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Low-dimensional convection dynamics in the Helimak configuration

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For the last three decades, chaotic dynamics in laboratory plasma has been studied for many different plasma configurations (see, e.g. [1, 2, 3]) and is still a very active research area. In this short article, we analyze the data from the Blaaman device operated in the Helimak configuration, i.e. a plasma confined by toroidal magnetic field with a weak vertical magnetic component superposed. We study time series of plasma potential and electron density, measured by Langmuir probes and sampled at a frequency of 100 kHz. In every location in a 1×1 cm^2 grid in the torus cross section 1×10^5 samples were collected by a probe and simultaneously by a reference probe located in the midde of the density slope on the low-field side.

In [4] data of this type were used to demonstrate the presence of unstable flute modes convected with the vertical plasma flow. These modes can be considered as a helical convection roll elongated along the helical magnetic field.

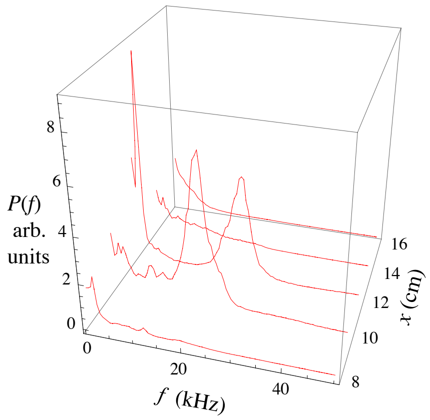

Figure 1: Power spectra.

There is a close analogy between the Rayleigh-Bénard (RB) instability in a fluid and the flute interchange instability in the magnetized plasmas, and it is well known that the RB instability can develop low-dimensional and chaotic dynamics. In [5] the analogy between RB and flute interchange instability was explored through the diffusionless Lorenz model for the flute interchange instability,

$$\begin{aligned} x' &= -x-y \\ y' &= -zx \\ z' &= xy+R, \end{aligned} \tag{1}$$

where x and y represent the potential and density amplitude of the flute mode, and z measures the strength of the electron pressure profile gradient driving the instability. The parameter R represents the strength of the plasma source that maintains the pressure gradient. Since the plasma (and the convection rolls) have a (vertical) drift perpendicular to the (horizontal) pressure gradient they will create a Doppler shifted spectral peak in the probe signals which is not described by these equations. In Fig. 1 we plot the power spectral density for the plasma potential in several different horizontal positions in the plasma. Two distinctive features are present in the spectra: one is localized on the low frequencies and represents oscillations of the profile, while the other one corresponds to the Doppler frequency of the propagating convection cells. The results for the electron density fluctuations are very similar.

In Fig. 2 (a), a conditional average technique (see [6]) is employed to map the average space-time evolution of the electron density fluctuations, and flute modes which move vertically (along the y-axis) are easy to recognize. In order to study the dynamics of the background profile we apply a moving average filter to extract the low-frequency component of the plasma potential and electron density.

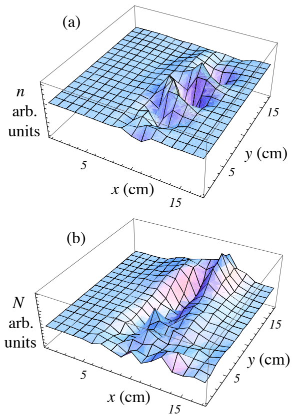

Figure 2: Conditionally averaged structures.

In Fig. 2(b) we show the conditionally averaged low-pass filtered electron density. In the animation of the same structure, one can see standing-wave like motion along the radial direction (x-axis), with a rapid loss of phase information. This irregular phase shift might be a symptom of chaotic dynamics. In order to characterize our system quantitatively, we first estimate the Lyapunov exponent for the electron density fluctuations, which is shown in Fig. 3 together with the solution of the Lorenz model for the flute interchange instability for $R = 3.3$. The Lyapunov exponents for both experimental (low-pass filtered electron density) and synthetical data converge to $\lambda \sim 4 \times 10^3\ \mathrm{s}^{-1}$, which is consistent with the low frequency spectral peak broadening in Fig. 1. Although the Lyapunov exponent is finite and positive, which is a characteristic of chaotic systems, it was pointed out in [7] that stochastic time series can also give positive Lyapunov exponents when calculated numerically. Because of this, we apply recurrence plot analysis (see [8, 9]) which is a powerful tool for visualization of the recurrences of phase-space trajectories.

