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Probabilistic-Statistical Approach to Description of the Fluctuational Particle Fluxes in the Plasma Edge

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1 Introduction

Studies of plasma turbulence are intimately related to plasma heating and particle transport in closed magnetic confinement systems. It is well known that heat transport in the edge plasma depends primarily on turbulent process. In connection with this, considerable recent attention has been focused on plasma turbulence. Studies of plasma density fluctuations δn, floating potential $\delta\varphi$, and turbulent particle fluxes $\tilde{\Gamma}$ have shown that their probability density function cannot be described by Gaussian distributions [1]. They have power tails $p(x,t) \propto x^{-\alpha-1}, x \to \infty$ and properties of self-similarity $p(x,t) = t^{-H} p(xt^{-H}, 1)$. Here, $p(x,1)$ is the distribution at the initial time and H is the Hurst parameter. The fact that the probability densities of amplitudes of plasma fluctuations possess these properties was established in [2, 3].

In this work fractional stable distribution was applied for description of fluctuations of plasma density δn, floating potential $\delta\varphi$, and turbulent particle fluxes $\tilde{\Gamma}$ in the edge plasma. The distributions also power tails, leptocurtic and possess self-similarity. For these reason, they can be applied to describing plasma density fluctuation and turbulent particle fluxes.

2 Generalized Diffusion Equation

Let us consider the following summation scheme. Let $X_1, X_2, \dots$ be independent, identically distributed random variables and $T_1, T_2, \dots$ be variables that are independent of both each other and also of the sequence $X_1, X_2, \dots$, which are identically distributed over the positive semi-axis. We consider the compound process

$$S(t) = \sum_{j=1}^{N(t)} X_j, \tag{1}$$

where $N(t)$ is the counting process $N(t) = \sum_{j=1}^{N(t)} T_j \leqslant t < \sum_{j=1}^{N(t)+1} T_j, \quad T_j > 0$. We assume that random variables X_j to the region of normal attraction of a strictly stable law with characteristic function $\hat{g}(k;\alpha,\theta)$ $(0 < \alpha \leqslant 2)$ and T_j belong to the region of normal attraction of a one-sided strictly stable law with characteristic function $\hat{g}(k;\beta,1)$ $(0 < \beta \leqslant 1)$. The characteristic function of the strictly stable law has the form

$$\hat{g}(k;\mu,\theta) = \exp\left\{-|k|^{\mu} \exp\left(-i\frac{\pi\theta\mu}{2}\operatorname{sign} k\right)\right\}, \tag{2}$$

where $0 < \mu \leqslant 2$, $|\theta| \leqslant \min(1, 2/\mu - 1)$ [4].

The sum (1) is physically interpreted as a random coordinate of a particle at time t that undergoes random walk in the CTRW model. X_j is then interpreted as a jump of

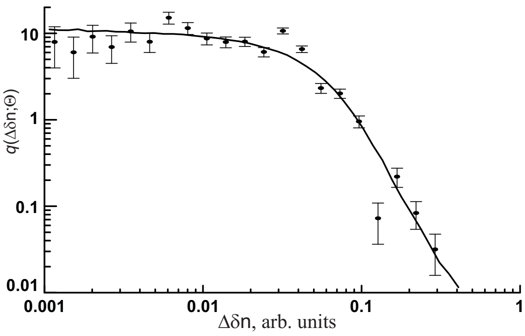

Figure 1. PDF of increments of amplitudes of δn_j^* in the ECR heated plasma. Points show the empirical distribution, a solid curves shows the FSD with parameters $\Theta = (\alpha, \beta, \theta, \lambda)$, where $\alpha = 1.66$, $\beta = 0.85$, $\theta = 0$, $\lambda = 0.031$.

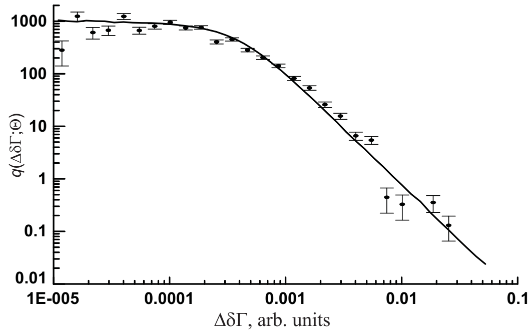

Figure 2. PDF of increments of amplitudes $\tilde{\Gamma}^*$ in the ECR heated plasma. Points show the empirical distribution, a solid curves shows the FSD with parameters $\Theta = (\alpha, \beta, \theta, \lambda)$, where $\alpha = 1.11$, $\beta = 0.96$, $\theta = 0$, $\lambda = 0.00033$.

the particle between two spatial points, whereas T_j is interpreted as a random waiting time of the particle between two successive jumps.

