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Gyrokinetic-Water-Bag modelling of low-frequency instabilities in a laboratory magnetized plasma column

R. Klein, P. Morel, N. Besse, E. Gravier, P. Bertrand

Laboratoire PMIA, UMR 7040 CNRS - Nancy Université, F-54500 NANCY, France

It is now widely believed that low frequency turbulence developing from small scale instabilities is responsible for the phenomenon of anomalous transport generally observed in magnetic confinement fusion experiments. Among these microinstabilities, drift waves and ITG (Ion Temperature Gradient) instabilities may play an important role in explaining the anomalous heat and particle transport observed in tokamaks. On the other hand, small scale linear devices, together with numerical simulations, can play an important role in understanding basic plasma processes. Solving 3D fluid equations is the most convenient way to compute the plasma response to the pertubed electromagnetic field when wave-particle interactions are neglected. However, for some devices a fluid description usually overestimates turbulent fluxes because the resonant wave-particle interactions can not be fully described with fluid equations.

A drastic improvement is given by the gyrokinetic theory [1, 2] for ions (ion-neutral collisions are neglected). Recently, an alternative approach has been proposed to solve the Vlasov equation, based on a water bag representation of the distribution function which is not an approximation but rather a special class of initial conditions allowing to reduce the full kinetic Vlasov equation into a set of hydrodynamic equations while keeping its kinetic character [3, 4]. Finite Larmor Radius (FLR) effects are taken into account by averaging over the gyromotion of ions and introducing the polarization drift. A fluid description is used for electrons [5], taking into account the collisions with the neutrals. We propose thus a kinetic model (called CGWB, Collisional Gyro Water Bag model in the rest of the paper), able to describe both the collisional drift waves and the ITG instabilities, taking into account the possible interactions between waves and ions. The goal of this work is to investigate the low frequency instabilities in a laboratory magnetized plasma column using the CGWB model.

In cylindrical geometry, with $\mathbf{B} = B\mathbf{u}_z$, the ion Vlasov equation takes the following form :

$$\frac{\partial f}{\partial t} - \frac{1}{rB}\frac{\partial \langle\phi\rangle}{\partial \theta}\frac{\partial f}{\partial r} + \frac{1}{rB}\frac{\partial \langle\phi\rangle}{\partial r}\frac{\partial f}{\partial \theta} - \frac{q}{m_i}\frac{\partial \langle\phi\rangle}{\partial z}\frac{\partial f}{\partial v_\parallel} + v_\parallel\frac{\partial f}{\partial z} = 0 \qquad (1)$$

and the quasi-neutrality equation can be written in the following way :

$$n_e = Z_i\left[\langle n_i\rangle + \nabla_\perp\cdot\left(\frac{n_i}{\omega_{Ci}B}\nabla_\perp\phi\right)\right] \qquad (2)$$

where $\langle . \rangle$ represents the gyro-averaged operator.

A linear study of the CGWB model provides the following linear dispersion relation:

$$\frac{\omega^* + i\gamma_{\parallel}}{\omega - \omega_0 + i\gamma_{\parallel}} + \delta - J_0^2 \sum_{j=1}^{N} \alpha_j \frac{Z_i \omega_{cs}^2 + \omega_j^* \omega}{\omega^2 - k_{\parallel}^2 a_j^2} = 0 = \varepsilon(\omega) \tag{3}$$

where

$$\delta = Z_i \rho_s^2 \left(\kappa(r) + k_\theta^2\right) \tag{4}$$

and $\omega_{cs}^2 = k_{\parallel}^2 c_s^2$, J_0 is the gyroaverage operator, $Z_i^* = Z_i/\tau$ with $\tau = T_i/T_e$, $c_s = \sqrt{KT_e/m_i}$, $\rho_s = c_s/\omega_{Ci}$, $k_\theta = m/r$, m is the azimuthal mode, and :

$$\kappa(r) = -\left[\frac{\partial^2 g}{\partial r^2} + \left(\frac{\partial g}{\partial r}\right)^2 + \left(\frac{\partial \ln n_0}{\partial r} + \frac{1}{r}\right)\frac{\partial g}{\partial r}\right] \tag{5}$$

where $g(r)$ depends on the radial profile of the potential:

$$\phi_{m\omega}(r) = \phi_{0_{m\omega}} \exp(g(r)) \tag{6}$$

a_j are the velocities at the equilibrium ($v_j^{\pm} = \pm a_j$, f_0 is an even function of v at the equilibrium).

