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Transport barriers in two dimensional magnetohydrodynamic turbulence

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Introduction

Predictive modelling of anomalous transport requires a theory of multi-scale interaction between turbulence and coherent structures, in particular, shear flows and magnetic fields. Given the ubiquitous nature of shear flows in magnetised plasmas, and in particular their propensity to reduce turbulent transport by shear flow suppression which may lead to the formation of transport barriers [1], the investigation of how a shear flow as well as a magnetic field affects turbulent transport is essential for understanding and prediction of transport in these systems.

The purpose of this work is to calculate turbulent transport coefficients in magnetised plasmas in the presence of mean shear flows as a means to determine the transport properties of these systems [2]. The calculation of turbulent viscosity and turbulent diffusivity will enable the quantitative prediction for turbulent transport reduction (or enhancement) due to shear flows and magnetic field strength separately. In this work we extend using numerical calculation a previous analytical study of a short correlated fluid forcing [3] to accommodate a forcing of finite frequency and memory time. Using this fluid forcing of finite correlation time allows us to consider real systems where turbulence is influenced by waves (e.g. the solar tachocline and fusion devices) [4]. With these predictions other physical processes for enhanced transport may be proposed to explain observed phenomena involving fast transport.

Model

We consider a visco-resistive incompressible MHD fluid stirred by an external small-scale forcing in the presence of a uniform guide magnetic field, B, aligned with a mean shear flow, V. The Navier-Stokes and induction equations and solenoidal and incompressible constraints are suitably non-dimensionalised using the Alfvén velocity and characteristic length scale of the system, and we use a unit magnetic Prandtl number. In a 2D domain with (x,z) coordinates these equations can be re-written in terms of the magnetic vector potential $\mathbf{b} = \nabla \times a\hat{\mathbf{y}}$ and the vorticity $\Omega\hat{\mathbf{y}} = \nabla \times \mathbf{v}(x,z)$. We assume clear scale separation between the mean fields and fluctuations and separate velocity (vorticity) and magnetic field (vector magnetic potential) as $\mathbf{v} = \mathbf{V} + \mathbf{v}'$ (for vorticity $\Omega = \Omega_0 + \Omega'$). Taking the governing equations we perform quasi-linear analysis, neglecting the local interaction of fluctuations compared to nonlocal interactions between small and large scale fields. The equations of the fluctuating fields are now

$$\left[\frac{\partial}{\partial t}+V\frac{\partial}{\partial x}-\nu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\right]\Omega' = -B\frac{\partial}{\partial x}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)a'+f, \quad (1)$$

$$\left[\frac{\partial}{\partial t}+V\frac{\partial}{\partial x}-D\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\right]a' = -v_z'\frac{\partial A}{\partial z}, \quad (2)$$

where ν and D are molecular viscosity and diffusivity respectively. The mean field equations for large scale velocity and magnetic field ($A = \langle a\rangle$) are similarly found to be

$$\left[\frac{\partial}{\partial t}+V\frac{\partial}{\partial x}-\nu\frac{\partial^2}{\partial z^2}\right]V = -\frac{\partial}{\partial x}\langle p+\frac{b'^2}{2}\rangle-\frac{\partial}{\partial z}\langle v_x'v_z'-b_x'b_z'\rangle, \quad (3)$$

$$\left[\frac{\partial}{\partial t}+V\frac{\partial}{\partial x}-D\frac{\partial^2}{\partial z^2}\right]A = -\frac{\partial}{\partial z}\langle a'v_z'\rangle. \quad (4)$$

In equation (3) the term for total stress can be seen to represent a turbulent viscosity ν_T by taking $\langle v_x'v_z'-b_x'b_z'\rangle = -\nu_T\partial V/\partial z = \nu_T\Omega_0$. Similarly in equation (4), the term for magnetic flux can represent a turbulent magnetic diffusivity D_T via $\langle a'v_z'\rangle = -D_T\partial A/\partial z = D_T B$. The computation of ν_T and D_T requires the solution of equations (1) and (2) for Ω' and a' and the evaluation of the correlation functions $\langle v_x'v_z'-b_x'b_z'\rangle$ and $\langle a'v_z'\rangle$. These solutions are found in [2] and [3] for the cases of a fluid forcing f of short and finite correlation time respectively. In the following we compute the dependence of the correlations $\langle v_x'v_z'-b_x'b_z'\rangle$ and $\langle a'v_z'\rangle$, and hence ν_T and D_T, on magnetic field strength and shear flow strength, for the latter case of a spatially homogeneous and temporally stationary fluid forcing with finite memory time and frequency.

