Self-organization of electromagnetic plasma edge turbulence

G. Fuhr1,2, S. Benkadda1,2, P. Beyer1,2, X. Garbet3, I. Sandberg4, H. Isliker5

1 France-Japan Magnetic Fusion Laboratory, LIA 336 CNRS
2 LPIIM, UMR 6633 CNRS-Université de Provence, Marseille, France
3 IRFM, Association EURATOM – CEA, CEA Cadarache, France
4 National Technical University of Athens, Association Euratom–Hellenic Republic, Department of Electrical and Computer Engineering GR-157 73 Athens, Greece
5 Department of Physics, Aristotle University of Thessaloniki, GR-541 24 Thessaloniki, Greece

Abstract. Experimental measurements of fluctuation levels on typical fusion devices reveal that magnetic perturbations are typically much smaller than electrostatic perturbations. However, as even small magnetic fluctuations (∼10^-4) can locally modify the topology of the magnetic surfaces, they play an important role with respect to the transport properties of the plasma. Here, we show results from numerical simulations of resistive ballooning turbulence in toroidal geometry. These simulations have been performed with EMEDGE3D, a three dimensional global code which calculates the evolution of the pressure and the electrostatic potential at the plasma edge and also includes self-consistent electromagnetic fluctuations. The competitive mechanisms (e.g. Reynolds stress, Maxwell stress) which are responsible for the flow generation in turbulent plasmas are investigated and the scaling laws for the turbulent transport are determined in terms of the value of plasma $\beta$ which also determines the strength of the magnetic fluctuations.

In hot magnetized plasmas, the cross-field transport is dominated by the presence of instabilities which give rise to both, electrostatic and magnetic fluctuations. Experimental measurements of fluctuation levels on typical fusion devices reveal that magnetic perturbations are typically much smaller than electrostatic perturbations. However, as even small magnetic fluctuations (∼10^-4) can locally modify the topology of the magnetic surfaces, they play an important role with respect to the transport properties of the plasma.

Electromagnetic plasma edge turbulence is simulated here using a resistive ballooning model. The latter is based on fluid equations for the normalized electrostatic potential $\phi$, electromagnetic flux $\psi$ and pressure $p$ [3].

\[
\partial_t \nabla^2_\perp \phi + \{\phi, \nabla^2_\perp \phi\} = -\frac{1}{\alpha_N} \nabla_\parallel \nabla^2_\perp \psi - G p + v \nabla^4_\perp \phi, \quad (1)
\]

\[
\partial_t p + \{\phi, p\} = \delta \cdot \nabla_\parallel p + \chi_\parallel \nabla^2_\parallel p + \chi_\perp \nabla^2_\perp p + S, \quad (2)
\]

\[
\partial_t \psi = -\nabla_\parallel \phi + \frac{1}{\alpha_N} \nabla^2_\perp \psi. \quad (3)
\]

Equation (1) corresponds to the normalized charge balance, Eq. (2) is the normalized energy...
balance and Eq. (3) corresponds to the Ohm’s Law. $\nabla_\perp$ and $\nabla_\parallel$ respectively correspond to the parallel and perpendicular gradients along field lines. $G$ is the curvature operator, $\nu$ represents the viscosity, $\chi_\parallel$ and $\chi_\perp$ are the parallel and perpendicular thermal diffusivities respectively. Time is normalized by the resistive interchange time $\tau_{int} = \sqrt{R_0 L_p/2}/c_S$, where $c_S$ is the sound speed, $R_0$ is the major radius and $L_p$ is the pressure gradient length. The perpendicular and parallel length scales are the resistive ballooning length $\xi_{bal} = \sqrt{\rho \eta_\parallel/\tau_{int} L_s/B_0}$ and the magnetic shear length $L_s$, respectively. Here, $\rho$ is the mass density and $\eta_\parallel$ the parallel resistivity. $q(r)$ stands for the safety factor which measures the magnetic field line pitch. $\alpha_N$ is expressed by $\alpha = q^2 \alpha_N$ where $\alpha$ is the normalized pressure gradient typically used to express the stability limit for ideal ballooning modes, $\alpha \simeq 1$ [1]. $\alpha_N$ is also related to the $\beta$ parameter (the ratio of kinetic pressure to the magnetic pressure) through $\alpha_N = \beta L_s^2/(R_0 L_p)$. The parameter $\delta_c$ is defined by $\delta_c = 2\Gamma L_p/R_0$, $\Gamma$ is the heat capacity ratio ($\Gamma = 5/3$). The last term of the r.h.s. of Eq. (2), $S(r)$ represents a constant energy source.

Magnetic flux surfaces are modeled by a set of concentric circular torii, where the coordinates $(r, \theta, \varphi)$ correspond to the minor radius, the poloidal and toroidal angles respectively. The Poisson bracket is $\{f,g\} = r^{-1}(\partial_r f \partial_\theta g - \partial_r g \partial_\theta f)$, the curvature operator is $G = \sin \theta \partial_r + \cos \theta r^{-1} \partial_\theta$, and the gradient along the field lines is given by $V_\parallel = R^{-1}(\partial_\varphi - 1/q \partial_\theta) - 1/B_0 \{\psi, \cdot \}$. Assuming a monotonically increasing safety factor $q(r)$, the simulations cover a domain between $q = 2$ and $q = 3$ in the vicinity of a reference surface $r_0$ at the edge. The values for the parameters used here are $r_0/\xi_{bal} = 500, R_0/L_s = 1, \delta_c = 0.04$ and $\nu = \chi_\perp = 2$. In these simulations, the parallel diffusivity is set to 0.

