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Critical Phenomena in Strongly Coupled Fine Particle Plasmas

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Introduction

Fine particle plasmas (dusty plasmas) are charge neutral mixtures of macroscopic fine particles (dust particles), ions, and electrons. Fine particles can be modeled by Yukawa particles with hard cores embedded in the ambient plasma composed of ions and electrons. When the latter is regarded as an inert background, we have the Yukawa one-component plasma (OCP).

The isothermal compressibility of OCP generally diverges with the increase of the Coulomb coupling. In order to observe this thermodynamic instability, however, it is necessary to take the background into account as a real physical entry to the system: This instability is suppressed when the density of the background is kept constant. In the system of Yukawa particles embedded in ambient plasma, it is shown that the total isothermal compressibility diverges when the coupling of Yukawa particles is sufficiently strong[1].

Equation of State of Yukawa Particles and Ambient Background Plasma

We consider the system in a volume V composed of N_i ions (i) with the charge e, N_e electrons (e) with the charge $-e$, and N_p fine particles (p) with the charge $-Qe$, satisfying the charge neutrality condition for densities

$$(-e)n_e + en_i + (-Qe)n_p = 0. \tag{1}$$

We assume that $n_i,\ n_e \gg n_p$ and take the statistical average with respect to electrons and ions to have an expression for the Helmholtz free energy of the system.The effective interaction energy for fine particles is given by[2]

$$U_p = U_{coh} + U_{sheath}, \tag{2}$$

$$U_{coh} = \frac{1}{2}\int\int d\mathbf{r}d\mathbf{r}'\frac{\exp(-|\mathbf{r}-\mathbf{r}'|/\lambda)}{|\mathbf{r}-\mathbf{r}'|}\rho(\mathbf{r})\rho(\mathbf{r}') - (self\text{-}interactions). \tag{3}$$

Here the charge density $\rho(\mathbf{r}) = \sum_{i=1}^{N_p}(-Qe)\delta(\mathbf{r}-\mathbf{r}_i) + Qen_p$ includes that of the background plasma Qen_p and $1/\lambda^2 = 4\pi n_e e^2/k_BT_e + 4\pi n_i e^2/k_BT_i$. The term U_{sheath} is the (free) energy associated with the sheath around fine particles. Fine particles interact via the Yukawa interaction and effectively confined by the average charge density of background plasma composed of ions and electrons. We explicitly take into account the contribution of the background plasma to the equation of state. We also take the finite radius of fine particles into account.

Our system is characterized by four parameters:

$$\Gamma = \frac{(Qe)^2}{ak_BT_p}, \quad \xi = \frac{a}{\lambda}, \quad \Gamma_0 = \frac{(Qe)^2}{r_pk_BT_p} = \Gamma\frac{a}{r_p}, \quad A = \frac{n_ek_BT_e + n_ik_BT_i}{n_pk_BT_p} \gg 1, \tag{4}$$

where $a = (3/4\pi n_p)^{1/3}$ is the mean distance between particles and r_p is the radius of core of fine particles. We assume three components have different temperatures, T_p, T_i, and T_e. The total pressure is approximately given by

$$\frac{p_{tot}}{n_pk_BT_p} \approx \frac{A}{1-\eta} + \frac{p_p}{n_pk_BT_p}, \tag{5}$$

where

$$\frac{p_p}{n_pk_BT_p} \approx \frac{1+\eta+\eta^2-\eta^3}{(1-\eta)^3} + a_1\tilde{\Gamma}e^{a_2\xi}\left(\frac{1}{3}+\frac{1}{6}a_2\xi+\frac{\tilde{r}_p^2}{1+\tilde{r}_p}\right)$$
$$+a_3\tilde{\Gamma}^{1/4}e^{a_4\xi}\left(\frac{1}{3}+\frac{2}{3}a_4\xi+\frac{\tilde{r}_p^2}{1+\tilde{r}_p}\right)+\frac{3}{2}\tilde{\Gamma}\xi^{-2}\frac{\tilde{r}_p^2}{1+\tilde{r}_p}(1+e^{-2\tilde{r}_p})-\frac{1}{4}\tilde{\Gamma}\xi e^{-2\tilde{r}_p}, \tag{6}$$

