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The inhibition of charged particle transport by a new streaming instability

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Diffusive shock acceleration (DSA) of cosmic rays (CR) occurs at a large range of astrophysical shocks. CR accelerated at the outer shock of supernova remnants (SNR) are thought to account for majority of CR arriving at the earth with energies less than 10^{15}eV, and may possibly account for galactic CR production in the range 10^{15} to 10^{17}eV. DSA depends upon rapid scattering of CR by magnetic field fluctuations in the plasma both upstream and downstream of the shock. In the case of SNR, even if the fluctations are well-developed ($\delta B \sim B$) the typical interstellar magnetic field of ~3μG is insufficient to accelerate CR to the break, known as the 'knee', in the galactic CR spectrum at 10^{15}eV (Lagage & Cesarsky 1983). If $\delta B \sim B$, the scattering mean free path can be as small as a CR Larmor radius, but the Larmor radius is too large for acceleration to 10^{15}eV in the lifetime of the SNR. However, it has been shown (Lucek & Bell 2000, Bell 2004) that CR in the shock precursor excite an instability that can amplify the magnetic field by orders of magnitude above its initial seed value. We discuss here whether this instability occurs in laser-produced plasmas with energetic electrons taking the role of the cosmic rays.

In the astrophysical case, CR are mainly highly relativistic protons which a Larmor radius much greater than the Larmor radius of thermal particles. The instability grows at wavelengths less than CR Larmor radius. The CR are more or less undeflected by the local magnetic field on the scale of a perturbation wavelength, but they carry an electric current j_{CR} and are subject to a force $j_{CR} \times B$ as they stream ahead of the shock in the precursor. Although undeflected themselves, the CR exert an equal and opposite force on the thermal plasma which behaves magnetohydrodynamically because of the small Larmor radius of the thermal electrons and ions. In its simplest form, the instability can be modelled by adding an extra $-j_{CR} \times B$ term to the momentum equation for the thermal plasma and treating j_{CR} as both uniform and unchanging. The thermal plasma responds by moving with velocity u and distorting the magnetic field which is frozen into the thermal plasma according to ideal MHD. In the simplest and most rapidly growing configuration, the zeroth order magnetic field B and the CR current j_{CR} are parallel or anti-parallel in the z direction and all quantities vary spatially only with z. The equations describing the instability reduce to

$$\frac{\partial B}{\partial t} = -u \times B \qquad \rho \frac{\partial u}{\partial t} = -B \times (\nabla \times B) - j_{CR} \times B$$

ρ, B_z and j_{CR} are all uniform and unchanging because the motion is transverse to the *z* direction. This equation has an exponentially growing solution with a growth rate

$$\gamma = \left(\frac{kBj_{CR}}{\rho} - k^2 \mathrm{v}_A^2 \right)^{1/2}$$

where $\mathrm{v}_A=(B^2/\rho\mu_0)^{1/2}$ is the Alfven speed in the unperturbed magnetic field B_z. The $k^2\mathrm{v}_A^2$ term limits the instability to wavelengths for which $j_{CR}>kB/\mu_0$. The maximum growth rate is $\gamma_{\max}=j_{CR}(\mu_0/2\rho)^{1/2}$ at a wavenumber $k_{\max} = \mu_0 j_{CR}/2B$.

If we replace the CR current by the current carried by fast electrons into a laser produced plasma and assume that the same analysis holds then

$$\gamma_{\max} = 2.5\, I_{19} T_{MeV}^{-1} \rho_{cgs}^{-1/2}\ \mathrm{psec}^{-1} \qquad k_{\max} = 2\pi\, I_{19} T_{MeV}^{-1} B_{100}^{-1}\ \mu m^{-1}$$

where I_{19} is the energy carried by the fast electrons in units of $10^{19}\mathrm{Wcm}^{-2}$, T_{MeV} is the fast electron energy in MeV, ρ_{cgs} is the mass density in gm cm^{-3}, and B_{100} is the magnetic field in units of 100MG.

