

On the quasilinear transport fluxes driven by microinstabilities in tokamaks

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Experimental results suggest that the particle transport in tokamaks, which is usually dominated by ITG and TEM driven turbulence, depends on the collisionality [1]. In the collisionless limit, numerical simulations of ITG-mode driven turbulence give an inward particle flux, both in fluid, gyrofluid and gyrokinetic descriptions. The inward flow is mainly caused by magnetic curvature and thermodiffusion. However, nonlinear gyrokinetic calculations show that even a amount of finite collisionality strongly affects the magnitude and sign of the particle flux [2]. The inward particle flow obtained in the collisionless limit is rapidly converted to outward flow as electron-ion collisions are included. The present paper addresses the collisionality dependence of the quasilinear flux due to ITG and TE-modes.

Perturbed electron density response

We consider an axisymmetric, large aspect ratio torus with circular magnetic surfaces. The nonadiabatic part of the perturbed distribution function of species a is given by the linearized gyrokinetic equation, [3]

$$\frac{v_{\parallel}}{qR} \frac{\partial g_a}{\partial \theta} - i(\omega - \omega_{Da})g_a - C_a(g_a) = -i \frac{e_a f_{a0}}{T_a} (\omega - \omega_{*a}^T) \phi J_0(z_a), \quad (1)$$

where θ is the extended poloidal angle, ϕ is the perturbed electrostatic potential, f_{a0} is the equilibrium Maxwellian distribution function, $\omega_{Da} = -k_{\theta} \left(v_{\perp}^2/2 + v_{\parallel}^2 \right) (\cos \theta + s\theta \sin \theta) / \omega_{ca} R$ is the magnetic drift frequency, $\omega_{*a}^T = \omega_{*a} [1 + (w/T_a - 3/2) \eta_a]$, η_a is the ratio of the density and temperature scale lengths, $w = m_a v^2/2$ is the kinetic energy, ω_{*a} is the diamagnetic frequency, $\omega_{ca} = e_a B / m_a$ is the cyclotron frequency, J_0 is the Bessel function of order zero and $z_a = k_{\perp} v_{\perp} / \omega_{ca}$, and the rest of the notation is standard.

The collisions are modeled by a pitch-angle scattering operator $C_e = v_e(v) 2\xi / B \partial_{\lambda} \xi \lambda \partial_{\lambda}$, where $v_e(v) = v_T / x^3$, $x = v / v_{Te}$, $\xi = v_{\parallel} / v$ and $\lambda = \mu / w$ with $\mu = m_a v_{\perp}^2 / 2B$. If the electron distribution is expanded as $g_e = g_{e0} + g_{e1} + \dots$ in the smallness of the normalized collisionality $v_{*e} = v_e / \varepsilon \omega_b \ll 1$ and $\omega / \omega_b \ll 1$, where ω_b is the bounce frequency, then in lowest order we have $\partial_{\theta} g_{e0} = 0$. The circulating electrons can assumed to be adiabatic, while in the trapped

region g_{e0} is given by the constraint obtained from the next-order equation

$$(\omega - \langle \omega_{De} \rangle) g_{e0} - \frac{i v_e}{\varepsilon \hat{\tau}_B} \frac{\partial}{\partial \kappa} \hat{J}(\kappa) \frac{\partial g_{e0}}{\partial \kappa} = - \frac{e \langle \phi \rangle}{T_e} (\omega - \omega_{*e}^T) f_{e0}, \quad (2)$$

where $\langle \dots \rangle$ is the orbit average, $\langle \omega_{De} \rangle = \omega_{D0} [E/K - 1/2 + (2rq'/q)(E/K + \kappa - 1)]$, with $\omega_{D0} = -k_\theta v^2 / \omega_{ce} R$, $\hat{J} = E(\kappa) + (\kappa - 1)K(\kappa)$, $\hat{\tau}_B = K(\kappa)$, and E and K are the complete elliptic integrals with the argument $\kappa = (1 - \lambda B_0(1 - \varepsilon)) / (2\varepsilon \lambda B_0)$.

We introduce a parameter $\hat{v} \equiv v_e / (\omega_0 \varepsilon)$, where $\omega_0 = \omega / y$, $y = \sigma + i\hat{\gamma}$, $\sigma = \text{sign}(\Re\{\omega\})$ denotes the sign of the real part of the eigenfrequency and $\hat{\gamma} = \gamma / \omega_0$ is the normalized growth rate. The equation for g_{e0} is

$$\hat{v} (g_{e0}'' + (\ln \hat{J})' g_{e0}') + i \frac{K}{\hat{J}} \left(y - \frac{\langle \omega_{De} \rangle}{\omega_0} \right) g_{e0} = i \frac{SK}{\omega_0 \hat{J}}, \quad (3)$$

where $S = - \frac{e \langle \phi \rangle}{T_e} (\omega - \omega_{*e}^T) f_{e0}$. The perturbed electrostatic potential is approximated by $\phi(\theta) = \phi_0 (1 + \cos \theta) / 2 [H(\theta + \pi) - H(\theta - \pi)]$, where H is the Heaviside function and then $\langle \phi \rangle = \phi_0 E(\kappa) / K(\kappa)$.

