*35th EPS Conference on Plasma Phys. Hersonissos, 9 - 13 June 2008 ECA Vol.**32D**, O-4.056 (2008)*

Cross field thermal transport in highly magnetized plasmas

M. Psimopoulos, S. Tanriverdi, G. Kasotakis, M. Tatarakis

Centre for Plasma Physics & Lasers, Department of Electronics, TEI. of Crete, Chania 73133, Crete, Greece

Abstract

By combining the Poynting vector with the resistive MHD equations it is shown that the cross field thermal diffusivity of electrons at the limit of very strong magnetic fields scales as $\chi_\perp^e \sim \nu_{ee}\lambda_s^2 \sim T_e^{-3/2}$ where T_e is the electron temperature, ν_{ee} is the *e-e* collision frequency and λ_s is the skin depth. This effect is independent of the magnetic field strength B and of the perpendicular electron diffusion, and is due to kinetic energy exchange between colliding electrons moving on parallel field lines in the presence of a cross field temperature gradient. For highly magnetized plasmas the present mode of heat transport is larger than the classical heat conduction $\chi_\perp^e \sim B^{-2}$ and provides with a microscopic mechanism that balances magnetic diffusion (paleoclassical hypothesis) leading to an explanation of the anomalous heat transport observed in tokamaks.

1. Introduction

Plasma fusion machines operate with increasingly strong magnetic fields $\vec{B}$ reaching the regime $r_e < \lambda_D$ and electron thermal transport exceeds the classical (and neoclassical) estimates by orders of magnitude, and it is usually called *anomalous*. Also, in classical theory it is accepted [1] that the conduction heat flux is carried by the plasma particles in the sense $\vec{q} = \int \frac{1}{2} m\tilde{o}^2 f(\vec{r},\vec{o})\vec{o}d\vec{o}$ where $f(\vec{r},\vec{o})$ is perturbed from its equilibrium form [2]. Therefore, restriction of the cross field motion of particles due to the effect of $\vec{B}$ restricts the classical heat transport coefficients $\kappa_\perp$. For example, in the regime $\omega_e\tau_e >> 1$ the thermal diffusivity of electrons $\chi_\perp^e = \kappa_\perp^e / n_e$ scales [1] as their diffusion coefficient $D_\perp^e$, viz.

$$\kappa_\perp^e \sim D_\perp^e \sim n_e \frac{e^2c^2m_e^{1/2}}{B^2T_e^{1/2}} \quad (cm^2/s) \tag{1}$$

and decreases at the limit $B \to \infty$. Recently, in order to resolve the problem of anomalous thermal transport, the "paleoclassical hypothesis" [PH] was advanced [3] based on the observation that electron heat flux would be of the correct order *if it was due* to magnetic diffusion: $\chi_\perp^e \sim D_\eta \sim T_e^{-3/2}$. However, no credible theory was found [4] to prove that magnetic diffusion alone can induce thermal transport in order to validate this hypothesis.

2. New mode of Heat Transport

In the present article we argue that a new mode of heat transport across $\vec{B}$ found to occur [5] in highly magnetized plasmas could form a basis for the [PH]. According to this mechanism at the limit $B \to \infty$ the electron Larmor radius becomes very small and the electron guiding centers move on parallel $\vec{B}$ lines. After collision two electrons may keep their parallel velocity components or they may interchange them depending on the distance of their respective trajectories. For every velocity exchange during the collision, energy is transmitted across $\vec{B}$ and contributes to the overall heat transport without any particle transport taking place. Note that unlike particles (*e,p*) always retain their velocity after collision and do not contribute to heat transport by this mechanism. The non-local kinetic equation describing (e-e) collisions at the limit $B \to \infty$ reads [5]

$$\frac{\partial f(\vec{r}_1,\upsilon_{1z},t)}{\partial t} = \int d\vec{r}_2 \int_{\upsilon_{1z}-\chi}^{\upsilon_{1z}+\chi} |\upsilon_{2z}-\upsilon_{1z}| \left\{ f(\vec{r}_2,\upsilon_{1z},t) f(\vec{r}_1,\upsilon_{2z},t) - f(\vec{r}_1,\upsilon_{1z},t) f(\vec{r}_2,\upsilon_{2z},t) \right\} d\upsilon_{2z} \quad (2)$$

where $\varepsilon = 2e/\sqrt{m_e |\vec{r}_2 - \vec{r}_1|}$ and the electron thermal diffusivity was obtained from Eq.(2) as:

$$\chi_\perp^e = \nu_{ee} L_\perp^2 \ ; \quad \nu_{ee} = \frac{3\sqrt{\pi}}{2} \frac{n_e e^4}{m_e^{1/2} T_e^{3/2}} \quad (3)$$

where ν_{ee} is the (*e-e*) parallel collision frequency at $B \to \infty$ and $L_\perp$ is a cut-off length representing the distance across $\vec{B}$ over which interactions can be sustained. The determination of $L_\perp$ has been a matter of controversy. Some authors [6,7] identified $L_\perp \sim \lambda_D = (T_o / 4\pi n_o e^2)^{1/2}$: Debye length. However, screening is impossible in single charged plasmas due to repulsive Coulomb potentials whereas for quasi-neutral highly magnetized plasmas inverse screening [5] or antishielding [8] may severely restrict the ability of particles to screen a test particle of opposite charge. In particular, the number density of untrapped electrons in thermal equilibrium at the distance $r < d \sim n_0^{-1/3}$ from a test proton where $\phi(r) \approx e/r$ ($x = r/\lambda_L$; $\lambda_L = e^2/T_0 \sim$ Landau length) reads:

(i) $B = 0$ (Electrons accelerate and focus): $$n_e = n_o \left\{ \frac{2}{\sqrt{\pi x}} + \left[1 - erf\left(1/\sqrt{x}\right)\right] e^{1/x} \right\} \quad (4a)$$

(ii) $B = +\infty$ (Electrons only accelerate): $$n_e = n_o \left[1 - erf\left(1/\sqrt{x}\right)\right] e^{1/x} \quad (4b)$$

in both cases the protons decelerate and obey Boltzmann's law: $n_p = n_0 e^{-1/x}$. (4c)

Note that trapped electrons were neglected in the above analysis. Numerical evaluation of Eqs.(4) shows that in the case $B \to +\infty$, $n_e < n_p$ for $r > 1.3\lambda_L$ confirming the existence of antishielding. According to this point of view, it was suggested in Ref.[5] that the skin-depth may be a more realistic choice for $L_\perp$ in Eq.(3) viz. $L_\perp \sim \lambda_s = \left(m_e c^2 / 4\pi n_o e^2\right)^{1/2}; \chi_\perp^e = \alpha \nu_{ee} \lambda_s^2$.

3. Particle and Energy Balance

In the present paper we show that macroscopic theory implies indeed that the latter choice may be correct and that the new mode of heat transport validates the [PH].

Consider first particle transport for a (*e-p*) plasma where $n_e \approx n_p$; $T_e \approx T_p$ in the presence of a strong $\vec{B}$. Adding up the two-fluid momentum equations in steady state the diamagnetic current (with or without collisions) reads:

$$\vec{j} = \frac{2c}{B^2}\left\{\vec{B} \times \left[T_e \vec{\nabla} n_e + n_e \vec{\nabla} T_e\right]\right\} \tag{5}$$

If collisions are present, electrons and protons will diffuse outwards with equal fluxes [1]

$$\vec{\Gamma}_{out}^e = -D_\perp^e \left\{\vec{\nabla} n_e + \frac{1}{4}\frac{n_e}{T_e}\vec{\nabla} T_e\right\}; \quad \vec{\Gamma}_{out}^p = -D_\perp^p \left\{\vec{\nabla} n_p + \frac{1}{4}\frac{n_p}{T_p}\vec{\nabla} T_p\right\} \tag{6}$$

For $\omega_e \tau_e >> 1$ we have $D_\perp^e = 2(m_e \nu_{ep})\dfrac{c^2 T_e}{e^2 B^2}$; $D_\perp^p = 2(m_p \nu_{pe})\dfrac{c^2 T_p}{e^2 B^2}$ so that $m_e \nu_{ep} = m_p \nu_{pe}$ and $T_e = T_p$ imply $D_\perp^e = D_\perp^p$. Also, an electric field $\vec{E} // \vec{j}$ should be present in this case in order to keep steady state by inward *E/B* fluxes: $\vec{\Gamma}_{in}^e = n_e c \dfrac{\vec{E} \times \vec{B}}{B^2}$; $\vec{\Gamma}_{in}^p = n_p c \dfrac{\vec{E} \times \vec{B}}{B^2}$ (7)

Balance of particle fluxes: $\vec{\Gamma}_{in}^e + \vec{\Gamma}_{out}^e = 0$ and $\vec{\Gamma}_{in}^p + \vec{\Gamma}_{out}^p = 0$ gives

$$n_e c \frac{\vec{E} \times \vec{B}}{B^2} = D_\perp^e \left[\vec{\nabla} n_e + \frac{1}{4}\frac{n_e}{T_e}\vec{\nabla} T_e\right] \tag{8}$$