$$\omega^* = -\frac{KT_e}{eB} k_\theta \partial_r \ln n_{e_0} \tag{7}$$

$$\gamma_{\parallel} = \frac{k_{\parallel}^2 KT_e}{m_e \nu_{en}} \tag{8}$$

$$\omega_0 = k_{\parallel} v_{\parallel 0} \tag{9}$$

where ω^* is the electron diamagnetic frequency, $k_{\parallel}$ is the parallel wave number and $v_{\parallel_0}$ is the drift of the electrons in the z-direction. Electron-neutral collisions are destabilizing while ion-neutral collisions are stabilizing. Thus neglecting ion-neutral collisions (for strong magnetic field) is equivalent to neglect their stabilizing effect.

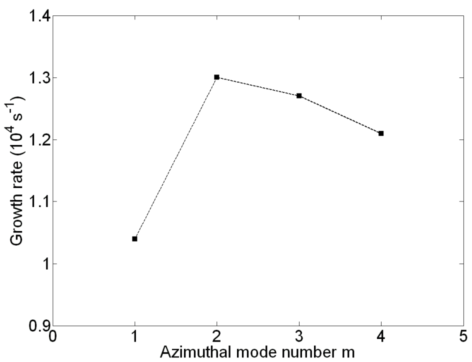

Figure 1: Instability growth rate versus azimuthal mode number for collisional drift waves, with parameters from [5].

First a comparison between results obtained by both CGWB and non-local fluid model given reference [5] are provided. Only one bag is chosen for the CGWB model, so that the resulting set of equations is equivalent to the fluid model. The density profile is assumed to be gaussian. The CGWB results are shown Fig. 1. The linear growth rate of the instability is plotted against the azimuthal mode m. We see a maximum rate for $m = 2$ equal to $\gamma = 1.3 \times 10^4 s^{-1}$. These results given by the CGWB model are in complete agreement with the values predicted by the non-local cylindrical fluid model.

We next validate the CGWB model by comparing the linear analytical investigation of collisional drift waves and the experimental results obtained from the laboratory magnetized plasma column Mirabelle [6]. The ion temperature is approximatively equal to 0.03 eV (no temperature gradient). The equation (3) with only one bag exhibits results shown in Fig. 2. We can see that the transition from $m = 2$ to $m = 1$ is well described for $v_{\|0}$ going from $2v_{Te}$ to $10v_{Te}$. This behavior is in a very good agreement with the experimental result, i.e. the mode is getting lower with an increasing electron parallel drift.

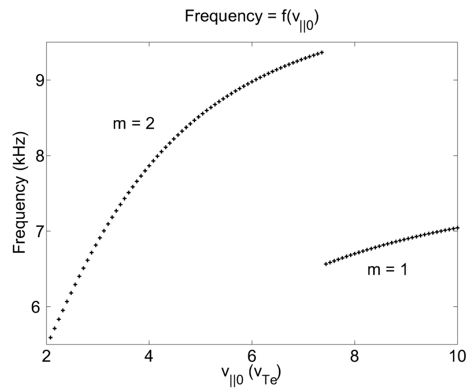

Figure 2: Frequency of the more unstable mode plotted against the parallel electron drift $v_{\|0}$, with FLR effects, and with one bag ($N = 1$).