$$\eta = \left(\frac{\Gamma}{\Gamma_0}\right)^3, \quad \tilde{\Gamma} = \Gamma\frac{\exp(2\tilde{r}_p)}{(1+\tilde{r}_p)^2}, \quad \tilde{r}_p = \frac{r_p}{\lambda} = \frac{a}{\lambda}\frac{r_p}{a} = \xi\frac{\Gamma}{\Gamma_0}, \tag{7}$$

and $a_1 = -0.896$, $a_2 = -0.588$, $a_3 = 0.72$, $a_4 = -0.22$.

Phase Diagrams

When the coupling of fine particles becomes sufficiently strong, the total isothermal compressibility diverges and we have a phase separation and related critical point[1]. Example of phase diagrams are shown in Figs.1(a) and 1(b). In the $(\Gamma,\ \xi)$ plane, we have a domain where phases with higher and lower densities coexist. In the $(\Gamma/\xi^2,\ p_{tot})$ plane, we have a line on which two phases coexist, terminating at the critical point. The former is analogous to the density-temperature diagram and the latter, to the pressure-temperature diagram.

Density Fluctuations

The static form factor of Yukawa particles $S(k)$ is related to the dielectric response function $\varepsilon(k,\omega=0)$ describing the response to external Yukawa particle density via the fluctuation-dissipation theorem as

$$S(k) = \frac{k^2+1/\lambda^2}{k_D^2}\left[1-\frac{1}{\varepsilon(k,\omega=0)}\right], \tag{8}$$

where $k_D^2 = 4\pi n_p(Qe)^2/k_BT_p$. In the limit of long wavelengths, the balance of the external force and the pressure gradient gives

$$S(k) \sim \left[-\frac{V}{n_pk_BT_p}\left(\frac{\partial p_{tot}}{\partial V}\right)_{T_i,T_e,T_p} + \mathscr{O}\left(k^2\right)\right]^{-1}. \tag{9}$$

Density fluctuations are thus enhanced near the critical point as shown in Fig.2.

Within the Yukawa OCP, the response to the external field is related to the 'screened response' and we have

$$S_{OCP}(k) \sim \left[k_D^2 \lambda^2 - \frac{V}{n_p k_B T_p} \left(\frac{\partial p_p}{\partial V} \right)_{T_p} + \mathscr{O}\left(k^2\right) \right]^{-1}. \tag{10}$$

The divergence of the isothermal compressibility of Yukawa OCP $-(\partial V/\partial p_p)_{T_p}$ is not directly related to the enhancement of the density fluctuation[3].

Applicability to Fine Particle Plasmas

There exist some aspects of fine particle plasmas which are not described by the Yukawa system embedded in ambient plasma; for example, (a) anisotropy of interaction, (b) possible existence of attractive interactions between particles[4], (c) nonlinear screening and deviation from Yukawa interaction, and (d) thermodynamic openness.

As for (a), we assume that we have a bulk, approximately isotropic three-dimensional fine particle plasmas which may be realized under microgravity or methods to effectively cancel the gravity. The possibility (b) has the effect to make the conditions for strength of coupling in favor of the instability. As for (c), collision experiments indicate that Yukawa repulsion works at least in some range of mutual distance[5]. Though the aspects (b), (c), and (d) may influence critical conditions quantitatively, we may expect our results may be applicable to fine particle plasmas semi-quantitatively.

Thermodynamics of fine particles plasmas are characterized by theoretical parameters and it is necessary to interpret them into experimental parameters. In fact, it is shown that characteristic parameters realized by fine particle plasmas should satisfy the condition

$$\xi^2/(\Gamma/A) \geq 3. \tag{11}$$

We have a domain of experimental parameters for the the observation of the critical point where the above condition is satisfied.

