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Water bag modelling of a multi-species plasma.

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The water bag model has been recently adapted to the context of cylindrical magnetized plasmas [1]. Such a representation gives an interesting connexion between fluid and kinetic models, using the conservation property of the Vlasov equation to provide a multi-fluid-like set of equations. The multi-water-bag distribution function takes a step-like form along the parallel (to the magnetic field) velocity direction, and dynamics of the system is entirely described by the evolution of water bag velocity contours. The present work consists in a generalization of the multi-water-bag description to the case of a multi-species plasma. As a first step, we assume cylindrical geometry, adiabatic electrons and a kinetic description of each ion species. With the aim of describing various populations (Carbon, Tungsten), our model keeps a fully kinetic moment-sense description of each ion species. A parametric linear study of Ion Temperature Gradient instability has been achieved, considering a Deuterium plasma mixed with different impurity populations.

The water bag model

Initially introduced by De Packh [2] and Hohl [3], the water bag model was generalized to the multi-water-bag one [4, 5]. The main idea of this description is to use the Liouville's conservation in phase space to reduce of one dimension a given problem. The water bag description is well adapted to problems with a unique velocity component. It consists in assuming a multi-step-like distribution function along the velocity component. The values of such a function are constant. The dynamics of the plasma is entirely supported by the velocity contours of each uncompressible "water bag" [6]. According to the gyrokinetic ordering and assumptions, whose essential result is to reduce the velocity space to the parallel velocity direction (in the sense that perpendicular velocity is not an independent variable), the concept of water bag has been recently extended to gyro-water-bag model [1]. By inserting a multi-water-bag distribution function into the set of cylindrical gyrokinetic equations, we get gyro-water-bag equations [7]:

$$\partial_t v^{\pm}_{\mu j} + (\mathscr{J}_\mu \mathbf{v_E}).\nabla_\perp v^{\pm}_{\mu j} + v^{\pm}_{\mu j}\partial_z v^{\pm}_{\mu j} = \frac{q_i}{M_i}\mathscr{J}_\mu E_\parallel \quad (1)$$

$$Z_i n_i + Z_i \nabla_\perp . \left(\frac{n_i}{B\Omega_{ci}}\nabla_\perp \phi\right) = n_{e0}\left(1 + \frac{e\phi}{T_e}\right) \quad (2)$$

where $\mu = M_i v_\perp^2/(2B)$ is the magnetic momentum (adiabatically invariant), $\mathscr{J}_\mu$ denotes the gyroaverage operator, $\mathbf{v_E} = \mathbf{E}\times\mathbf{B}/B^2$ is the electric drift. q_i and M_i are respectively mass and charge of an ion (i) population described with the water-bag contours $v_{\mu j}^{\pm}$. The ion density n_i is defined relatively to the water bag parameters $n_i = \sum_\mu \sum_{j=1}^{N} A_{\mu j} \mathscr{J}_\mu (v_{\mu j}^+ - v_{\mu j}^-)$. We do use the following assumptions:

- ⋆ Adiabatic electrons with small potential energy $e\phi << T_e$.
- ⋆ Constant magnetic field.
- ⋆ Typical lengths greater than the Debye one: quasi-neutral plasma.

Water bag equations for a multi-species plasma

To describe a multi-species plasma, we introduce a water-bag-like distribution function for each species we consider:

$$f_{s\mu}(\mathbf{r}, v_\parallel, t) = \sum_{j=1}^{N_{s\mu}} A_{s\mu j} \left[\Upsilon(v_\parallel - v_{s\mu j}^-) - \Upsilon(v_\parallel - v_{s\mu j}^+)\right] \quad (3)$$

The water bag parameters are chosen with moment-sense equivalences [1] relatively to continuous maxwellian equilibrium distribution functions. The relative thickness of each one is linked to the mass number through the thermal velocity $v_{Ts}^2 = T_s/M_s$. In cylindrical geometry, we get the multi-species multi-water-bag set of equations:

$$\partial_t v_{s\mu j}^{\pm} + (\mathscr{J}_{s\mu}\mathbf{v_E}).\nabla_\perp v_{s\mu j}^{\pm} + v_{s\mu j}^{\pm}\partial_z v_{s\mu j}^{\pm} = \frac{q_s}{M_s}\mathscr{J}_{s\mu} E_\parallel \quad (4)$$

$$\sum_s \left[Z_s n_s + Z_s \nabla_\perp.\left(\frac{n_s}{B\Omega_{cs}}\nabla_\perp \phi\right)\right] = n_{e0}\left(1 + \frac{e\phi}{T_e}\right), \quad (5)$$

where each ion density n_s is defined relatively to the water bag parameters:

$$n_s = \sum_\mu \sum_{j=1}^{N} A_{s\mu j} \mathscr{J}_{s\mu}(v_{s\mu j}^+ - v_{s\mu j}^-) \quad (6)$$

In order to obtain the linear stability threshold of a multi-species plasma, we assumed small perturbations around an homogeneous (θ, z), symmetric relatively to parallel velocity equilibrium, without any equilibrium electric field:

$$v_{s\mu j}^{\pm}(\mathbf{r}, t) = \pm a_{s\mu j}(\mathbf{r}) + w_{s\mu j}^{\pm}(r) e^{i(m\theta + k_\parallel z - \omega t)} \quad (7)$$

$$\phi(\mathbf{r}, t) = 0 + \delta\phi(r) e^{i(m\theta + k_\parallel z - \omega t)} \quad (8)$$

