

Chaotic Magnetic Fields due to Asymmetric Current Configurations – Modeling Cross-Field Particle Diffusion in Cosmic Rays

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The observed propagation of cosmic rays in the interplanetary space cannot be explained unless there is diffusion of the energetic particles across the interplanetary magnetic field [1]. The cross-field diffusion of cosmic rays is assumed to be due to the chaotic nature of the interplanetary/intergalactic magnetic fields. Among the classic works on this subject have been those of Parker [2] and Jokipii [3]. Parker considered the passage of cosmic ray particles and energetic solar particles in a large scale magnetic field containing small scale irregularities. In the context of cosmic ray propagation, Jokipii considered a small fluctuating component, added on to a uniform magnetic field, to study the spatial transport of particles. In these studies the irregular component of the magnetic field is prescribed in an *ad hoc* fashion. In contrast, we consider asymmetric, nonlinear, steady-state magnetic fields, in three spatial dimensions, generated by currents flowing in circular loops and straight lines [4]. From this fully three-dimensional analysis we show that a particle moving in the static magnetic fields of such current systems, or in combination with a superimposed uniform magnetic field, can undergo cross-field transport. An important fact that emerges from our calculations is that the motion of a particle in a three-dimensional chaotic magnetic field is not necessarily chaotic and, thus, cannot undergo cross-field diffusion. However, in a variety of cases the particle motion can be chaotic, leading to spatial diffusion across the magnetic field. For cross-field diffusion it is necessary that the magnetic field lines be three dimensional.

Magnetic Fields of a Current Loop and a Current Wire

For a circular current loop with its center located at the origin of a cartesian coordinate system (x, y, z) and lying in the x-y plane the vector potential, in spherical coordinates, is

$$A_{\phi}(r, \theta) = \frac{\mu_0 I_L}{2\pi} \frac{a}{(a^2 + r^2 + 2ar \sin \theta)^{1/2}} F(k) \quad (1)$$

where μ_0 is the free space permeability, I_L is the current in the loop, r is the radial coordinate, θ is the zenith angle, and ϕ is the azimuthal angle. The vector potential has only the azimuthal

component, $F(k) = [(2 - k^2) K(k) - 2E(k)] / k^2$, $k^2 = (4ar \sin \theta) / (a^2 + r^2 + 2ar \sin \theta)$, and $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind, respectively. The magnetic field components are obtained from

$$B_r = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\phi), \quad B_\theta = -\frac{1}{r} \frac{\partial}{\partial r} (r A_\phi) \quad (2)$$

For a current carrying straight wire along the z -direction, the vector potential has just the z -component

$$A_z(r) = \frac{\mu_0 I_W}{2\pi} \ln(r) \quad (3)$$

where I_W is the current in the wire and $r = \sqrt{x^2 + y^2}$ is the radial distance from the wire. The magnetic field is along the azimuthal angle

$$B_\phi = -\frac{\partial}{\partial r} A_z = \frac{\mu_0 I_W}{2\pi} \frac{1}{r} \quad (4)$$

The equations, in spherical coordinates, for the magnetic field lines $\vec{B} = (B_r, B_\theta, B_\phi)$ are

$$\frac{dr}{B_r} = \frac{rd\theta}{B_\theta} = \frac{r \sin \theta d\phi}{B_\phi} = \frac{ds}{|\vec{B}|} \quad (5)$$

where s is the path length along the field line.

There are two particular current systems we have been studying: a current loop combined with a straight current wire, and two current loops. In either case the geometrical setup is as follows. A reference current loop is assumed to be lying in the $x - y$ plane with the center of the loop located at the origin of the coordinate system.

In the first current system which includes a straight wire, we will assume that the wire intersects the plane of the reference current loop. The point of intersection in the $x - y$ plane could be displaced relative to the center of the loop. In the second system we assume that the center of the second current loop is on the z -axis. We assume that the center is displaced relative to the reference system and that the plane of the second loop is inclined relative to the plane of the first loop. The angle of inclination is measured relative to the z -axis.

Current Loop and a Straight Current Wire

The loop-wire system is an interesting example to study since it provides an understanding of the more complicated loop-loop system. Figure 1a shows the Poincaré surface-of-section, in the $x - z$ plane, of magnetic flux surfaces for the loop-wire system where the wire is passing through the center of the loop. The ratio of the currents are $I_L/I_W = 5$. The distances are normalized to the radius a of the loop. The flux surfaces are similar to those one would expect for a dipolar field.

Figure 1b shows the flux surfaces, in the surface-of-section plot, when the wire is displaced by $\Delta r/a = 0.001$ relative to the origin. The wire is still parallel to the z -axis. The initial conditions for the magnetic field lines in this figure are the same as in Fig. 1(a) and the color scheme is identical. The inner two surfaces are still well defined as in Fig. 1(a) but the outer surfaces have broken up into islands. The order of the islands increases as the field lines move away from the outer edge of the current loop. The formation of islands is indicative of the fact that the field lines are no longer confined to a surface but occupy a finite volume in space.

Figure 2a is the same as Figs. 1 except that $\Delta r/a = 0.01$. Here we see that the inner most flux surface is still well preserved but the outer flux surfaces have disintegrated to occupy large volumes of space. The outer magnetic field lines are chaotic.