It was shown that, on the above assumptions, asymptotic (at $t \to \infty$) distribution of the sum (1)) is described by the FSD

$$q(x; \alpha, \beta, \theta) = \int_0^\infty g(xy^{\beta/\alpha}; \alpha, \theta) g(y; \beta, 1) y^{\beta/\alpha} dy, \quad (3)$$

where $g(x; \alpha, \theta)$ is the strictly stable probability density, whereas $g(y; \beta, 1)$ is the one-sided strictly stable probability density with characteristic function (2).

It can be shown (see [5]) that the asymptotic coordinate distribution is described by the equation

$$\frac{\partial^\beta p(x,t)}{\partial t^\beta} = -D(-\Delta)^\alpha p(x,t) + \frac{t^{-\beta}}{\Gamma(1-\beta)} \delta(x), \quad (4)$$

where $\partial^\beta / \partial t^\beta$ is the Riemann–Liouville fractional derivative and $(-\Delta)^\alpha$ is the fractional-order Laplacian [6]. In [7], it is shown that the solution to this equation has the form

$$p(x,t) = (Dt)^{-H} q\left(|x| \, (Dt)^{-H}; \alpha, \beta, 0 \right), \quad (5)$$

where $q(x; \alpha, \beta, 0)$ is the FSD and $H = \beta/\alpha$ is the Hurst parametr. It follows herefrom that the density $p(x,t)$ possesses self-similarity.

3 Approximation of turbulent particle fluxes

Turbulent particle fluxes were measured in the edge plasma of the L-2M stellarator with the use of a set of three Langmuir probes. It is well known that the flux $\tilde{\Gamma}_j$ is defined as,

$$\tilde{\Gamma}_j = \delta n_j \delta v_j^r, \qquad j = 1 \ldots N,$$

where δn_j are plasma density fluctuations, δv_j^r are fluctuations of the radial velocity, and N is the number of points in the time sample being processed.

Hence, the problem reduces to finding estimates $\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\lambda}$ of parameters of the FSD from measured time sequences $\alpha, \beta, \theta, \lambda$ by using the algorithm described in Appendix A.

However, it is incorrect to apply formulas (8) immediately to time sequences $\delta\varphi_j^1$, $\delta\varphi_j^2$, δn_j, $\tilde{\Gamma}_j$. First, we deal with real physical variables that vary in time at a random but finite rate. The finite rate implies that two successive variables in the measure time sample are not statistically independent. Second, an analog-to-digital converter (ADC) introduces additive noise into the measurement results. This noise disturbs homogeneity of the sample and, consequently, we cannot use the algorithm for estimation of the FSD parameters. We have to sample statistically independent variables and to eliminate the noise from the time sequences.

We assume that the observed plasma turbulence is due to the presence of structures that have an enhanced density of charged particles in comparison with the surrounding plasma. We assume that the density of charged particles in a given structure does not depend on the density in the neighboring structures. Choosing only maximum and minimum values of the probe current and passing to their increments, we obtain a sample of independent, identically distributed random variables $\Delta\tilde{\Gamma}'_k = \tilde{\Gamma}'_k - \tilde{\Gamma}'_{k+1}, k = 1, \dots, N' - 1$.

The procedure for exclude ADC noise from data series is follows: before the heating pulse, we determine values $C_{\max}$ and $C_{\min}$ and compute $\Delta C = |C_{\max} - C_{\min}|$. From the sample $\Delta\tilde{\Gamma}'_k$ we exclude increments that satisfy the condition $|\Delta\tilde{\Gamma}'| < \Delta C$, and proceed to the new sample $\Delta\tilde{\Gamma}''_m, m = 1, \dots, N''$:

$$\Delta\tilde{\Gamma}^*_l = \Delta\tilde{\Gamma}''_{3l-2} - 1/2\left(\Delta\tilde{\Gamma}''_{3l-1} + \Delta\tilde{\Gamma}''_{3l}\right), \qquad (6)$$

where $l = 1, \dots, [N''/3]$. Here, $[A]$ is the integer part of A. This transformation translates the random variable $Y(\alpha, \theta)$ in (7) from the class of stable laws into a class of strictly stable laws; in so doing, the values of characteristic exponents α and β do not change [4]. From the sample $\Delta\tilde{\Gamma}^*_l$ and formulas (8) we can find estimates $\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\lambda}$ of the parameters $\alpha, \beta, \theta, \lambda$.