Solving Eq.(8) with respect to $\vec{E}$ and using Eq.(5) we obtain Ohm's law in the form

$$\vec{E} = \frac{1}{\sigma_\perp}\left\{\vec{j} - \frac{3}{2} c n_e \frac{\vec{B} \times \vec{\nabla} T_e}{B^2}\right\} \tag{9}$$

where $\sigma_\perp = \dfrac{n_e e^2}{m_e \nu_{ep}}$ is independent of *B* as $B \to +\infty$. Combining Eqs.(5,9) we find that the $\vec{E}$ necessary to balance particle diffusion by *E/B* drifts is given by

$$\vec{E} = \frac{c}{\sigma_\perp B^2}\left\{\vec{B} \times \left[2T_e \vec{\nabla} n_e + \frac{1}{2} n_e \vec{\nabla} T_e\right]\right\} \tag{10}$$

Consider next electron energy transport across $\vec{B}$ due to Ohmic heating:

(i) Outward heat flux: $$\vec{q}_e = -n_e \chi_\perp^e \vec{\nabla} T_e \tag{11}$$

(ii) Inward Pointing vector: $$\vec{S} = \frac{c}{4\Gamma}(\vec{E} \times \vec{B}) \tag{12}$$

We observe that if $B \to +\infty$, $\vec{q}_e$ cannot balance $\vec{S}$ if the classical heat transport coefficient Eq.(1) is used because in this case $\chi_\perp^e \sim 1/B^2$ whereas using Eq.(10), $\vec{S}$ is given by

$$\vec{S} = \frac{c^2}{4\pi\sigma_\perp}\left[2T_e\vec{\nabla}n_e + \frac{1}{2}n_e\vec{\nabla}T_e\right] \tag{13}$$

so that S does not vanish at $B \to +\infty$. The coefficient $D_\eta = c^2/4\pi\sigma_\perp$ is the magnetic diffusivity. We observe therefore that for ohmic heating, in the case of a tokamak satisfying profile consistency $n_e \sim T_e^{1/2}$ [9] we have $T_e\vec{\nabla}n_e = \frac{1}{2}n_e\vec{\nabla}T_e$, $\vec{S} = \frac{3c^2}{4\pi\sigma_\perp}n_e\vec{\nabla}T_e$ so that for $\nu_{ee} \perp \nu_{ep}$ balance of energy fluxes viz. $\vec{q}_e + \vec{S} = 0$ gives $$\chi_\perp^e = \frac{3c^2}{8\pi\sigma_\perp} = \frac{3}{2}\nu_{ee}\lambda_s^2 = \frac{9\sqrt{\pi}}{4}\frac{m_e^{1/2}e^2c^2}{T_e^{3/2}} \tag{14}$$

Note that $\chi_\perp^e$ in the form of Eq.(14) was attributed [10] to turbulent transport and was shown to explain well the Alcator scaling in tokamaks. In conclusion, relation $\chi_\perp^e \sim D_\eta$ forming the [PH] has now a clear microscopic justification valid at the limit $B \to +\infty$ if $\chi_\perp^e$ is identified with the new mode of heat transport Eq.(3) where $L_\perp \sim \lambda_s$.

References

[1]S.I.Braginskii, Reviews of plasma physics, Vol.1, Consultants Bureau (1965).

[2] E.M. Epperlein and M.G. Haines, *Phys. Fluids* 29, 1019 (1986).

[3] J. D. Callen, *Phys.Rev.Lett.* 94, 055002 (2005); *Phys.Plasmas* 12, 092512 (2005).

[4] J. W. Connor, R. J. Hastie and J. B. Taylor, *Phys.Plasmas* 15, 014701 (2008);
A. Thyagaraja and M. Roach, *Phys.Plasmas* 15, 014703 (2008).

[5] M. Psimopoulos and D.Li, *Proc.R.Soc.Lond.* 437, 55 (1992).

[6] M.G. Haggerty, *Can.J. Phys.* 43, 122 (1965).

[7] D. Dubin and T. M. O'Neil, *Phys.Rev.Lett.* 78, 3868 (1997).

[8] C. Hansen and J. Fajans, *Phys.Rev.Lett.* 74, 4209 (1995);
C. Hansen, A. B. Reimann and J. Fajans, *Phys.Plasmas* 3(5), 1820 (1996).

[9] M. Psimopoulos, *Phys.Lett.* A162, 182 (1992).

[10] T. Ohkawa, *Phys.Lett.* A67, 35 (1978).