Langevin equation:

$$x(t) = x_0 + \int_0^t \xi_2(t')dt', \qquad \text{with } \langle \xi_2(t)\xi_2(t) \rangle = 2D\,\delta(t-t'). \tag{1}$$

ξ_2 is a Gaussian, uncorrelated noise. The macroscopic transport equation for the particle density that is derived from Eq. (1) is then the standard diffusive equation: $\partial_t n = D\partial_x^2 n$. The propagator for Eq. (1) is also easily found to be [G(x) is a Gaussian law with unit variance]:

$$P(x,t|x_o,t_0) = G\left[(x-x_0)/\sqrt{Dt}\right] \tag{2}$$

That is, if motion is diffusive, the propagator is a Gaussian law with variance growing as $\sigma^2 \sim t$. In order to test whether the nature of transport is diffusive or not, we can compare the measured propagator along any direction of interest with Eq. (2). In our case, we will construct radial ion propagators using the kinetic ion orbits and use them to probe the nature of radial transport in the presence of sheared poloidal mean flows. We have run three different UCAN simulations using the aforementioned parameters but with different flow implementations. In the first case, denoted by **SCF**, the ITG modes drive nonlinearly a poloidal (and toroidal) sheared flow which, after the initial linear phase, acts back on the turbulence until saturation is reached. In the second case, denoted by **NoF**, the back-reaction of the driven mean flow on the fluctuations is neglected by artificially suppressing the angular (poloidal and toroidal) average of the electrostatic potential. In the third case, denoted by **EXT**, the average self-consistent mean flow is also removed, but it is substituted by an external flow with strength equal to the time-average (over the saturated phase) of the self-consistent mean flow. Time traces of the electrostatic energies for all cases are shown in Fig. 2(left). The propagators are constructed using ions close to the radius of stronger turbulence at an initial time t_0 chosen well within the saturated phase (marked in Fig. 2(left) with arrows).

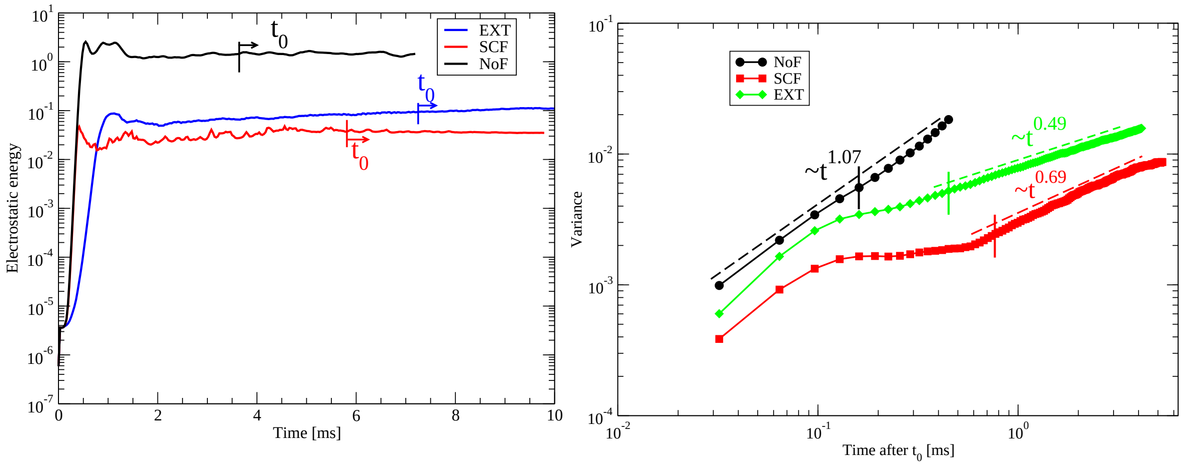

Fig 2.- (Left) Electrostatic energies for the three UCAN runs; (right) variance of propagators after t_0.

