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Construction of the equilibrium generating function and an area-preserving map for the DIII-D shot 115467 at 3000 ms

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We can express the magnetic field in the form $2\pi B = \nabla\psi\times\nabla\theta + \nabla\varphi\times\nabla\chi$. θ and φ are, respectively, any poloidal and toroidal angle. ψ and χ are the toroidal and poloidal magnetic flux, respectively [1-2]. The poloidal flux, $\chi=\chi(\psi,\theta,\varphi)$, is the Hamiltonian of the magnetic field lines, θ is generalized position, ψ is generalized momentum, and φ is generalized time. We call the canonical variables (θ,ψ) the natural coordinates for magnetic field line. In an axisymmetric field, such as equilibrium field in divertor tokamaks, one can always transform to action-angle coordinates by defining a new toroidal flux Ψ such that $\partial\Psi/\partial\psi=1-\partial\lambda/\partial\theta$ with χ a function of θ alone. The magnetic field then has the representation $2\pi B = \nabla\Psi\times\nabla\theta_m + \nabla\varphi\times\nabla\chi(\Psi)$. The angle $\theta_m=\theta+ \lambda(\theta,\psi)$ is called a magnetic angle. The quantities (Ψ,θ_m) are always ill-behaved functions of position at a separatrix. The action-angle canonical variables (Ψ,θ_m) are called magnetic coordinates. If we define a generating function by $S(x,\theta)=(x^2/2)\tan(\theta)$, then we can give a canonical transformation by $\partial S/\partial x=y$ and $\partial S/\partial\theta=\psi_t/ B_0$, where B_0 is some arbitrary, constant magnetic field, $x=R-R_0$, $y=Z-Z_0$, (R_0,Z_0) is the location of magnetic axis in the cylindrical (R,Z,φ) coordinates. This canonical transformation has the explicit forms $x=\sqrt{(2\psi/B_0)}\cos(\theta)$, $y=\sqrt{(2\psi/B_0)}\sin(\theta)$. The Hamiltonian in the new variables is χ/B_0 expressed as a function of (x,y,θ). We call the canonical variables (x,y) the physical coordinates for magnetic field line. We can also calculate the magnetic coordinates from the physical coordinates. So, generally we can integrate the trajectories of magnetic field lines in any one of these three canonical coordinates – natural (ψ,θ), action-angle or magnetic (Ψ,θ_m), and physical (x,y) or (R,Z). In all three canonical representations, when φ is canonical time, poloidal magnetic flux χ is the Hamiltonian function, expressed as a function of canonical coordinates and canonical time. Each set of canonical coordinates has its own advantages and disadvantages. In action-angle or magnetic coordinates, the equilibrium safety factor $q(\Psi)$ has a logarithmic singularity on the separatrix surface. So an area-preserving map in action-angle coordinates cannot be integrated across separatrix surface. It is generally not possible to invert the action-angle or magnetic coordinates (Ψ,θ_m) to the physical coordinates (x,y) or (R,Z) or to natural coordinates (ψ,θ).

Effort in this paper is: (1) to construct the equilibrium Hamiltonian for magnetic field lines in DIII-D [3] in the canonical natural coordinates (ψ,θ) from the experimental data in the DIII-D, (2) to construct an area-preserving map for trajectories of magnetic field lines in the DIII-D in natural coordinates, and (3) to use this map to calculate the effects of magnetic perturbation in the DIII-D. The first step in our efforts is to calculate a highly accurate equilibrium Hamiltonian $\chi_{EQL}(\psi,\theta)$ for magnetic field lines in the DIII-D in natural coordinates from the axisymmetric equilibrium magnetic surfaces data in the DIII-D. We use the $\chi_{EQL}(R,Z)$ data from the DIII-D shot 115467 at 3000 ms from the equilibrium fitting code EFIT [4,5]. We form the functions $u(\psi,\theta)=\surd(\psi)\cos(\theta)=\surd(B_0/2)(R\text{-}R_0)$ and $v(\psi,\theta)=\surd(\psi)\sin(\theta)=\surd(B_0/2)(Z\text{-}Z_0)$. Equilibrium poloidal flux inside the separatrix is χ_{SEP}=0.283 Webers. Magnetic field on the magnetic axis is B_{axis} =1.589 Tesla. The magnetic axis is at (R_0,Z_0)=(1.758 m, -0.023 m). We set $B_0=B_{axis}$. The poloidal flux is $\chi=\chi_{EQL}$. We use the $(R\text{-}R_0)$, $(Z\text{-}Z_0)$, data for χ_{EQL}/χ_{SEP} =0.9, 0.95, 1.0, 1.02, and 1.04. The surface χ_{EQL}/χ_{SEP}=1 is the equilibrium separatrix surface. We calculate the corresponding *u* and *v* data for these five equilibrium surfaces from the EFIT $\chi_{EQL}(R\text{-}R_0,Z\text{-}Z_0)$ data points. We set χ_{EQL} =constant. We express the function $F(\psi,\theta)$ as a bivariate polynomial in *u* and *v*,

$$\chi_{EQL}(u(\psi,\theta),v(\psi,\theta)) = \sum a_i u^i + \sum b_j v^j + \sum c_{ij} u^i v^j, \qquad i=1 \text{ to } N;\ j=1 \text{ to } M. \qquad (1)$$