Conclusion

In order to observe phenomena near the critical point, it is necessary to have a bulk isotropic three-dimensional system of fine particle plasmas. Though it is difficult to realize such a system on the ground due to gravity on fine particles, we expect the experiment under microgravity may provide a chance of observation.

References

[1] H. Totsuji, J. Phys. A: Math. Gen. **39**, 4565(2006); *Non-Neutral Plasma Physics VI, Workshop on Non-Neutral Plasmas 2006*, ed. M. Drewsen *et al.*, AIP Conference Proceedings 862, American Institute of Physics, New York, 2006, p.248.

[2] H. Totsuji *et al.*, Phys. Rev. E **71**, 045401(R)(2005); H. Totsuji *et al.*, Phys. Rev. E **72**, 036406(2005); T. Ogawa *et al.*, J. Phys. Soc. Japan **75**, 123501(2006).

[3] H. Totsuji and K. Tokami, Phys. Rev. A **30**, 3175(1984).

[4] For example, M. Nambu *et al.*, Phys. Lett. A **230**, 40(1995); V. N. Tsytovich *et al.*, Commun. Plasma Phys. Control. Fusion, **17**, 249(1996).

[5] R. Kompaneets *et al.*, Phys. of Plasmas **14**, 052108(2007).

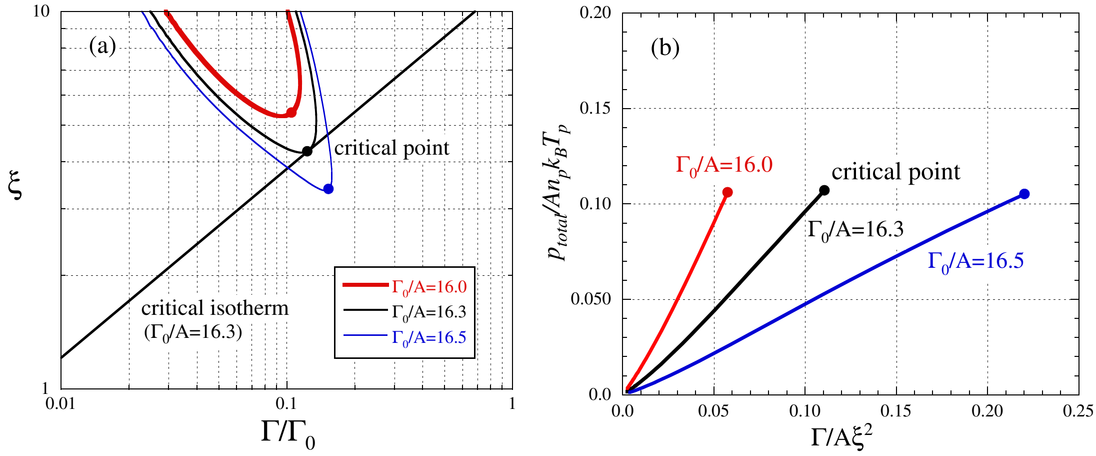

Figure 1: Examples of phase diagrams in $(\Gamma/\Gamma_0,\ \xi)$-plane(a) and $(p_{tot}, \Gamma/\xi^2)$-plane(b).

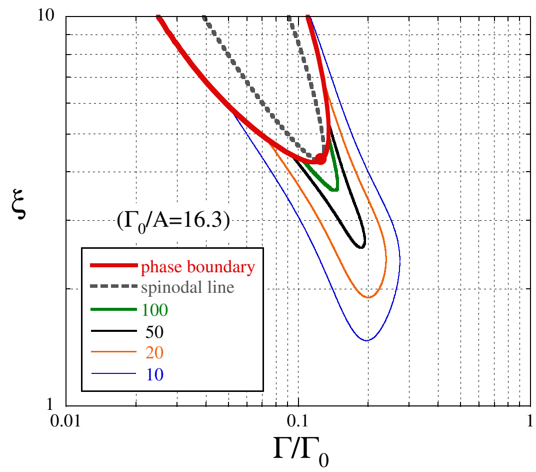

Figure 2: Density fluctuation enhancement. Numbers are enhancement in amplitude squared.