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Selective ion capture instability for ion-particle interactions in weakly ionized gas

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Task formulation

An aerosol ensemble with a given constant concentration, N_q, is considered to move through bi-polar ion media under the external uniform electric field. Particles are assumed to be monodisperse and spherical. They move with a fixed terminal velocity, V_q, relative to neutral gas. Ions are generated by homogeneous volume source with constant rate, I. Together with ion-ion recombination and interactions with aerosols the source forms a quasi-state ionic concentration, n_0, and media conductivity σ. Particles start acquiring charge q by SIC - selective ion capture charging mechanism.

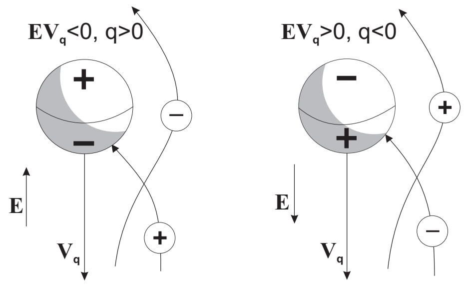

Figure 1: Selective ion capture charging for a single particle.

Single SIC effect

When a particle moves in a weakly ionized gas and external electric field with terminal velocity V_q lower than ion drift velocity, then SIC takes place, see [1,2,3]. One of important peculiarity of SIC that is particle acquire charge even ion conductivities are equal. The sign of charge is determined by the direction of external electric field , see Figure 1. Charging currents can be simply written in a case of weak diffusion and collinear $\mathbf{E}$ and $\mathbf{V_q}$, see [2,3]. For $\mathbf{E} \uparrow\uparrow \mathbf{V_q}$ we have

$$J_+ = \frac{1}{4} q_U \nu_{i+} \begin{cases} -4\tilde{q}, & \tilde{q} < 0; \\ 0, & \tilde{q} > 0; \end{cases} \quad J_- = \frac{1}{4} q_U \nu_{i-} \begin{cases} 0, & \tilde{q} < -1; \\ (1+\tilde{q})^2, & |\tilde{q}| < 1; \\ 4\tilde{q}, & \tilde{q} > 1. \end{cases} \qquad (1)$$

At the opposite case $\mathbf{E} \uparrow\downarrow \mathbf{V_q}$ the currents are

$$J_+ = \frac{1}{4} q_U \nu_{i+} \begin{cases} -4\tilde{q}, & \tilde{q} < -1; \\ (1-\tilde{q})^2, & |\tilde{q}| < 1; \\ 0, & \tilde{q} > 1; \end{cases} \quad J_- = \frac{1}{4} q_U \nu_{i-} \begin{cases} 0, & \tilde{q} < 0; \\ 4\tilde{q}, & \tilde{q} > 0. \end{cases} \tag{2}$$

where $\tilde{q} = q/q_U$ is the normalized particle charge, $q_U = 3ER^2$ is the maximum charge acquired by ideal conducting sphere with ions due to the SIC, $\nu_{i\pm} = 4\pi e B_{i\pm} n_{i\pm}$ is ionic relaxation frequency, where $B_{i\pm}$, $n_{i\pm}$ are the ion mobility and concentration.

Collective SIC effects

The 1D model is assumed and z axis, $\mathbf{E}$ and $\mathbf{V_q}$ are directed along $\mathbf{g}$ - free fall acceleration. And after we suggest that formula (1) works (assuming that SIC is realized: $V_q > EB_{i-}$) the initial system is:

$$\partial_t q + V_q \partial_z q = -q\nu_{i+} - \frac{3}{4}\nu_{i-} ER^2 (1 + \frac{q}{3ER^2})^2; \partial_t N_q + V_q \partial_z N_q + N_q \partial_z V_q = 0 \tag{3}$$

$$\partial_t V_q + V_q \partial_z V_q + \alpha_q V_q^2 = g + \frac{Eq}{m};\ \partial_z E = 4\pi (N_q q + en_+ - en_-); \tag{4}$$

$$\partial_t n_+ + V_{i+} \partial_z n_+ + n_+ \partial_z V_{i+} = I + 4\pi q N_q n_+ B_{i+} - \lambda n_+ n_-; \tag{5}$$

$$\partial_t n_- + V_{i-} \partial_z n_- + n_- \partial_z V_{i-} = I - \frac{3}{4}\nu_{i-} ER^2 (1 + \frac{q}{3ER^2})^2 \frac{N_q}{e} - \lambda n_+ n_-, \tag{6}$$

where q, N_q, V_q are the particle charge, concentration and fall velocity; $V_{i\pm} = B_{i\pm} E$ are the ion drift velocities. At first, let us examine the particle charge and ions concentration as a function of external electric field and ions initial concentration, $n_0 = \sqrt{I/\lambda}$.

