

## Attraction of dust clusters and formation of super-crystals

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Dust clusters (structures) with large number of grains (that can be either in gas, liquid or crystal states) are shown to have spherical equilibrium states of finite size  $R_{str}$  of the order and larger than the ion-neutral mean free path and a to have negative charges  $-Z_{str}e$  that have a simple estimate  $Z_{str}e^2/R_{str}T_i \approx 1 - 3$ . They can serve as "super-dust grains" which can form a "super-crystals" if the equilibria are stable. Both stability of equilibria and possible over-screening of fields of these clusters are investigated numerically. The equilibrium states of the structures are determined by single parameter-the total plasma flux on the surface of the structure. The negative charge at the structure radius  $R_{str}$  is due to collection of not fully screened individual grain charges caused by ion-neutral collisions (for distances larger than the non-linear screening radius of individual grains the flux velocity on grains is  $u \propto 1/r^2$  and  $E \propto u \propto 1/r^2$  as for Coulomb field). For distances larger than  $R_{str}$  the total plasma flux on the structure consists of the convection flux and the diffusion flux and is created by the structure self-consistently. It is sufficient to confine dust grains, electrons and ions. The perturbation of the structure state are shown also to depend on the single parameter determined by variations of the external flux at the surface of the structure. Only low frequency perturbations that can displace the grains are included in investigation of stability. Frequencies much less than the inverse time for establishing the electron are considered. The additional forces in the perturbations are related to dust friction in neutral gas related with appearance of dust velocities in perturbations and the dust inertia. The problem of linear perturbations is considered and the perturbations are expanded in Fourier components in time. The frequency dependence of the structure responses is found. The dimension-less parameters used are : the ion density  $n$ , electron density  $n_e$  and Havnes parameter  $P$  normalized with respect to critical density  $n_{cr} = T_i/4\pi e^2\lambda_{in}^2$  ( $\lambda_{in}$  is the ion neutral mean free path) :  $n \rightarrow n_i/n_{cr}$ ;  $n_e \rightarrow n_e/n_{cr}$ ;  $P \rightarrow n_d Z_d/n_{cr}$ ; the ion drift velocity  $u$  is normalized with respect to ion thermal velocity  $u \rightarrow v_i\sqrt{2}v_{Ti}$  and the total flux  $\Phi$  as  $\Phi \rightarrow \Phi/n_{cr}\sqrt{2}v_{Ti}$ . Important is the normalization of the dust velocity  $v_d \rightarrow v_d/v_{cr}$  where for Epstein force  $v_{cr} = v_{Tn}(3T_n/4T_i)(\sigma_{in}/\pi a^2)(aT_e/e^2)$ , ( $\sigma_{in}$  is the cross-section of ion-neutral collisions); the frequency is normalized using this effective dust velocity  $\omega \rightarrow \omega\lambda_{in}/v_{cr}$  and the dust mass is normalized as  $m = m_d(v_{cr}^2/T_i)(e^2/aT_e)$  where  $T_n, T_i$  and  $T_e$  are the neutral atom, ion and electron thermal velocities respectively. Notice that the normalized dust mass is practically

independent on dust size  $m = (63/A)(\sigma_{in}/3 \times 10^{-15} \text{ cm}^2)(T_n/T_i)^2(\rho_d/3 \text{ g/cm}^3)$ . Using the small parameter  $T_d/T_i Z_d \ll 1$  it is possible to find two equations for perturbation (denoted by symbol  $\delta$ ) - the balance of forces on grains and grain continuity equation

$$\delta (\alpha_{dr}(\beta, u)\beta\sqrt{nu} - E) - \frac{v_d}{z} (1 - im\omega) = 0 \quad -i\omega\delta \left(\frac{P}{z}\right) + \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{Pv_d}{z}\right) = 0$$

where  $E$  is the electric field,  $\alpha_{dr}(\beta, u)$  is the drag coefficient depending on drift velocity  $u$  and the non-linear parameter  $\beta = az\sqrt{n}/\tau; a \rightarrow a/\lambda_{in}; \tau = T_i/T_e$  (the drag force is  $F_{dr}\lambda_{in}/T_i = Z_d\alpha_{dr}(\beta, u)\beta$  and was investigated in [1,2]);  $z$  is the equilibrium charge of the grains in the structure  $z = Z_d e^2/aT_e$ . Other equations for disturbances of ions, electron, ion drift velocity, fluxes and grain charges are simply the variations of equilibrium equations [3]. Using the equilibrium stationary distributions of the self-organized structures [3] the responses of the structure to the total flux perturbations have been found numerically. The final result is the ratio of the perturbation  $\delta n(r, \omega), \delta n_e(r, \omega), \delta u(r, \omega), \delta P(r, \omega), \delta z(e, \omega), v_d(r, \omega)$  to the "total" flux perturbations  $\delta\Phi'_{ext}(\omega) \equiv \delta\Phi(R_{str}, \omega)R_{str}^2$  which are named as **global structure susceptibilities** and are denoted by  $\alpha_{\delta n} \dots$ ; for example

$$\alpha_{\delta n}(r, \omega) = \frac{\delta n(r, \omega)}{\delta\Phi'_{ext}(\omega)}$$

and similar for  $\alpha_{\delta n_e}(r, \omega); \alpha_{\delta u}(r, \omega); \alpha_{\delta P}(r, \omega)$

