Towards an improved first principle based transport model

A. Casati\textsuperscript{1}, C. Bourdelle\textsuperscript{1}, X. Garbet\textsuperscript{1}, F. Imbeaux\textsuperscript{1},
J. Candy\textsuperscript{2}, F. Clairet\textsuperscript{1}, G. Dif-Pradalier\textsuperscript{1}, G. Falchetto\textsuperscript{1}, T. Gerbaud\textsuperscript{1}, V. Grandgirard\textsuperscript{1},
P. Hennequin\textsuperscript{3}, R. Sabot\textsuperscript{1}, Y. Sarazin\textsuperscript{1}, L. Vermare\textsuperscript{3} and R. Waltz\textsuperscript{2}

\textsuperscript{1}Association EURATOM-CEA, CEA/DSM/IRFM, CEA-Cadarache St Paul-Lez-Durance 13108, France
\textsuperscript{2}General Atomics, P.O. Box 85608, San Diego, California 92186-5608, USA
\textsuperscript{3}Laboratoire de Physique et Technologie des Plasmas, CNRS-Ecole Polytechnique,
91128 Palaiseau Cedex, France

In order to gain reliable predictions on turbulent fluxes in tokamak plasmas, physically comprehensive transport models have to be improved. Unfortunately nonlinear gyrokinetic simulations are still too costly in terms of computing time; on the other hand, the quasi-linear approximation can retain the relevant physics for fairly reproducing both experimental results [1] and nonlinear gyrokinetic simulations [2]. This work aims at improving the recently developed quasi-linear model QuaLiKiz [3], based on a fast linear gyrokinetic code. Three main issues are presented: i) test the validity of the quasi-linear approach using the Eulerian $\delta f$ code GYRO [4] for coupled ITG-TEM turbulence and the semi-Lagrangian full-f code GYSELA [5] for ITG only, ii) optimize the QuaLiKiz assumptions on the saturated electric potential by comparison with turbulence measurements on Tore Supra and nonlinear simulations, iii) successful comparison of both energy and particle diffusivities predicted by GYRO and QuaLiKiz for several tokamak relevant conditions.

1. Validating the fundamentals of quasi-linear approach

The quasi-linear predictions of turbulent fluxes have an intrinsic dependence on the choices adopted for the structure of the saturated turbulent electric field, which is not self-consistently solved in the quasi-linear approximation. Hence, the validation of the quasi-linear approach has to be done apart from any hypothesis on the saturation spectrum. Therefore we study the radial $\mathbf{E} \times \mathbf{B}$ turbulent flux from the k mode $\Gamma_k = \langle \delta n_k \delta n^*_{E_k B_k} \rangle \cong \langle i k_\rho c / B \delta n_k \delta n^*_{E_k B_k} \rangle$ (example of particle flux, brackets stand for a statistical average). The flux is proportional to the sinus of the cross-phase $\Delta \Phi_k$ between the fluctuating electric potential and the transported quantity (density or energy fields). Firstly, local (with no radial profiles variations) electrostatic gyrokinetic simulations of coupled ITG-TEM turbulence have been performed with the Eulerian $\delta f$ code GYRO; the probability density function (PDF) of these cross-phases
from each k-mode has been calculated for the nonlinear saturation phase and compared to the linear cross-phases from a GYRO simulation of the linear most unstable mode. Fig. 1a shows a very good agreement between the nonlinear and the linear cross-phases in the plane \( \theta = 0 \), where the interchange instability is supposed to be dominant. In the case of pure ITG turbulence, i.e. with adiabatic electrons, the nonlinear de-phasing between \( \delta p_i \) and \( \delta v_{E,B} \) has been directly studied through global nonlinear full-f gyrokinetic simulations with GYSELA, and compared to the \( \delta p_i - \delta \phi \) de-phasing from local \( \delta f \) GYRO simulations (Fig. 1b). The two codes predict coherent total ion heat fluxes; nevertheless, phase shifts more peaked towards low \( k_\theta \) scales are obtained by the global GYSELA simulations in the plane \( \theta = 0 \) with respect to the local ones by GYRO.

![Cross-phase ion energy vs potential](image1.png)

**Fig 1a:** Local GYRO simulation (kinetic electrons) with \( R/L_T = R/L_e=9.0, R/L_n = 3.0, R/a = 3.0, r/a=0.5, q^i=2.0, s=1.0, \rho^* = 1/400 \) (GASTD case); PDF of the nonlinear \( \Delta \Phi_k \), white line is the linear \( \Delta \Phi_k \).

**Fig 1b:** Local GYRO versus global GYSELA simulation of pure ITG turbulence: \( \sin(\Delta \Phi_k \delta p_i - \delta \phi_k) \) for GYRO and \( \cos(\Delta \Phi_k \delta p_i - \delta \phi_k) \) for GYSELA are shown.

![Cross-phase ion energy vs potential](image2.png)

**Fig 2a:** Local GYRO and global GYSELA simulations (adiabatic electrons) with \( R/L_T = 8.28, R/L_e = 2.78, r/a=0.4, q^i=1.23, s=0.6, \rho^* = 1/256 \). The ratios of the quasi-linear and nonlinear transport weights versus \( k_\theta \), are shown.