The stationary particle charge can be found suggesting that $\partial/\partial z = 0$, and $\partial/\partial t = 0$. The simple system, describing net particle charge and ion concentrations, may be written as:

$$\tilde{n}_{+s} - \tilde{n}_{-s} + \varepsilon_E \tilde{q}_s = 0, \quad 1 - \tilde{n}_{+s}\tilde{n}_{-s} + \varepsilon_E \tilde{q}_s \varepsilon_{1+} \tilde{n}_{+s} = 0, \quad 4p\frac{\tilde{n}_{+s}}{\tilde{n}_{-s}}\tilde{q}_s + (1+\tilde{q}_s)^2 = 0 \tag{7}$$

$$\text{where } \ \tilde{n}_{\pm s} = \frac{n_{\pm s}}{n_0},\ \varepsilon_{1\pm} = \frac{4\pi B_{i\pm} e}{\lambda},\ p = \frac{B_+}{B_-},\ \varepsilon_E = \frac{N_q}{n_0}\frac{3ER^2}{e},\ \tilde{q}_s = \frac{\tilde{\rho}_{qs}}{\varepsilon_E} = \frac{q_s}{q_U} \tag{8}$$

$\tilde{q}_s$ is the stationary particle charge normalized on $3ER^2$. The value ε_E plays role of a control parameter and determines the degree of ions large scale perturbation. The general solution of (7) is simply to obtain numerically, see Figure 2, while the analytics may be examined in two important cases 1) $\varepsilon_E \ll 1$ and 2) $\varepsilon_E \gg 1$:

$$1)\ \tilde{q}_s = \tilde{q}_0 \left(1 + \tilde{q}_0 \varepsilon_E \frac{2p}{1+2p}\right),\ \tilde{n}_{\pm s} = 1 \mp \frac{1}{2}\tilde{q}_0 \varepsilon_E (1 \mp \varepsilon_{1\pm}), \tag{9}$$

$$2)\ \varepsilon_E \tilde{q}_s = \tilde{\rho}_{qs} = -\frac{1}{\sqrt{\varepsilon_{1+}}} + \frac{2p(1+\varepsilon_1)}{\varepsilon_{1+}^2 \varepsilon_E},\ \tilde{n}_{+s} = \frac{1}{\sqrt{\varepsilon_{1+}}} - \frac{2p(1-\varepsilon_{1+})}{\varepsilon_{1+}^2 \varepsilon_E},\ \tilde{n}_{-s} = \frac{4p}{\varepsilon_{1+}\varepsilon_E} \tag{10}$$

where

$$\tilde{q}_0 = 1 + 2p - \sqrt{(1+2p)^2 + 1} \tag{11}$$

is the stationary charge of single particle - in a weak electric field it is the maximum particle charge provided by SIC $|\tilde{q}_0| = 3 - 2\sqrt{2} \simeq 0.17$ for $p = 1$ and it is less than the charge which is acquired in unipolar ionized air, $|\tilde{q}_s| = 1$. For large field the particle net charge is equal to $-en_0/N_q\sqrt{\varepsilon_1}$.

Let us estimate ε_E for development tropospheric cloud stage: $E \simeq 0.01kV/cm$, $R \simeq 100\,\mu$, $N_q \simeq 0.05\,cm^{-3}$, $n_0 \simeq 5\cdot 10^3\,cm^{-3}$, and we have $3ER^2/e \sim 2\cdot 10^4$, $N_q/n_0 = 10^{-5}$ so $\varepsilon_E = 0.2$. For thunderstage we suppose $E \simeq 1\,kV/cm$, $R \simeq 1\,mm$, $N \simeq 0.03\,cm^{-3}$, $n_0 \simeq 3\cdot 10^4\,cm^{-3}$, and $3ER^2/e \sim 2\cdot 10^8$, $N_q/n_0 = 10^{-6}$ so $\varepsilon_E \simeq 200$.

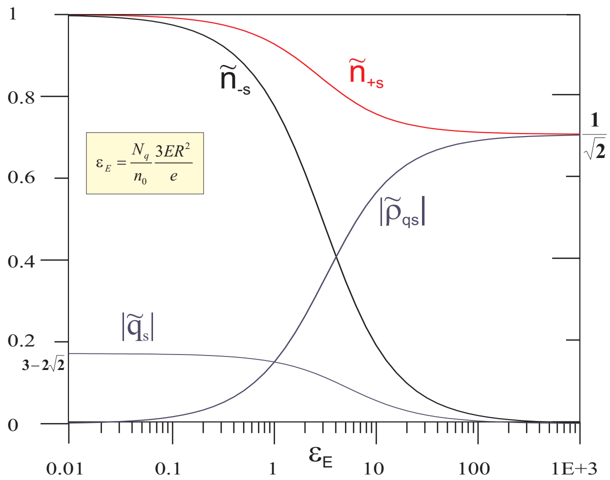

Figure 2: Ion concentration and droplet charge for p=1 and $\varepsilon_{1\pm} = 2$