The results are discussed next. The variances of the propagators are shown in Fig. 2(right). Snapshots of the propagators at different times (after t_0) are shown in Fig. 3. Clearly, the variance only increases linearly with time if there is no sheared mean flow. In the cases with a sheared poloidal mean flow (either self-consistent or external), the variance grows sub-linearly, which points to sub-diffusive motion (after an initial phase dominated by the bounce-averaged banana motion [2]). The propagator shape is found to be Gaussian in the absence of the mean flow (see Fig. 3(left)). This result, combined with the linear increase of the variance reveals that radial transport is diffusive in this case. In the case of the externally driven mean flow, the propagator is still Gaussian, but the sub-linear scaling of the variance rules out diffusive dynamics. The self-consistently driven flow case is even more interesting, since the propagator exhibits a fat-tailed distribution law which decays with exponent *~-2.4*. To interpret these results, it must be considered that sub-diffusion is usually observed when finite typical transport scales are absent. In this situation, transport is not describable in terms of diffusion. A Langevin description may still be possible, but it must be generalized to:

$$x(t) = x_0 + \frac{1}{\Gamma(H+1/\alpha-1)}\int_0^t (t-t')^{H-1/\alpha}\,\xi_\alpha(t')\,dt' . \qquad (3)$$

Here, α varies between *0* and *2*, and *H* between 0 and 1. For $\alpha = 2$ and $H = 1/2$, Eq. (2) is recovered and transport is then diffusive. However, if $H \neq 1/2$, Eq. (3) describes fractional Brownian motion [4], and the propagator is still Gaussian but with a variance increasing as $\sigma^2 \sim t^{2H}$. Finally, if $\alpha < 2$, the uncorrelated noise is not Gaussian, but distributed as a Levy law with an algebraic tail of exponent $-(1+\alpha)$. The resulting motion, known as fractional Levy motion, has a propagator which is a Levy law of the same exponent. In an infinite system its variance diverges. But in a finite system, it can be computed and grows as $\sigma^2 \sim t^{2H}$.

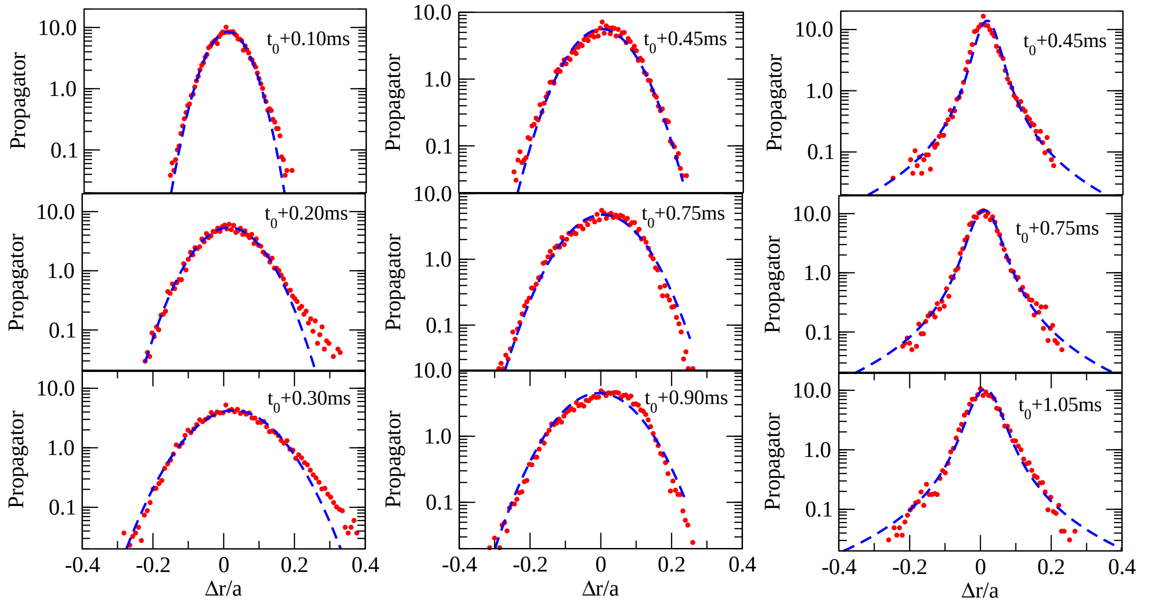

Fig 3.- Radial ion propagators (and best fits in dashed blue) at different times for: (left) NF case; (middle) EXT case; (right) SC case. NF and EXT fits are Gaussian. SC fits are Levy laws with $\alpha \sim 1.4$.

Clearly, our simulations show that radial turbulent transport becomes sub-diffusive when a sheared poloidal mean flow is present, driven either externally (*H~0.25*) or self-consistently by the turbulence (*H~0.35*). In the last case, the change is even more dramatic, becoming also non-Gaussian ($\alpha \sim 1.4$). Although the value of the exponents would probably be set by the shear and flow amplitudes, our results suggest that effective diffusivities are indeed inadequate to capture the dynamics and thus to model transport across sheared flows.

References

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