$\alpha_{\delta z}(r, \omega); \alpha_{\delta v_d}(r, \omega); \alpha_{\delta n_d}(r, \omega); \alpha_{\delta \Phi}(r, \omega)$

These responses were calculated numerically for different dust masses ( $m = 1, 3, 10$ ) in the range of frequencies where they are not vanishing and for different equilibrium dust structures founded in [3]. As an example we give here the responses for medium size structure [3]  $R_{str} = 0.689\lambda_{in}$ . For this structure the most important range of frequencies is  $0 < \omega < 1.8$ . The results for imaginary and real parts of some responses are presented on Fig.1. Here and below the solid lines correspond to the distance from the center of the structure  $0.2R_{str}$ , dotted lines correspond to the distance  $0.4R_{str}$ , dash-dotted lines to  $0.6R_{str}$  and dashed lines to  $0.8R_{str}$ .

For perturbations of frequency  $\omega_0$  the ion density can be written as  $\delta n(r, t) = |\alpha_{\delta n}(r)|\Phi_0 \times \cos(\omega_0 t + \phi_n(r))$  and similar to other responses.

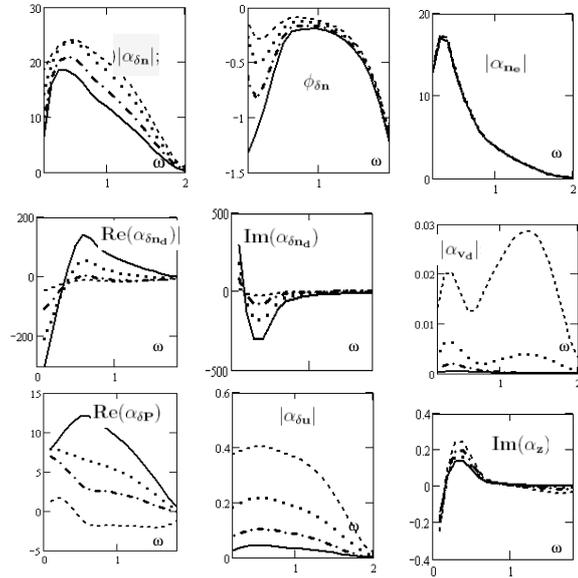


Figure 1: Some responses,  $m = 3, R_{str} = 0.689$

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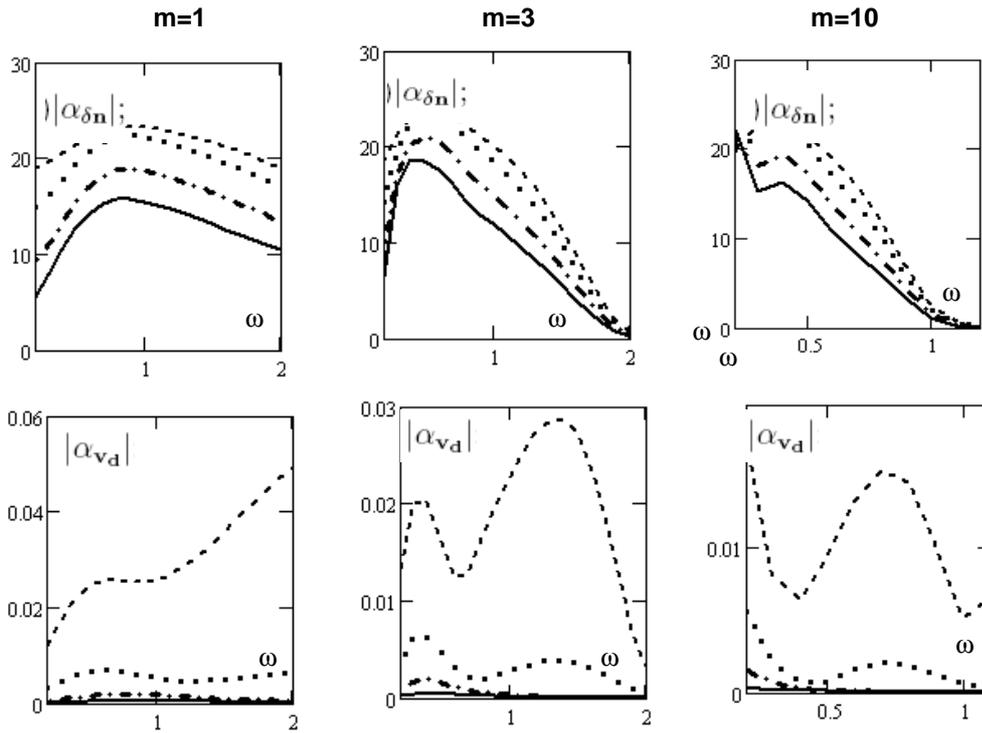


