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Enhanced turbulent mixing in an oscillatory zonal flow

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Introduction The use of zonal flows to regulate turbulent transport is now widely recognised as a vitally important transport process with applications to a variety of physical systems. Of particular interest are the edge of fusion tokamaks [1] and the solar tachocline [2]. Given the importance of Zonal flows it is imperative that we have an accurate understanding of the fundamental physics.

The purpose of this paper is to perform a detailed quantitative study of the effect of oscillatory shear flow on the turbulent transport of a passive scalar field [3]. In particular, we derive the asymptotic scalings of turbulence amplitude and transport with the rms shear strength via numerical integrations of the theoretical results derived by Kim [4]. We distinguish three different scaling regimes depending on the characteristic time scales of turbulence and oscillatory shear flow, identify the two types of resonances (Doppler and parametric resonances), and then obtain asymptotic scalings with rms shearing rate valid in each regime by numerical computations. We show that turbulent diffusion depends crucially on the competition between suppression due to shearing and enhancement due to the resonances, thereby suggesting that oscillatory shear flows can enhance the value of turbulent diffusion, accelerating turbulent mixing. We also show that an oscillatory shear flow is significantly less efficient at moderating turbulent transport than an ordinary non-oscillating flow.

Governing equations We consider a passive scalar field model where a passive scalar n is advected by a given turbulent flow $\overline{v}$ and shear flow $\mathbf{U}_0$ while being diffused by molecular dissipation D. By quasi-linear analysis, the fluctuatingscalar field n' evolves according to the following equation:

$$\{\partial_t + \mathbf{U}_0 \cdot \nabla\} n' = -v_x \partial_x N_0 + D\nabla^2 n'. \tag{1}$$

Here, $N_0 = \langle n \rangle = N_0(x)$ is the large-scale component. We assume that the shear flow is in the y direction, varying linearly in x, and is oscillatory in time of the form $\mathbf{U}_0 = -x\Omega_m \sin(\omega_z t)\,\mathbf{y}$; ω_z and Ω_m are the frequency and rms shearing rate of the oscillating shear flow. Note that since both N_0 and $\mathbf{U}_0$ depend only on x while $\mathbf{U}_0$ is in the y direction, there is no direct effect of the shear flow on the mean field N_0 (i.e. $\mathbf{U}_0 \cdot \nabla N_0 = 0$). That is, the shear flow influences the mean field only indirectly through its effect on turbulence.

The flux and amplitude are obtained by assuming that the statistics of the turbulent flow v_x

are spatially homogeneous and temporally stationary with the following correlation function:

$$\langle \tilde{v}_x(\mathbf{k}_1,t_1)\tilde{v}_x(\mathbf{k}_2,t_2)\rangle = (2\pi)^2\delta(\mathbf{k}_1+\mathbf{k}_2)\phi(\mathbf{k}_2,t_2-t_1)\,, \tag{2}$$

where ϕ is the correlation function of v_x in Fourier space. Further, the random turbulent flow is taken to have characteristic frequency ω and correlation time $\tau_c = 1/\gamma$. Specifically, ψ is taken to have Lorentzian frequency spectrum centered around ω with width γ as $\phi(\mathbf{k}_2,t_2-t_1) = \psi(\mathbf{k}_2)\int(d\omega'/\pi)e^{-i\omega'(t_2-t_1)}\gamma/[(\omega'-\omega)^2+\gamma^2]$. It can be readily shown that the velocity amplitude is related to the power spectrum ψ as $\langle v_x^2\rangle = \int d^2k\psi(\mathbf{k})/(2\pi)^2$.