By neglecting any second order term in perturbation, we get the linearized gyro-water-bag set of equations:

$$(\omega \mp k_\parallel a_j) w^{\pm}_{s\mu j} = \left[k_\parallel \frac{q_s}{M_s} \mp \frac{k_\theta}{B} d_r a_{s\mu j} \right] \mathscr{J}_{s\mu} \phi \quad (9)$$

$$\sum_s Z_s \left[\delta n_s + \nabla_\perp . \left(\frac{n_{s0}}{B\Omega_{cs}} \nabla_\perp \phi \right) \right] = \frac{e n_{e0}}{T_e} \delta\phi, \quad (10)$$

where the perturbed density is given by the perturbed velocity contours:

$$\delta n_s = \sum_\mu \sum_{j=1}^{N_{s\mu}} A_{s\mu j} \mathscr{J}_{s\mu} (w^+_{s\mu j} - w^-_{s\mu j}) \quad (11)$$

The electric potential is assumed to take the form $\delta\phi(r) = \delta\phi_0 e^{g(r)}$. Now we introduce $\kappa(r) = d_r^2 g + (d_r g)^2 + d_r g/r$. In eliminating water bag velocity contours $w^{\pm}_{s\mu j}$, we finally obtain the multi-species plasma dielectric function:

$$\varepsilon(\omega) = 1 - \sum_s Z_s^\star \frac{Z_s n_{s0}}{n_{e0}} v^2_{Ts} \left[\frac{\kappa(r) + \kappa_{ns} d_r g - k_\theta^2}{\Omega^2_{cs}} + \mathscr{J}^2_{s\mu} \sum_{j=1}^{N_{s\mu}} \alpha_{s\mu j} \frac{k_\parallel^2 - \omega k_\theta \kappa_{s\mu j}/\Omega_{cs}}{\omega^2 - k_\parallel^2 a^2_{s\mu j}} \right], \quad (12)$$

where the typical water bag parameters $\alpha_{s\mu j} = 2A_{s\mu j} a_{s\mu j}/n_{s\mu 0}$, $\kappa_{s\mu j} = d_r a_{s\mu j}/a_{s\mu j}$ are determinate using a moment sense equivalence. Especially, we assume radial dependencies $\alpha_{s\mu j}\kappa_{s\mu j} = \beta_{s\mu j}\kappa_{Ts\mu}/2 + \gamma_{s\mu j}\kappa_{ns\mu}$. The linear stability threshold is given by a parametric approach relatively to ω, solving the conditions $\varepsilon(\omega) = 0$ and $d_\omega \varepsilon(\omega) = 0$. In the case of two ion species, such a threshold is a surface in a four dimensions space (both temperature and density gradients), so do we use assumptions we will discuss in the following, to finally obtain relevant results.

ITG stability and dilution effects

In the case of a single ion species plasma, the ITG instability occurs without including finite Larmor radius effects. As a first step, we neglect them in the following analysis. The temperature radial profiles are assumed to be equal for both main and impurities populations. Dilution effects are discussed in the following. An important and quite open issue in the conception of the ITER reactor is the one about the first components in the wall. The best candidates are Carbon and Tungsten, so do we study here, as an example, their influence on the stability of a mainly Deuterium plasma. We report in Fig. (1) the stability diagrams in the case of a D-C (up) or a D-W (down) two-species plasma. The control parameter is the relative density of impurities at a radial given point of the plasma column. In order to consider only dilution effects, we assume here a flat radial density profile of impurities.

We observe in Fig. (1) a global destabilizing effect of the plasma with an increasing relative ratio of impurities. Such an effect could strongly affect the well-known $\eta \to 2$ branch of the stability diagrams. It occurs for low values of the density of impurities (1% for C, $0,1\%$ W). The intensity of destabilization is clearly proportional to the mass and charge of impurities. In the case of the Tungsten population, we can notice that the stability threshold exhibits a bi-lobe-like structure. Such a fact could corroborate the idea to depict such an impurity population with only one bag. It is due the large mass ratio between W and D, which implies a very thick distribution function of Tungsten.

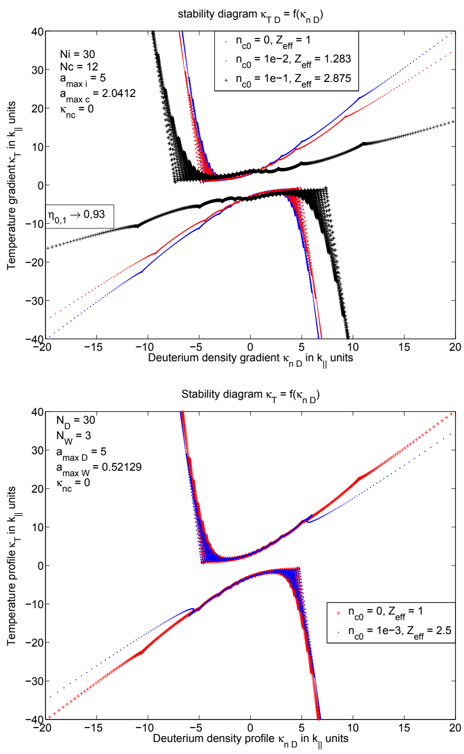

Figure 1: Comparison between Carbon and Tungsten impacts on the ITG stability of a Deuterium plasma (κ_{nD}, κ_{TD}) plane in $k_{||}$ units.

Conclusion

This study has pointed out the ability of the giro water bag model to describing a multi-species plasma. Thanks to the linear analysis, we are able to obtain an analytical threshold of the ITG instability. Some interesting results have been obtained. The density ratios are strongly related to the ion mass and charge numbers. A destabilizing effect has been shown in the case of a flat density profile.

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