We find that N=5 and M=6 gives the most satisfactory representation of the EFIT data for DIII-D shot 115467 at 3000 ms. We call the equilibrium poloidal flux, given by eqn. (1) with N=5, M=6, the equilibrium generating function (EGF) for the field lines in the DIII-D; and we denote it by $\chi_{EGF}(\psi,\theta)$. We calculate the equilibrium magnetic surfaces from $\chi_{EGF}(\psi,\theta)$. We compare the equilibrium magnetic surfaces from $\chi_{EGF}(\psi,\theta)$ with those from the EFIT. See Figs. 1 to 5. The comparison shows that the equilibrium generating function reproduces the EFIT surfaces with high accuracy. The distance between the two X-points is 4.24 cm. χ_{SEP_EGF} is 0.14% smaller than χ_{SEP} for the shot. Difference in poloidal angular position of the X-point is 1.02 %. Difference in distance from O-points to X-points is 2.78 %. So magnetic surfaces, positions of the O-point, X-point, and the poloidal flux inside the ideal separatrix are calculated from $\chi_{EGF}(\psi,\theta)$ quite accurately. We calculate the equilibrium safety factor $q(\chi_{EGF})=(1/2\pi)(d/d\chi_{EGF})[(1/2\pi)\int_C\psi(\theta,\chi_{EGF})d\theta]$, and show it in Fig. 6. The difference in the q profiles is because the equilibrium parameters in the EFIT data do not include the fast ion pressure (in the standard EFIT reconstructions we assume that $T_i = T_e$) or bootstrap current. Near separatrix, we see that $q(\chi)=-0.3056\ln(1-\chi/\chi_{SEP\text{-}EGF})+2.3215$. In the SOL of the DIII-D, the safety factor first decreases to the value 3.8, and then increases as the poloidal field from the plasma current decays with increasing radius. Safety factor in the DIII-D SOL calculated from the EGF first decreases to 4.03, and then increases. It also increases logarithmically as we approach the separatrix surface.

We use canonical transformation to construct the map

$$\psi_{n+1} = \psi_n - k(\partial/\partial\theta_n)(\chi(\psi_{n+1}, \theta_n, \varphi_n), \qquad \theta_{n+1} = \theta_n + k(\partial/\partial\psi_{n+1})(\chi(\psi_{n+1}, \theta_n, \varphi_n) \qquad (2)$$

This map preserves area, since $|\partial(\psi_{n+1},\theta_{n+1})/\ \partial(\psi_n,\theta_n)|=1$. k is map parameter. We set $k=2\pi/72$. If we take 72 iterations of map to be equals to a single toroidal circuit of DIII-D, we reproduce the equilibrium safety factor. k represents asymmetries. These asymmetries destroy the ideal separatrix surface, and create a layer of chaotic field lines surrounding where the ideal separatrix surface was. Width of stochastic layer $w \cong 7.25\text{X}10^{-6}$ m. Loss of poloidal flux from inside the ideal separatrix surface is $\chi_{LOSS} \cong 1.7\text{X}10^{-11}$ Webers. We apply this map to calculate stochastic broadening due to topological noise [6] and field errors [7]. The field-errors and statistical topological noise together give the magnetic perturbation $\chi_1(\theta,\varphi) = \delta\Sigma_{(m,n)}\cos(m\theta\text{-}n\varphi)$. We ignore the radial dependence of the Fourier modes. Fourier mode numbers are (m,n) = {(3,1), (4,1), (6,2), (7,2), (8,2), (9,3), (10,3), (11,3)}. The common amplitude δ can vary from $0.8\text{X}10^{-5}$ to $2.0\text{X}10^{-5}$ [8]. We consider the case when these modes are locked. The main effect of the field-errors and noise is to create islands and chaos. For this noise and field errors, the width of stochastic layer near the X-point in principal plane of the DIII-D varies from 5.8 cm to 12.0 cm; and the loss of poloidal flux from inside the ideal separatrix is from about 0.001 to 0.005 Webers or 0.4% to 1.6%. See Figs. 7 to 9. This work is supported by the US Department of Energy grants DE-FG02-01ER54624 and DE-FG02-04ER54793.