**Fig 2b:** \( \langle |\delta \phi_k|^2 \rangle \) saturation spectra from local GYRO and global GYSELA simulations.

The following transport weight has been moreover defined,

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w^{QL,NL}_k = \left\{ \left( \Gamma_k (r, \theta) \right), \left( Q_k (r, \theta) \right) / \left( \delta \phi_k (r, \theta) \right)^2 \right\} \quad (\Gamma_k \ and \ Q_k \ are \ respectively \ the \ particle \ and \ energy \ fluxes, \ while \ brackets \ stand \ for \ a \ flux-surface \ and \ radial \ average), \ such \ that \ this \ quantity \ can \ be \ calculated \ in \ the \ full \ nonlinear \ as \ well \ in \ the \ linear \ regimes. \ The \ ratio \ between \ the \ linear \ and \ the \ nonlinear \ transport \ weights \ has \ been \ studied \ by \ mean \ of \ local \ \delta f \ (with \ GYRO) \ and \ global \ full-f \ (with \ GYSELA) \ gyrokinetic \ simulations. \ Fig. \ 2 \ refers \ to \ pure \ ITG \ turbulence, \ where \ scales \ up \ to \ k_\theta \rho_s = 1.48 \ have \ been \ resolved \ (results \ corresponding \ to \ k_\theta \rho_s > 1.0 \ are \ omitted \ for \ GYSELA \ since \ a \ simplified \ gyro-averaging \ operator \ is \ applied \ on \ these \ ranges). \ Both \ the \ local \ and \ the \ global \ simulations \ agree \ in \ recognizing \ a \ systematic \ over-prediction \ of \ the \ linear \ transport \ with \ respect \ to \ the \ nonlinear \ regime. \ The \ most \ relevant \ evidence \ is \ that \ these \ linear/nonlinear \ ratios \ stay \ reasonably \ constant \ when \ changing \ the
plasma parameters and over the $k_\theta$-spectrum, especially at low $k_\theta$ scales, where the driven transport is maximum. A remarkable agreement is observed between the local GYRO and the global GYSELA $\langle|\delta\phi_{k\theta}|^2\rangle$ saturated spectra, exhibiting a power law shape clearly peaking at low $k_\theta\rho_s \approx 0.2$. Consequently, the following relation for the quasi-linear expectations on all the turbulent fluxes can then be applied \( \Gamma^{0l}, Q^{0l} = C_0 \sum_{i,j} W_{ij}^{0l} \langle |\delta\phi_i|^{2} \rangle^{NL} \) (the index \( i \) eventually stands for the \( i \)-th linear unstable mode): the saturated $|\delta\phi|^{2}$ spectrum is externally imposed from the nonlinear saturated regime and $C_0 \approx 1/1.55$ is the only arbitrary constant needed to properly renormalize the quasi-linear over-prediction to the nonlinear fluxes. Results from Figs. 1, 2 provide then a very relevant justification of the quasi-linear approach, if suitable hypotheses on the saturated $\delta\phi$ spectrum are assumed.

2. $k_\theta$-spectra from experimental measurements and nonlinear simulations

The quasi-linear gyrokinetic transport model QuaLiKiz has been recently improved against linear GYRO simulations of the most unstable mode. Peculiar focus has been devoted to the hypotheses adopted in the model on the saturated $|\delta\phi_{k\theta}|^2$ spectrum. A power law of the type \((k_\theta\rho_s)^\alpha\) is generally able to fit very well both the potential and the density fluctuations $|\delta n_{k\theta}|^2$ spectrum obtained by nonlinear GYRO simulations for $k_\theta > k_{\theta, nl-max}$. A slope $3 < \alpha < 3.5$ has been typically observed, reproducing reasonably well the laser scattering turbulence measurements in the medium $k_\theta$ range observed on Tore Supra plasmas [6] (Fig. 3). On the other hand, the transition towards $\alpha \approx 6$ seen by the measurements at high $k_\theta$ scales has not been reproduced by GYRO nor GYSELA simulations within the explored $k_\theta$ ranges (up to $k_\theta\rho_s \approx 2$): this issue should be deepened in further studies.

In QuaLiKiz, we have consequently assumed a power law $S_k = (k_\theta\rho_s)^3$ spectrum symmetric around a $k_{ql-max}$; $k_{ql-max}$ is chosen in order to maximize the mixing length factor $\gamma_s / \langle k_1^2 \rangle$ on the most unstable mode, where $\langle k_1^2 \rangle = k_\theta^2 \left( 1 + s^2 \langle \theta^2 \rangle \right)$ [2, 3]. All the unstable
modes \( i \) are accounted in the model, and each of them is retained with a different weight factor \( S_k \max \left( \frac{\gamma_{k,i}}{\langle k^2 \rangle_k} \right) \).

3. QuaLiKiz versus GYRO and application to experimental discharges

QuaLiKiz expectations have been compared against a large number of nonlinear GYRO simulations with kinetic electrons, and only one renormalisation factor \( C_0 \) (as defined in section 1) for both particle and energy fluxes has been used in order to get the best fit to the nonlinear fluxes. Fig. 4a shows an example of the ion/electron energy and particle turbulent diffusivities: the scan of the plasma thermal gradients reveals that the quasi-linear approach respects the nonlinear ratios between the different transport channels.

A second example is a direct application to an experimental collisionality \( (\nu^*) \) scan realised on Tore Supra plasmas [7]. The \( \nu^* \) scaling of transport is particularly challenging for quasi-linear models [8]. Indeed, the nonlinear collisional damping of the zonal flows dynamics could enhance turbulence and hence transport, while linearly the opposite effect (transport decrease) is expected, driven by the collisional quenching of TEM. The latter effect is dominant in the range of the experimental parameters here studied, as shown by the decrease of transport with increasing collisionality. Fig. 4b demonstrates that, for realistic values of \( \nu_{ei} \), QuaLiKiz is able to well reproduce the nonlinear diffusivities predicted by comprehensive GYRO simulations (performed with pitch-angle scattering operators on both electrons and ions). Hence the zonal flows dynamics driven by \( \nu^* \) variations plays a subdominant role, whereas the coupled dynamics between ion and electron non-adiabatic responses is crucial for both GYRO and QuaLiKiz, in order to account for the TEM quenching.

References