Prior to constructing a recurrence plot the phase space is reconstructed by time-delay embedding [10], where vectors $\mathbf{x}_i$ $(i = 1, \dots, T)$ are produced. Then a $T \times T$ matrix consisting of elements 0 and 1 is constructed. The matrix element (i, j) is 1 if the distance is $\|\mathbf{x}_i - \mathbf{x}_j\| \leq r$ in the reconstructed phase space, and otherwise it is 0. The recurrence plot is simply a plot where the points (i, j) for which the corresponding matrix element is 1 is marked by a dot. The radius r is fixed and chosen

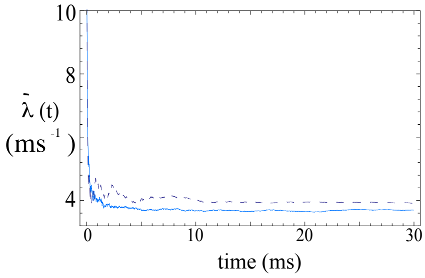

Figure 3: Lyapunov exponent.

such that a sufficient number of points are found to reveal the fine structure of the plot. If the recurrence plot displays lines parallel to the main diagonal, recurrences of the trajectories in the phase space occur. If these lines are long and continuous the dynamics is periodic. Short diagonal lines indicate a chaotic state where trajectories recur for a short time and then diverge exponentially again. Recurrence plots for the low-pass filtered electron density fluctuations is shown in Fig. 4 and display a structure indicative of chaotic dynamics. Finally, we need to estimate dimensionality of the chaotic attractor obtained from the low-pass filtered plasma potential and electron density. In this case, we apply the mean-field dimensional method [11], which has proven effective in identifying low-dimensionality in experimental time series. This method has an advantage over the standard correlation dimension method [12], since it estimates the attractor dimension of the averaged phase space and thereby reduces the effect of the noise which dominates the small scales. This method is based on the time-delay reconstruction [10], where both plasma potential and electron density data are reconstructed. In this way, we exploit all the available data and obtain more accurate phase-space reconstruction. The aim of the method is to give us the probability density function $P(D)$, which shows the minimum embedding dimension of the average phase space that approximates the actual dynamics. We have shown in [13] that the minimum embedding dimension was $D \sim 7$. According to Takens embedding theorem [10], in the absence of noise, the embedding dimension D is approximately equal to $2C+1$, where C is attractor dimension. This further implies that $C \leq 3$ for our plasma configuration.

In this short paper, we have shown that the data obtained from the Helimak device are low-dimensional and chaotic. The attractor dimension obtained from the plasma potential and electron density fluctuation is consistent with the number of equations which constitute the Lorenz model for the flute interchange instability. This has encouraged us to compare the diffusionless Lorenz model (with driving parameter $R = 3.3$) and the low-pass filtered electron density fluctuations, and we have obtained that both of them exhibit Lyapunov exponent of the same magnitude. Comparison of their power spectra as well as recurrence plots are presented in [14] and also indicate similar dynamics. From the analysis done, we conclude that the diffusionless Lorenz model with a parameter $R \sim 3$ could be a valid representation of the dynamics present in the

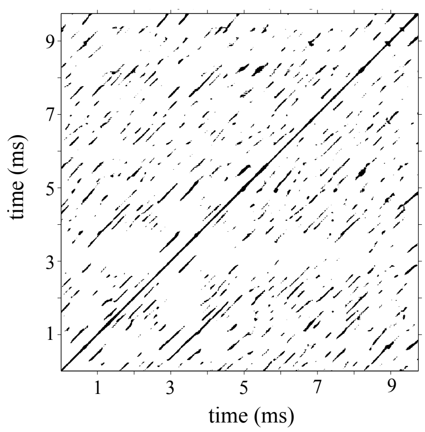

Figure 4: Recurrence plot.

Helimak plasma. Of course, the choice of $R = 3.3$ was done because the solution fits the experimental data well, and the comparison can only give an indication that a dynamical model of this structure can provide an adequate description of the profile-transport dynamics in the Helimak configuration.

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