The flux and amplitude of fluctuation of scalar fields, averaged over one oscillation of the shear flow ($2\pi/\omega_z$) as well as over the statistics of the turbulence follow from Eqs. (9)-(10) in [4] and can be expressed in the following dimensionless form:

$$\langle n'v_x\rangle_t = -\frac{\partial_x n_0}{(2\pi)^2}\int d^2k\psi(\mathbf{k})\int_0^{2\pi}\int_0^{\tau} d\tau d\tau_1 A(\beta,\overline{\omega},\tau,\tau_1)\times e^{-\overline{\gamma}(\tau-\tau_1)-\tau_D^{-1}B(\alpha,\tau,\tau_1)}, \tag{3}$$

and

$$\langle n'^2\rangle_t = \frac{(\partial_x n_0)^2}{(2\pi)^2}\int d^2k\psi(\mathbf{k})\int_0^{2\pi}\int_0^{\tau}\int_0^{\tau} d\tau d\tau_2 d\tau_1 A(\beta,\overline{\omega},\tau_1,\tau_2) \times e^{-\overline{\gamma}(\tau_1-\tau_2)-\tau_D^{-1}(B(\alpha,\tau,\tau_1)+B(\alpha,\tau,\tau_2))}. \tag{4}$$

Here,

$$A(\beta,\overline{\omega},\tau,\tau_1) = \cos\{\overline{\beta}\alpha(\cos(\tau)-\cos(\tau_1))-\overline{\omega}(\tau-\tau_1)\},\ \text{and}$$

$$B(\alpha,\tau,\tau_1) = (\tau-\tau_1)\{1+\alpha^2(1+\frac{1}{2}\cos 2\tau_1)+\frac{\alpha^2}{4}(\sin 2\tau+3\sin 2\tau_1-8\cos\tau_1\sin\tau)\}.$$

The dimensionless variables in Eqs. (3) and (4) are defined using ω_z^{-1} as a unit of time (i.e., $\tau = \omega_z t$) as follows: $\alpha = \frac{\Omega_m}{\omega_z} \propto$ rms shearing rate, $\overline{\gamma} = \frac{\gamma}{\omega_z} = \frac{1}{\tau_c\omega_z} \propto$ decorrelation rate of turbulence, $\overline{\beta} = kx \propto$ scale separation between the mean and fluctuations, $\overline{\omega} = \frac{\omega}{\omega_z} \propto$ frequency of turbulence and $\tau_D = \frac{\omega_z}{Dk^2} \propto$ molecular diffusion time scale. It is important toote that $k = k_y$ in $\overline{\beta}$ is the typical wavenumber of turbulence in the y direction while $k_x(t) = k_x(t_1)+k_y\int_{t_1}^{t} dt'\Omega_m\sin(\omega_z t')$ evolves in time due to shearing.

Results An initial analytical examination showed that for turbulence with a sufficently long correlation time, there is the possibility of Doppler resonance. This occours when the Doppler shifted turbulent frequency is zero, i.e.

$$\omega_d \approx \omega - ck_y\sqrt{\langle U_0(x,t)^2\rangle_t} = \omega - c\overline{\beta}\Omega_m \approx 0.$$

Note that a similar resonance condition in terms of rms velocity also holds in the case of random shear flow, while it was overlooked in the analysis of an oscillatory shear flow [4]. In the case of a steady shear flow $U_0\hat{y}$, this resonance condition will become exact as $\omega_d = \omega - k_yU_0 = 0$.

The numerical investigation identified the three distinct scaling regimes with rms shearing rate, depending on the value of turbulence decorrelation time. These are the short, medium and long correlation time limits: *Short correlation time* ($\overline{\gamma} \rightarrow \infty$): there is no resonance due to too rapid change in turbulence characteristics, with the flux decaying as $\overline{\gamma}^{-1}$. *Medium correlation time* ($1 < \overline{\gamma} \ll \infty$): turbulent transport is enhanced around the Doppler resonance where the wave phase speed matches the rms velocity of oscillatory shear which is clearly seen in Fig.1. Increasing either the shear or the characteristic frequency from the Doppler resonance point acts to quench turbulent transport. Our numerical results with $\overline{\gamma} = 5$ show that if the flux is normalized to the case of $\alpha = 0$ and $\overline{\varpi} = 0$, the flux, thereby turbulent diffusion, scales as, $\propto \Omega_m^{-0.5343}$ at resonance points (see Fig. 2). The dependence on Ω_m at resonance point is thus weaker than that in the strong shear limit ($\propto \Omega_m^{-1.015}$). Furthermore, the value of flux at resonance point is larger than that in the absence of the shear flow, *manifesting the enhancement of turbulent transport due to the oscillating shear flow*. If the flux is normalized to the case $\alpha = 0$, the flux scales as $\propto \Omega_m^{0.831}$ for all possible values of turbulence frequency ω, highlighting the increase in turbulent diffusion due to oscillatory shear flow! *Long correlation time* ($0 < \overline{\gamma} \ll 1$): there is parametric (harmonic) resonance of the form $\omega = n\omega_z$ where $n \in \mathbb{N}$ in addition to Doppler resonance [4]. At these resonance points, the numerically computed flux and amplitude show significant enhancements as shown in Fig.3, with their maximum values provided by Doppler resonance peaks.