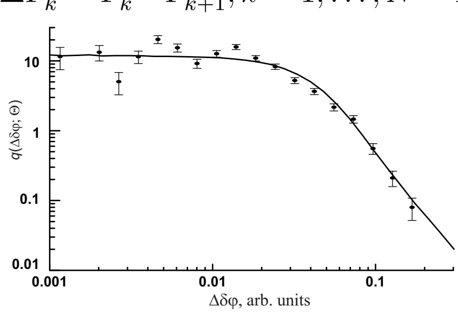

Figure 3. PDF of increments of amplitudes of $\delta\varphi_j^*$ in the ECR heated plasma. Points show the empirical distribution, a solid curves shows the FSD with parameters $\Theta = (\alpha, \beta, \theta, \lambda)$, where $\alpha = 1.55$, $\beta = 1.0$, $\theta = 0$, $\lambda = 0.018$.

4 Results and conclusions

Figures 1 - 3 show the empirical probability densities of increments of amplitudes of plasma density fluctuations (the saturation ion current) $\Delta\delta n^*$, floating potential $\Delta\delta\varphi^*$ and turbulent particle fluxes $\Delta\tilde{\Gamma}^*$. Comparing these distributions with FSDs shows that, within the statistical errors, the FSDs give good fit to the empirical distributions. Estimated parameters of FSDs are also shown.

The FSD, it will be remembered, is a solution to the generalized diffusion equation (4), hence the coincidence of FSD with the empirical density means that the increments of fluctuation amplitudes obey the generalized diffusion equation. This allows the following conclusions: stochastic process underlying the fluctuations of plasma density, floating potential, and turbulent particle fluxes can formally be described in the CTRW model. It should be borne in mind that the walk occurs in the phase space (the time is a characteristic of interest). We use the space $(t, \delta\varphi))$ for Fig. 3, $(t, \delta n)$ for Fig. 1, and $(t, \tilde{\Gamma})$ for Fig. 2. Accordingly, the coordinate x in equation (4) for each of these cases will take one of three physical variables: $\delta\varphi, \delta n, \tilde{\Gamma}$.

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A Estimation of the FSD parameters

Let $Z_1, Z_2, \ldots, Z_n$, $n \leqslant 4$ be independent, identically distributed random variables with density (3). The problem is to determine estimates $\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\lambda}$ of unknown parameters $\alpha, \beta, \theta, \lambda$. This problem was solved in [8], where a factional stable stochastic variable was represented in the form

$$Z(\alpha,\beta,\theta,\lambda) = \lambda \frac{Y(\alpha,\theta)}{[S(\beta,1)]^{\beta/\alpha}} \tag{7}$$

where $Y(\alpha,\theta)$ and $S(\beta,1)$ are strictly stable $(0 < \alpha \leqslant 2)$ and one-sided strictly stable $(0 < \beta \leqslant 1)$ stochastic variables with characteristic function (2).

Here, we only present the final result. The formulas for estimates $\hat{\alpha}$, $\hat{\beta}$, $\hat{\theta}$, $\hat{\lambda}$ of parameters $\alpha, \beta, \theta, \lambda$ has the form

$$\begin{aligned} &\hat{\theta} = 1 - \tfrac{2}{n} \textstyle\sum_{j=1}^{n} \mathbf{I}(X_j < 0), \quad && \hat{\alpha} = \frac{2\pi}{\sqrt{12V_n + \pi^2\left(2Z_n + 3\hat{\theta}^2 - 1\right)}}, \\ &\hat{\beta} = Z_n \hat{\alpha}, && \hat{\lambda} = \exp\left\{U_n - \mathbb{C}(Z_n - 1)\right\}, \end{aligned} \tag{8}$$

where $Z_n = \left(1 + \frac{M_n}{2\zeta(3)}\right)^{1/3}$, U_n, V_n, M_n are sample centered logarithmic moments

$$U_n = \frac{1}{n}\sum_{j=1}^{n} \ln|X_j|, \quad V_n = \frac{1}{n}\sum_{j=1}^{n} \left(\ln|X_j| - U_n\right)^2, \quad M_n = \frac{1}{n}\sum_{j=1}^{n} \left(\ln|X_j| - U_n\right)^3,$$

$\mathbf{I}(A)$ is the indicator of event A, $\mathbb{C} = 0.5772156649015325\ldots$ is the Eulerian constant, and $\zeta(3)$ is the Riemann function at point 3.

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