Results

The equations for the correlations of the magnetic flux, the Reynolds stress $\langle v_x'v_z'\rangle$ and the Maxwell stress $\langle b_x'b_z'\rangle$ (comprising the total stress) and the kinetic and magnetic energies of the fluctuations, $\langle v'^2\rangle$ and $\langle b'^2\rangle$, form double integrals, denoted $I_{MF,RS,MS,KE,ME}$, which are numerically computed. Each is computed for a range of values of shear strength given by $\xi \propto \Omega_0^{-1}$ for fixed magnetic field strength given by $M \propto B$, and vice versa, ensuring $\xi M \gg 1$ (strong shear limit). The turbulent frequency ω is chosen to be the value which produces the maximum transport due to resonance in the system (i.e. $\omega = k_x\Omega_0 \pm k_x B$).

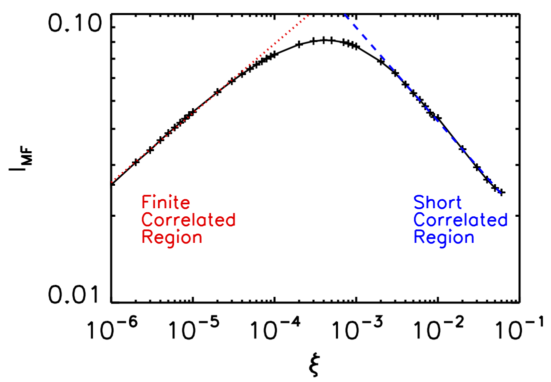

Figure 1: Magnetic flux integral with varying shear and magnetic field strength.

Figure 1 show the results for the integrals of magnetic flux, up to a factor of $(B\Omega_0)^{-1}$ which multiplies the result. No dependence on M for fixed ξ is found for the integrals of any of the individual correlation functions, therefore any reduction due to magnetic field strength comes from factors outside the integral. We observe two distinct regions in figure 1 corresponding to a strong shear strength region having a fluid forcing of finite correlation time, and a weak shear strength region having a fluid forcing of short correlation time. The power law scalings observed in these regions gives a reduction of turbulent diffusivity like $D_T \propto B^{-2}\Omega_0^{-5/4}$ and $D_T \propto B^{-2}\Omega^{-2/3}$ respectively.

The results for the integrals of Reynolds and Maxwell stress against varying shear strength is shown in figures 2, up to a multiplying factor of Ω_0^{-2}. Two distinct regions are again observed, corresponding to a strong shear strength region having a fluid forcing of finite correlation time, and a weak shear strength region having a fluid forcing of short correlation time. Both panels of figure 2 are seen to be qualitatively and quantitatively similar, their difference resulting in a small total stress. In the case of a fluid forcing of short correlation time this provides a reduction of turbulent viscosity like $\nu_T \propto B^{-2}$ (for a fluid forcing of finite correlation time the difference of the values is too small to permit exact determination the scaling of ν_T).

For the kinetic and magnetic energy integrals the results against varying shear strength are displayed in figure 3, up to a multiplying factor of Ω_0^{-2}. Fig. 3 exhibits the same distinct regions as Figs. 1 and 2 corresponding to fluid forcings of finite and short correlation time. A reduction of kinetic and magnetic energies in these two regimes like $\Omega_0^{-4/3}$ and $\Omega_0^{-2/3}$ respectively results. It is interesting to note that the kinetic and magnetic energies are qualitatively and quantitatively similar in both panels of figure 3, due to Alfvénization.