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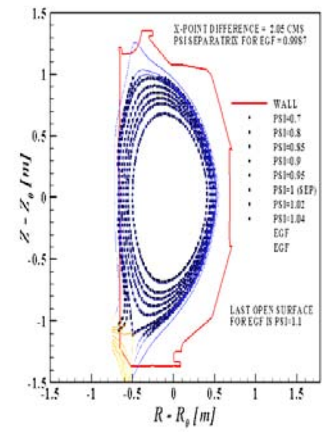

Fig. 1. Comparison of equilibrium surfaces.

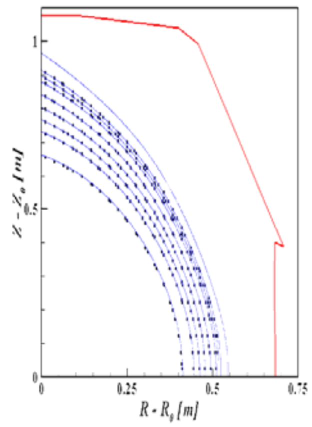

Fig. 2. An enlarged view of Fig. 1 in the first quadrant.

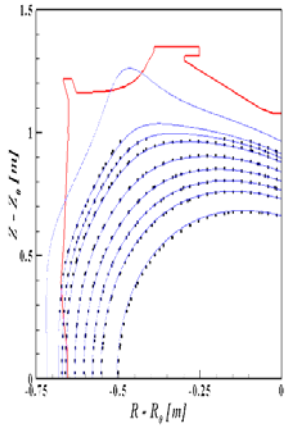

Fig. 3. An enlarged view of Fig. 1 in the second quadrant.

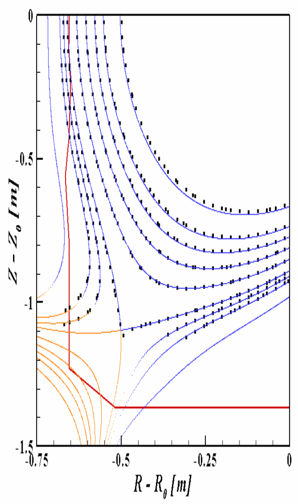

Fig. 4. An enlarged view of Fig. 1 in the third quadrant.

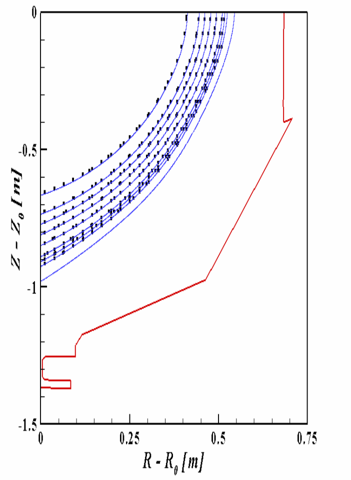

Fig. 5. An enlarged view of Fig. 1 in the fourth quadrant.

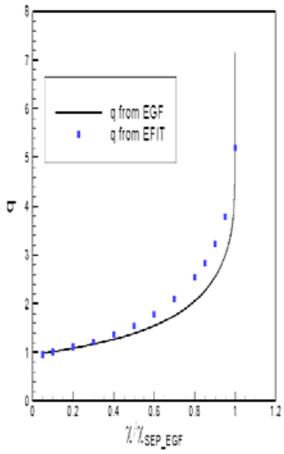

Fig. 6. Equilibrium safety factor from $\chi_{EGF,}$ and comparison with q from EFIT.

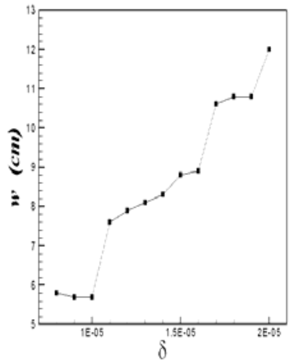

Fig. 7. Width of stochastic layer as a function of δ for noise and field errors.

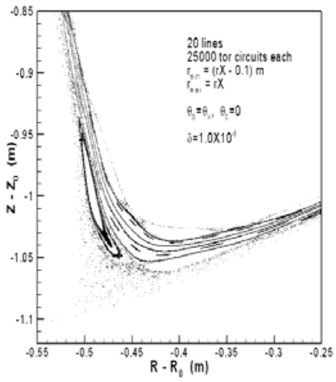

Fig. 8. Phase portrait near the X-point in the principal plane of DIII-D when $\delta=10^{-5}$.

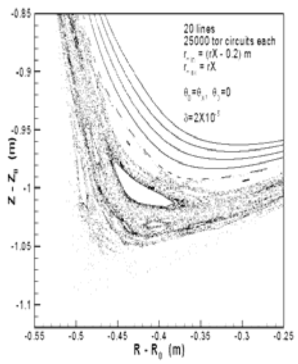

Fig. 9. Phase portrait near the X-point in the principal plane of DIII-D when $\delta=2X10^{-5}$.