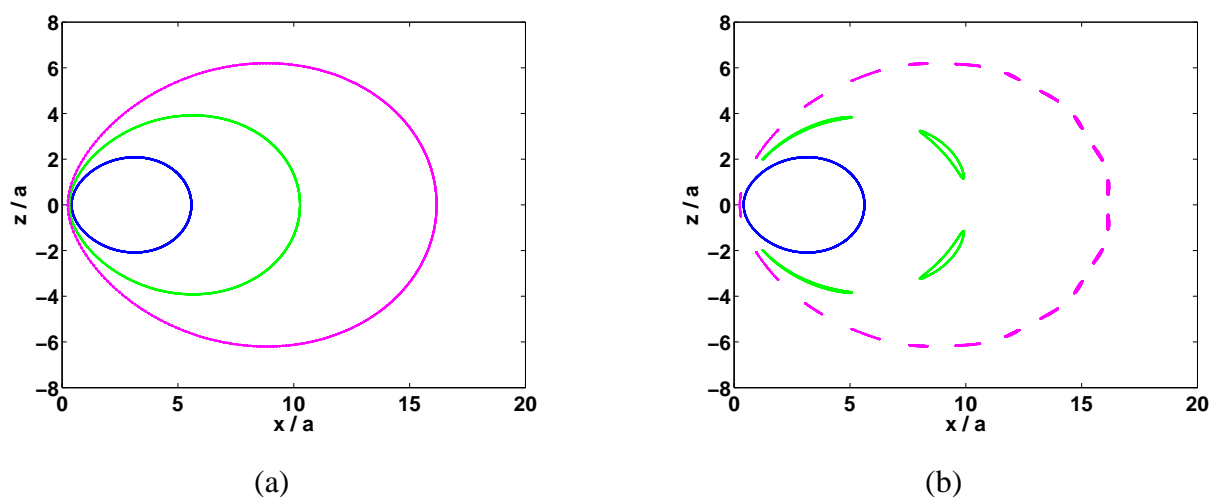


Figure 1: (a) In this surface-of-section plot the current loop intersects the surface at $x/a = 1$ and $z/a = 1$. The current wire is along the z -axis. (b) Surface-of-section when the wire is displaced by a distance $0.001a$ relative to the center of the loop.

Charge Particle Orbit in the Loop-Wire System

Figure 2b shows the orbit of a charge particle in three-dimensions in the case corresponding to Fig. 2a. The particle is started off in the red region at a location where the magnetic field line is chaotic. It is evident that the particle orbit is periodic.

Two Current Loops

The limitations on the length of this paper limit our subsequent discussion. For a system consisting of two current loops, if the plane of the second loop is tilted relative to that of the first loop, the magnetic field lines are three-dimensional and chaotic in space. The particle motion in such a field is also chaotic which leads to spatial transport.

Current Loop in an Ambient Uniform Magnetic Field

If we assume a circular current loop in an uniform background magnetic field, the motion of

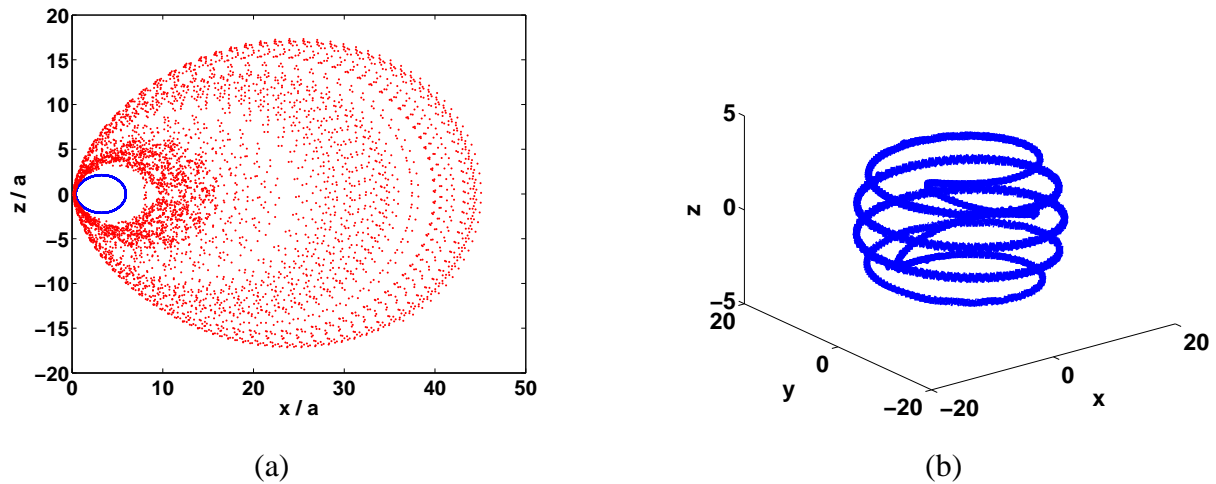


Figure 2: (a) The wire is displaced by a distance $0.01a$ relative to the center of the loop. (b) Charged particle orbit in three spatial dimensions. The initial location of the particle is in the red zone of (a).

a particle interacting with the combined fields can, for certain conditions, undergo cross-field diffusion. Suppose the background field is pointed in the y -direction. If the plane of the loop lies in the $x - y$ plane, or is perpendicular to the y -direction, there is no cross-field transport [4]. However, if the plane of the loop is in any other direction relative to the y -axis, the magnetic field lines near the center of the loop acquire a three-dimensional character, leading to cross-field diffusion of the particles.

In conclusion, we have shown that not all chaotic magnetic fields lead to chaotic particle motion. There are certain requirements for the fields in order to induce chaotic particle motion and cross-field transport. However, importantly, it is possible to have cross-field transport of cosmic rays without invoking *ad hoc* fluctuating magnetic fields.

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