In the limit of weak collisionality, $\hat{v} \ll 1$, the WKB solution of the homogeneous part of the equation can be derived. To make further progress analytically we need to approximate the elliptic functions with their asymptotic limits for small arguments, as was done in Ref. [4, 5]. Having calculated the inhomogeneous solution by means of variation of parameters, using the weak collisionality assumption for further simplification, taking the boundary conditions from regularity and continuity properties we can obtain an approximate solution for the perturbed distribution [5]. The above analysis is not valid in a small boundary layer at $\kappa \simeq 1$. The effect of the boundary layer reduces the collisional term, with a factor $\pi/2 \sqrt{\log \hat{v}^{-1/2}}$, which is less than 20% in practice, and in the following analysis this will be neglected.

Quasilinear particle fluxes

The non-adiabatic part of the perturbed trapped electron density can be obtained by taking an integral in velocity space that can be evaluated in terms of ${}_2F_0$ hypergeometric functions. In order to derive simple expressions for the quasilinear particle fluxes, it is instructive to expand in the limit of small $\hat{\omega}_D = \omega_{D0} / \omega_0$ and small $\hat{\gamma}$, keeping terms to the first order. Introducing $\hat{\omega}_{Dt} = \hat{\omega}_D / x^2$, $\hat{\omega}_{*e} = \omega_{*e} / \omega_0$ and $\hat{v}_t = \hat{v} x^3$, we obtain the quasilinear particle flux due to ITG

modes

$$\begin{aligned} \Gamma_e^{\text{ITG}} = \frac{np_e}{2e} \left| \frac{e\phi_0}{T_e} \right|^2 & \left\{ -\sqrt{\frac{\varepsilon}{2}} \left[1 - \frac{3\hat{\omega}_{Dt}}{2}(1 + \eta_e) \right] \hat{\gamma}\hat{\omega}_{*e} + \sqrt{\frac{\varepsilon}{2}} \frac{3\hat{\omega}_{Dt}\hat{\gamma}}{4} \right. \\ & + \frac{\Gamma(3/4)\sqrt{\varepsilon}\hat{v}_t}{\sqrt{2\pi}} \left\{ -1 + \frac{\hat{\gamma}}{2} + \left(\frac{3}{4}\eta_e - 1 \right) \left(1 - \frac{3\hat{\gamma}}{2} \right) \hat{\omega}_{*e} \right. \\ & \left. \left. + \frac{9\hat{\omega}_{Dt}}{16} \left[1 - \frac{3\hat{\gamma}}{2} + \left(1 + \frac{\eta_e}{4} \right) \left(1 - \frac{5\hat{\gamma}}{2} \right) \hat{\omega}_{*e} \right] \right\} \right\}. \end{aligned} \quad (4)$$

In the absence of collisions, the flux is inwards if the curvature and thermodiffusive fluxes dominate over diffusion. If collisions are included, the particle flux may be reversed, if the part of the flux that is dependent on the collisionality is positive. This reversal happens for instance for $\hat{\omega}_{Dt} = 0.6$ and $\eta_e = 3$, see Fig. 1/a (red, dotted curve).

However, if the ITG-instability growth rate is weak ($\hat{\gamma} \ll 1$) and η_e is large enough, the situation is completely different. Figure 1/c shows the normalized quasilinear electron flux for the same parameters as in Fig. 1/a, but for $\hat{\gamma} = 0.1$, representing a case close to marginal stability. The term proportional to the $\sqrt{\hat{v}_t}$ will change sign and now this will also lead to an inward flux. If the magnetic drift is high enough to give an inward flux for zero collisionality, then collisions will enhance this and the flux will therefore never be reversed. If the magnetic drift is very small, the flux is outwards for $\hat{v}_t = 0$. Then collisions may reverse the sign of the flux, but now from outwards to inwards.