Unstable electrostatic waves

First, for the simplicity let us examine the case of weak electrification $\varepsilon_E \ll 1$ and equal ion mobilities, $p = 1$. From this conditions it is following that $|\tilde{\rho}_{qs}| \ll 1$ and $n_{+s} \sim n_{-s}$ and $\nu_{i+} \simeq \nu_{i-} \simeq \nu_i$. Linearizing (3)-(6) one can obtained the system for normalized deviations $q' = (q-q_s)/q_U$, $n'_\pm = (n_\pm - n_{s\pm})/n_0$, $E' = (E-E_0)/E_0$, assuming that the $q', n'_\pm, E' \propto exp\{i\omega t - ikz/V_q\}$:

$$q'\left(\frac{i(\omega-k)}{\nu_i} + \frac{3}{2}\right) = -\frac{E'}{4}; \tag{12}$$

$$\rho'_i i\omega = 2ik\xi_i E' + \nu_i \varepsilon_E \left(\frac{3}{2}q' + \frac{1}{4}E'\right); \tag{13}$$

$$4f_N\left(q' + \frac{\rho'_i}{\varepsilon_E}\right) = -ikE',\ \rho'_i = n'_+ - n'_- \tag{14}$$

where k is the normalized scalar wave number and we take into account that $\xi_i = E_0 B_{i\pm}/V_q \ll 1$ and $|\tilde{\rho}_{qs}| \ll 1$ and, as consequence, $|E'| \gg |\rho'_i|$, $Re\omega \gg k\xi_i$, $|\tilde{\rho}_{qs}| \ll \varepsilon_1^{-1}$. After elementary calculations in (12) - (14) the dispersion equation is

$$1 - i\frac{2\nu_i}{\omega} + f_N \frac{\nu_i}{\omega(\omega - k - i3\nu_i/2)} = 0, \quad f_N = 3\pi R^2 N_q V_q \tag{15}$$

The analyzes of (15) show that unstable electrostatic waves ($Im\omega = \gamma < 0$) may exists in the system. Increment achieves the maximum at the range of long waves, $k \to 0$:

$$\gamma_2^0 = \frac{7\nu_i}{4} - \sqrt{(\frac{7\nu_i}{4})^2 + \nu_i(f_N - 3\nu_i)}, \quad \omega_2 = \frac{k}{2}\frac{\gamma_2^0 - 2\nu_i}{\gamma_2^0 - 7\nu_i/4}, \tag{16}$$

With decreasing the wave length the increment is decreasing too and in case of $\lambda = 2\pi/k_t$ SIC instability is inhibited. The magnitudes of k_t and ω_t are determined by the expressions:

$$k_t^2 = \frac{49}{12}\nu_i\{f_N - 3\nu_i\}, \quad \omega_t = \frac{4}{7}k_t \tag{17}$$

It is obvious that threshold of instability are determined by the expression: $f_N = 3\nu_i$, and in the case $f_N < 3\nu_i$ all harmonics of electrostatic waves are damping. Figure 3 shows the dispersion curves of SIC instability $f_N > 3\nu_i$, the group and phase velocities are also presented. The similar unstable waves exist in the media with two sort of particles, which are charging by collisions in external electric field, see [4].

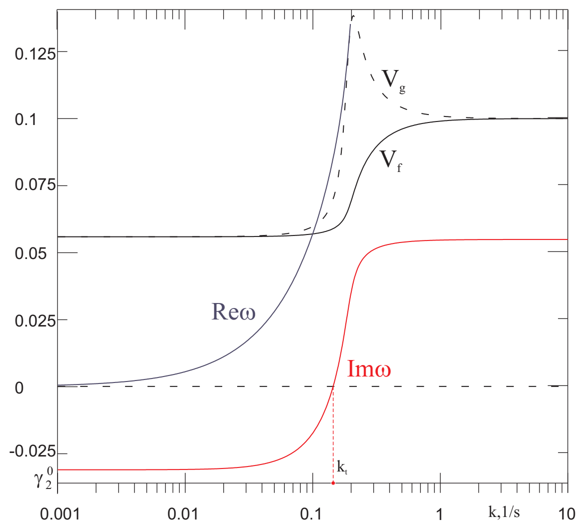

Figure 3: SIC instability dispersion curves, $N_q = 0.6\,cm^{-3}$, $R = .015\,cm$, $f_N = 0.173\,s^{-1}$, $\nu_i = 0.039\,s^{-1}$

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