Fig. 1 shows the time evolution of the edge confinement time $\tau_{conf}$, the energy associated with the mean electrostatic potential $E_\Phi$ as well as the energy associated with electromagnetic fluctuations $E_\Psi$ for different values of the $\alpha_N$ parameter, ranging from $\alpha_N = 0.01$ to $\alpha_N = 0.3$. Here, the edge confinement time is defined as

$$\tau_{conf} = \frac{1}{\Gamma_{tot}} \int_V p(r, \theta, \varphi, t) \, dV,$$

where $\Gamma_{tot} = \int_V S(r) \, dV$ is the constant energy flux coming from the plasma core and $V$ is the
simulated volume at the plasma edge. The energies $E_{\phi}$ and $E_{\psi}$ are defined by

$$E_{\phi}(t) = \sqrt{\int_R |\phi(r,t)|^2 \, dr} \quad E_{\psi}(t) = \sqrt{\int_V |\psi(r,\theta,\phi,t)|^2 \, dV}$$

(4)

A strong impact of the $\alpha_N$ parameter on the confinement time is observed. In a statistically stationary state, the level of $\tau_{\text{conf}}$ is lower by nearly 40% in the case of the highest value of $\alpha_N$ compared to the one with the lowest value of $\alpha_N$. The level of magnetic fluctuations is found to increase with increasing $\alpha_N$ (Fig. 1, right column). However, as the parallel diffusivity is set to zero in these simulations ($\chi_\parallel = 0$), the magnetic perturbations do not contribute to the evolution of the pressure [i.e. $\Gamma_{\delta B} = 0$ in Eq. (6) presented below] and therefore the decrease of $\tau_{\text{conf}}$ with $\alpha_N$ can not be attributed directly to the increase of the magnetic fluctuations and a different mechanism has to be invoked. We therefore study the energy associated with the mean electric potential, $E_{\phi}$, which is linked via the ExB drift to a mean poloidal rotation of the plasma. As can be seen from Fig. 1, middle column, the level of $E_{\phi}$ in the stationary state is decreasing with $\alpha_N$ and therefore the mean plasma rotation is decreasing. As this rotation typically has a stabilizing effect on the turbulence via a shearing of convective cells, the decrease of $E_{\phi}$ is expected to lead to a higher turbulence level and therefore a lower confinement time.

The competing mechanism for the generation of the mean poloidal rotation can be studied by averaging poloidally and toroidally the equation for the electrostatic potential, Eq. (1). The corresponding equation of the poloidal flow is :

$$\partial_t \bar{v}_\theta = -\partial_r (T_R + T_M + T_V) - \langle Gp \rangle_{\theta \phi},$$

(5)

where the notation $<>_{\theta \phi}$ stands for the average in the poloidal and toroidal directions; $\bar{v}_\theta = \langle v_\theta \rangle_{\theta \phi}$ is the flow profile and $\bar{v}_{r,\theta} = v_{r,\theta} - \bar{v}_{r,\theta}$ is the fluctuating part of the radial (respectively poloidal) velocity, $\bar{\psi}$ is the fluctuating part of the magnetic flux. The Reynolds stress $T_R = \langle \bar{v}_\theta \bar{v}_r \rangle_{\theta \phi}$ and the Maxwell tensor $T_M = \langle \partial_\theta \bar{\psi} \partial_r \bar{\psi} \rangle_{\theta \phi}$ are represented by the first two terms of Eq. (5). $T_V$ corresponds to the viscosity effects.

Following the equation for the pressure profile,

$$\partial_t \bar{p} = -\partial_r (\Gamma_{\text{conv}} + \Gamma_{\text{coll}} + \Gamma_{\delta B}) + \delta_c \langle G\phi \rangle_{\theta \phi} + S,$$

(6)

the total energy flux is decomposed into three components : the convective flux $\Gamma_{\text{conv}} = \langle \bar{p} \bar{v}_r \rangle_{\theta \phi}$, the collisional damping $\Gamma_{\text{coll}} = -\chi_\perp \partial_r \bar{p}$ and the flux due to the parallel diffusivity $\Gamma_{\delta B} = -\chi_\parallel \langle \partial_\theta \bar{\psi} \nabla \parallel \bar{p} \rangle_{\theta \phi}$.
To compare the relative importance of the stresses $T_R$ and $T_M$ in the transport dynamics, we study the time average of the norm of each of these tensors, defined by

$$< F^2 >_t = \sqrt{\left< \int_R |F(r,t)|^2 \, dr \right>_t},$$  \hspace{1cm} (7)

The results are plotted in Fig. 2. Both, Maxwell tensor and Reynolds stress are found to increase with $\alpha_N$. However, the ratio between $< T_M^2 >_t$ and $< T_R^2 >_t$ decreases with $\alpha_N$ such that for low values of $\alpha_N$, the Reynolds stress is dominant, and for $\alpha_N \geq 0.1$, Reynolds and Maxwell stresses have similar amplitudes. As the Reynolds stress is known to drive the poloidal flow [Eq. 5], we conclude that the decrease of the poloidal flow energy with increasing $\alpha_N$ is linked to a competition between Reynolds and Maxwell stresses.

In conclusion, turbulence simulations of a tokamak edge plasma have been realized focusing on the impact of electromagnetic fluctuations on transport and confinement time. The role of the $\alpha_N$ parameter has been analyzed by a study of the different stresses present in the equation for the poloidal plasma flow. A modification of the relative importance of Reynolds stress and Maxwell tensor depending on the value of $\alpha_N$ has been observed, leading to a decrease of the confinement time with increasing $\alpha_N$.

References