Conclusion

We have investigated the effect of large scale shear flow and magnetic field on magnetic flux (and hence D_T), Reynolds and Maxwell stress (hence ν_T) and kinetic and magnetic energy in

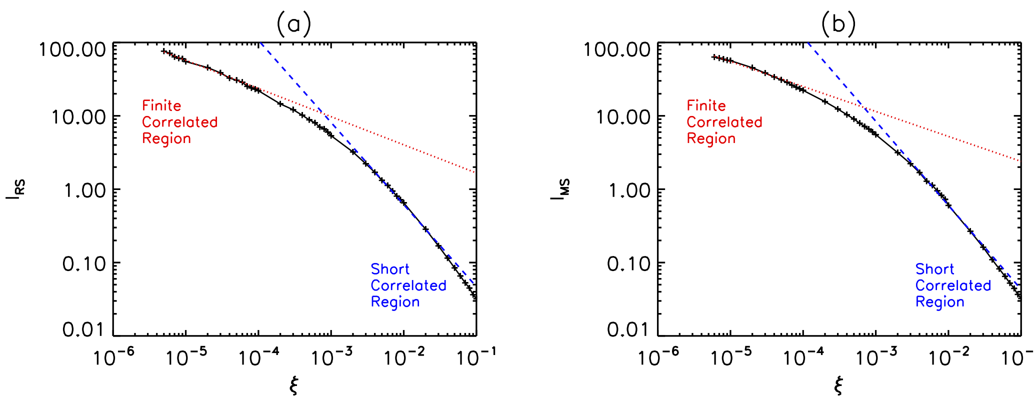

Figure 2: Reynolds/Maxwell stress integrals for varying shear flow strength.

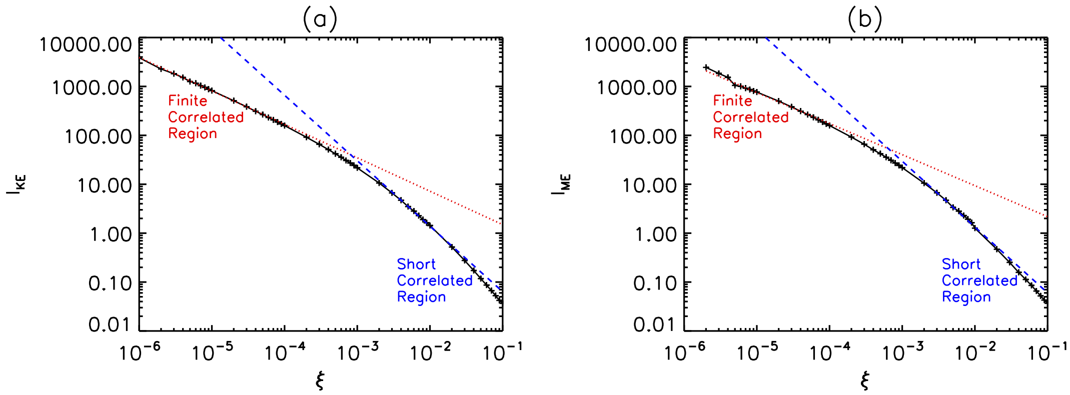

Figure 3: Kinetic/magnetic energy integrals for varying shear flow strength.

2D MHD turbulence, confirming Ref. [3] for a fluid forcing of short correlation time and finding new scalings for a fluid forcing of finite correlation time. In particular, we found that while shear flows suppress both transport and turbulent intensity, magnetic fields suppress transport only. This result not only demonstrates the crucial effect of shear flows and magnetic fields on turbulence regulation but also highlights the important difference in their effects. Furthermore, in all cases a stronger reduction by flow shear was found for the case of the forcing with finite correlation time, despite resonance with this type of forcing. This suggests the importance of the property of the turbulent flow in flow shear suppression. The stronger suppression due to a forcing of finite correlation time is due to the more consistent fluid motion that results, which allows shear flows more time to distort and disrupt the turbulent eddies responsible for enhanced transport. The similar results observed for the Reynolds and Maxwell stress and the equipartition of kinetic and magnetic energies is a result of the magnetic back reaction causing Alfvénization of the flow, where $\mathbf{v}' \sim \pm\mathbf{b}'$ and wave-like turbulence forms. The strong reduction of ν_T and D_T could be problematic in understanding fast magnetic dissipation/reconnection and momentum transport observed in astrophysical and laboratory plasmas, and suggests that additional physical mechanisms should be explored to explain the observations. Extensions of this work include incorporating other physical effects such as inhomogeneous turbulence, non-uniform magnetic fields or time-dependent shear flows to create a more applicable model.

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