Figure 2: Some responses for  $m = 1, 3, 10$

All figures show the existence of **resonances due to global structure modes** in the range where the linear theory gives only strongly damping or unstable modes. The responses for dust velocity and ion drift velocity are the largest at the periphery of the structures. Some dependencies of responses on dust masses are illustrated by Fig.2. With increasing of the dust masses the global

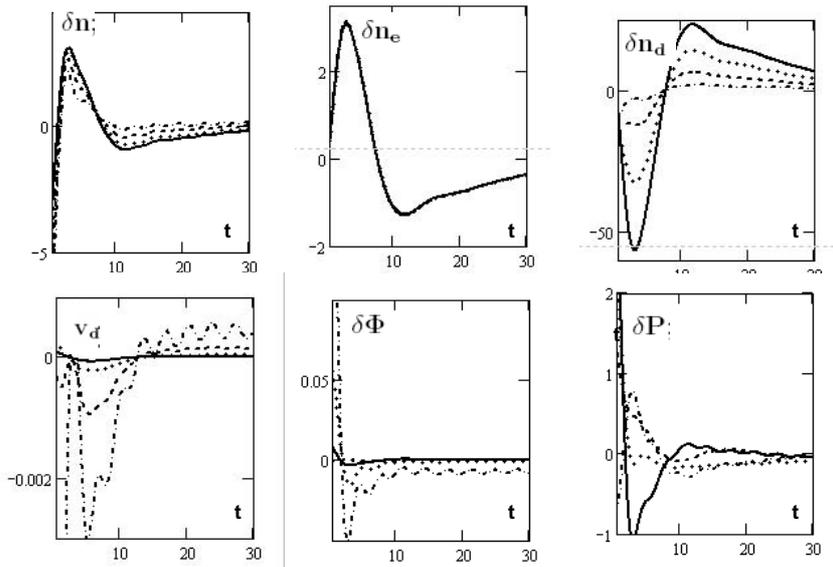


Fig.3

Figure 3: Illustration of structure stability;  $m = 3, R_{str} = 0.689$

mode resonances are shifted toward lower frequencies. The stability problem is solved by assuming that initial flux perturbations are constant in time for certain time interval (from  $-t_0$  to  $+t_0$ ) and vanishes for  $t > t_0$ . Then the asymptotic behavior in time of the perturbations is investigated. We present an example of numerical calculations for the structure used above with

$m = 3, t_0 = 0.5$  following the time development of the perturbations from  $t = 0.5$  up to  $t = 3$  (Fig.3). All perturbations are decaying in time and only for some of them (dust velocity, flux, ion drift velocity) there are left small amplitude oscillation with period of order of global mode period. In general it was **proved that the structures are stable**. This allows us, to consider the structures as some kind of "super-grains" surrounded by plasma fluxes, electron and ion polarizations the distributions of which is determined by the parameters at the structure surface.

It is found that in the dust free space, surrounding the structures, a large over-screening could be present. The over-screening is largest for small and medium structures The screening is not trivial since an important role is played by competition between convective and diffusion contributions to the total plasma self-consistent flux. Each of these two components is large as compared to the total flux and they are oppositely directed. This causes a decrease of ion densities with distance for  $r > R_{str}$  till the convection flux became small and the ion density starts to increase rapidly. The screening of the negative charge appears at about  $R_{scr} = (2 - 4)R_{str}$ . For  $r > R_{scr}$  further large increase of ion density creates positive charges or an over-screening with subsequent decrease of the field at large distances. Figure 4 shows the dependence of normalized electric field  $E_N = eE\lambda_{in}/T_i$  for  $r > R_{str}$  for a structure with  $R_{str} = 0.689\lambda_{in}$ . The structures can be separated by distances comparable with several structure sizes and the effect of shadow attraction is much larger than for usual grains. Both the presence of positive charges in the surroundings of the structures and shadow effects causes can create attraction of different structures. The attraction energy is estimated to be  $\approx 10^2 Z_{str} T_i$  with  $e^2 Z_{str} \approx (1 - 3)R_{str} T_i$  and is much larger than the attraction energy responsible for formation of an ordinary plasma crystals [1].

## References

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Fig.4 Illustration of cluster over-screening