The quasilinear flux driven by TEM modes is

$$\begin{aligned} \Gamma_e^{\text{TEM}} = \frac{np_e}{2e} \left| \frac{e\phi_0}{T_e} \right|^2 & \left\{ -\sqrt{\frac{\varepsilon}{2}} \left[1 + \frac{3\hat{\omega}_{Dt}}{2}(1 + \eta_e) \right] \hat{\gamma}\hat{\omega}_{*e} + \sqrt{\frac{\varepsilon}{2}} \frac{3\hat{\omega}_{Dt}\hat{\gamma}}{4} \right. \\ & + \frac{\Gamma(3/4)\sqrt{\varepsilon}\hat{v}_t}{\sqrt{2\pi}} \left\{ 1 - \frac{\hat{\gamma}}{2} + \left(\frac{3}{4}\eta_e - 1 \right) \left(1 - \frac{3\hat{\gamma}}{2} \right) \hat{\omega}_{*e} \right. \\ & \left. \left. + \frac{9\hat{\omega}_{Dt}}{16} \left[1 - \left(1 + \frac{\eta_e}{4} \right) \left(1 - \frac{5\hat{\gamma}}{2} \right) \hat{\omega}_{*e} \right] \right\} \right\}. \end{aligned} \quad (5)$$

There are two main differences compared with the ITG driven flux. First, the part of the flux that is driven by the curvature has opposite sign compared with ITG, and therefore contributes to the outward flux instead of driving an inward pinch. Second, the part of the flux that arises due to collisions is different and may have opposite sign compared with the ITG case, depending on parameters. Figure 1/b shows the normalized quasilinear flux for three different $\hat{\omega}_D$ if the plasma is far from marginal stability ($\hat{\gamma} = 0.7$) and Fig. 1/d shows the same for $\hat{\gamma} = 0.1$.

Starting from the gyrokinetic equation for the electrons but instead modeling the collisions by an energy-dependent Krook operator gives $g_{e0} = -\frac{e\phi_0}{T_e} \frac{\omega - \omega_{*e}^T}{\omega - \langle \omega_{De} \rangle + i\nu_{\text{eff}}} f_{e0}$, where $\nu_{\text{eff}} = \nu_T/\varepsilon x^3$.

The velocity-space integral of the perturbed electron distribution can be used to determine the quasilinear flux. If the plasma is far from marginal stability, the results for the pitch-angle scattering and Krook operator are qualitatively same, as shown in the upper figures of Fig. 1. However, as the lower figures in Fig. 1 show, as we approach marginal stability, the form of the collision operator matters more and more (mainly for ITG), and both the sign and the magnitude of the flux may be very different.

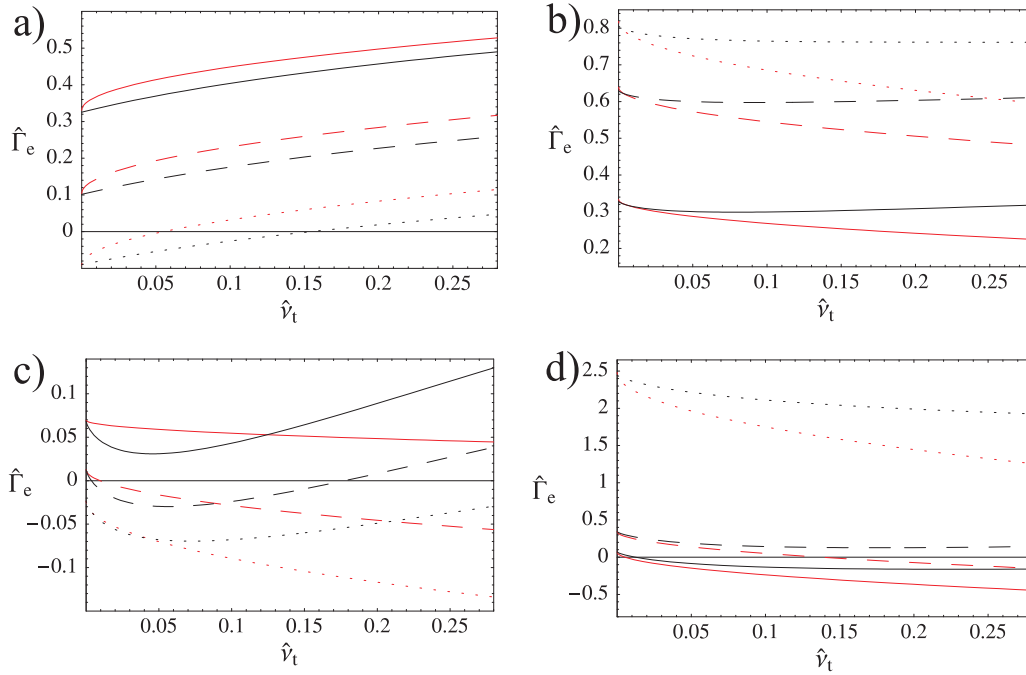


Figure 1: Normalized quasilinear electron flux as function of normalized collisionality for $\hat{\omega}_{*e} = 1$, $\eta_e = 3$. ITG (a, c), TEM (b, d). Red curves: pitch-angle scattering operator, black curves: Krook operator. $\hat{\omega}_{Dt}$ is 0 (solid), 0.2 (dashed), 0.6 (dotted). $\hat{\nu}_t$ is 0.7 (a, b), 0.1 (c, d)

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