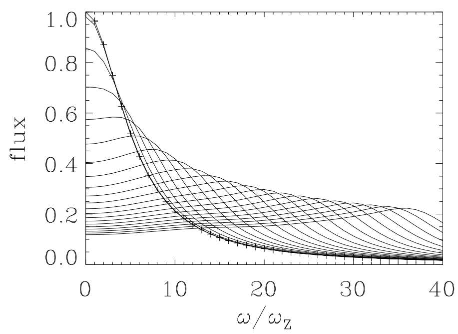

Figure 1: Flux as a function of $\overline{\varpi} = \omega/\omega_z$ with α increasing from top to bottom: top line no shear, bottom line strong shear

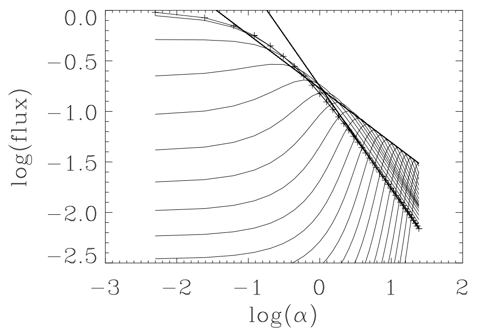

Figure 2: Flux as a function of α for $\overline{\varpi} = \omega/\omega_z$ with $\overline{\varpi}$ increasing from top (random turbulence) to bottom (wave like turbulence)

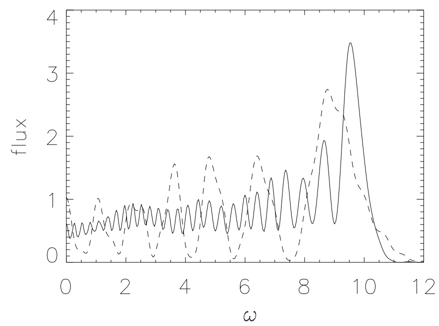

Figure 3: Flux as a function of ω for $\omega_z = 0.5$ (dotted line) and $\omega_z = 0.125$ (solid line) when $\overline{\gamma} = 0.01$, showing multiple harmonic resonance in addition to Doppler resonance

Conclusion Our results suggest that oscillatory shear flows can enhance the value of turbulent diffusion, speeding up turbulent mixing, depending on the characteristics of shear flow and turbulence. Enhancement of turbulent mixing can then have either welcoming or unwelcoming consequences in turbulent mixing. A typical example of the latter can be found in laboratory plasmas where the confinement is a critical issue. These results thus suggest that for the understanding and predictive modelling of turbulent transport in plasmas, it is necessary to determine frequency and power spectra of shear flows and turbulence. Our results have implications for turbulent mixing in many other fields such as geophysics, oceanography, atmospheric physics, solar physics, and magnetohydrodynamics where shear flows and turbulence are main players in transport. In particular, similar results [5] obtained in the passive scalar fields model are expected to be valid for the transport of magnetic fields in the 2D magnetohydrodynamic turbulence as long as the backreaction of magnetic fields is negligible (i.e. in the kinematic regime). An interesting question is then what happens to the transport/diffusion of large-scale magnetic fields when the backreaction is sufficiently strong to modify the characteristics of turbulence. In particular, it would be interesting to study whether oscillatory shear flows can weaken the severe quenching in the amplification of magnetic fields (the so-called dynamos) in 3D and their diffusion rate in 2D due to the magnetic backreactions. Furthermore, the backreactions of turbulence on shear flows will also have an importance consequence on the evolution of shear flows, for instance, leading to complex temporal and spatial dynamics, thereby dynamically determining the frequency and power spectra of shear flows, which have been assumed to be given in this paper. These problems will be addressed in future works.

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