The goal in this part is now to study the kinetic effects on the collisional drift waves using a multiple water-bag description of the ion distribution function. Here we only consider an increase of the ion temperature (without any temperature gradient), and consequently its kinetic effects on the collisional drift waves. Results are shown Fig. 3. The instability growth rate is plotted against the mode m, with $T_i = 2$ eV which corresponds to a ratio $v_{Ti}/v_\varphi = 0.43$, where v_φ is the phase velocity. Then wave-particle interaction effects can be expected. Two curves are presented. The first one with only one bag is the result of a model equivalent to a fluid one, and indeed is not able to take into account the kinetic effects. The second one with $N = 20$ bags, $v_{max} = 5v_{T_i}$, is equivalent to a kinetic model. We note that the instability growth rate clearly decreases with an increasing bag number. This result confirms that the kinetic phenomena play a stabilizing role when the thermal velocity is close to the phase velocity.

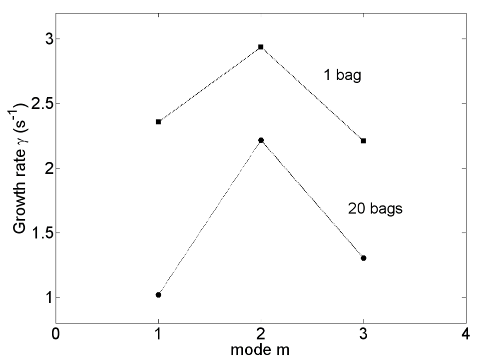

Figure 3: Instability growth rate plotted against the mode m, in the one bag case (equivalent to a fluid model), and in the 20 bag case (equivalent to a kinetic model), with $v_{\|0} = v_{Te}$ and $T_i = 2$ eV.

As a matter of fact it is convenient to compare our theoretical CGWB model with the CLM experimental results [7]. The parameter $\eta = \kappa_T/\kappa_n = \partial_r \ln T_i / \partial_r \ln n_{e_0}$ has to exceed a critical value to observe an ITG (Ion Temperature Gradient) instability. This parameter is increased by both flattening the density gradient and increasing the ion-temperature gradient. The CGWB model shows that the $m = 2$ mode is always dominant for parameters given in [7], with a real frequency in the range 2-11 kHz. Furthermore the perturbation propagates in the ion diamagnetic direction as expected for ITG instabilities. This ITG mode is confirmed in the CLM device where a $m = 2$ mode is obtained with a finite parallel wavelength, and the mode propagates azimuthally in the ion diamagnetic drift direction. The real frequency of the mode is in the 6-10 kHz range, which is in fairly good agreement with the CGWB predictions.

Finally we have investigated the transition from the collisional drift waves to the ITG instability when the parameter $\kappa_T = \partial_r \ln T_i$ increases. The CGWB results are shown Fig. 4. For $|\kappa_T|$ in the range 0 - 42 m^{-1}, the linear growth rate is about 10^3 s^{-1} and corresponds to collisional drift waves. ITG instability occurs when $|\kappa_T|$ exceeds the critical value $|\kappa_T| = 42$ m^{-1}, for which the ITG growth rate is greater than that of drift waves. Moreover, the perturbation propagates in the electron diamagnetic drift direction as expected for drift waves, or in the ion diamagnetic drift direction as expected for ITG instabilities.

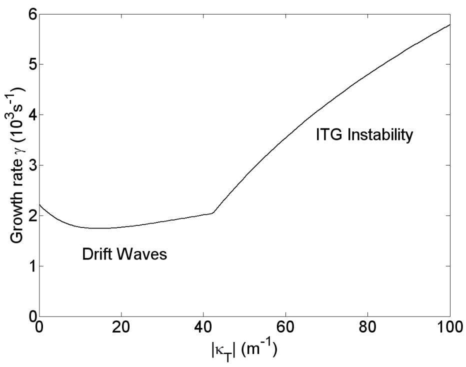

Figure 4: Transition from collisional drift waves to ITG instability. The growth rate of the more unstable mode is plotted against the parameter $\kappa_T = \partial_r \ln T_i$.

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