However, the assumptions regarding the thermal plasma, although suitable in astrophysics, do not always apply in laser-plasmas and we need to re-derive the dispersion relation to include non-MHD behaviour. The ideal MHD Ohm's law does not apply, so we choose another set of equations which do not assume that the thermal particles are magnetised and collisionless. If we replace the Ohm's law by a more general electron momentum equation and include electron-ion collisional scattering, the resulting linearised dispersion relation is

$$\omega^2 + \left(\frac{\gamma_0^2}{\omega_{ci}} \pm \frac{k^2 \mathrm{v}_A^2}{\omega_{ci}} - \frac{i\nu_{ei} k^2 c^2}{\omega_{pe}^2} \right)\omega + \left(\pm \gamma_0^2 - k^2 v_A^2 \right) = 0$$

where γ_0 is the growth rate for ideal MHD $\gamma_0^2=kBj_{CR}/\rho$, $\omega_{ci}=ZeB/m_i$ is the ion Larmor frequency and ω_{pe} is the electron plasma frequency. $\nu_{ei}k^2c^2/\omega_{pe}^2 = 0.01 T_{keV}^{-3/2} k_{\mu m}^2 Z$ psec^{-1} is a collisional damping rate where $k_{\mu m}$ is the wavenumber in $\mu\mathrm{m}^{-1}$ and the Coulomb logarithm is set equal to 4. $\nu_{ei}k^2c^2/\omega_{pe}^2$ is small enough to be negligible in many cases of interest, but it should be estimated for its significance in any particular calculation. The terms including the ion Larmor requency ω_{ci} and representing the effects of ion magnetisation, are the most significant non-ideal effect in laser-plasmas and we concentrate on this. Neglecting collisions and magnetic field line tension, the frequency satisfying the dispersion relation is

$$\omega = -\frac{\gamma_0^2}{\omega_{ci}} \pm_1 i \left(\pm_2 \gamma_0^2 - \frac{\gamma_0^4}{4\omega_{ci}} \right)^{1/2}$$

where $\pm_1$ and $\pm_2$ are chosen independently. With $\pm_1 = -$ and $\pm_2 = +$, instability occurs if $\omega_{ci} > \gamma_0/2$. The frequency has a real component so the instability propagates and is not purely growing. For instability, the ions must be magnetised in the sense that the ion Larmor radius must exceed half the ideal MHD growth rate. The maximum growth rate, and corresponding wavenumber k_{max} is

$$\gamma_{max} = \omega_{ci} = \frac{Z}{A} B_{100} \text{ psec}^{-1} \qquad\qquad k_{max} = 2\omega_{ci} \left(\frac{j_{CR}}{n_e e} \right)^{-1}$$

where B_{100} is the magnetic field in units of 100MG and m_i is A times the proton mass. Comparison with the ideal MHD growth rate given above shows that ion magnetisation is usually a more restrictive limitation on the maximum growth rate than the tension in the magnetic field, although this depends on the density and the magnitude of the magnetic field.

The quantity $j_{CR}/n_e e$ is the mean velocity v_0 of thermal electrons carrying the return current to balance the electric current carried by the high energy electrons. The growth rate can be recast in terms of this quantity since $\gamma_0^2 = k v_0 \omega_{ci}$, giving

$$\omega = -k v_0 \pm \left(k^2 v_0^2 - k v_0 \omega_{ci} \right)^{1/2} \qquad\qquad \gamma = \left(1 - \frac{k v_0}{\omega_{ci}} \right) \left(k v_0 \omega_{ci} \right)^{1/2}$$

The magnetisation limit on the growth rate can be re-interpreted as being the result of de-phasing due to electron motion when $k v_0 > \omega_{ci}$. The magnetic field is frozen in to the thermal electrons rather than the unmagnetised ions. If the return current carries the magnetic field a distance greater than one wavelength $2\pi/k$ during one wave period $2\pi/\omega_{ci}$, then the phase of the wave is changed and the feedback mechanism driving the instability is disrupted. Examination of the electron and ion momentum equations shows that the difference from the ideal MHD equations is that the Hall current in included, and the Hall current represents the difference between the electron and ion velocities as they appear in the $j \times B$ term in the electron momentum equation.

The de-phasing effect is removed if k is perpendicular, instead of parallel, to the fast electron current since the return current no longer transports fluctuations in the direction of variation. If the derivation is repeated with k perpendicular to j_{CR} but still with k parallel to the zeroth order magnetic field, the de-phasing term is removed. The growth rate is then restored to its ideal collisionless MHD value, $\gamma = (k\mathrm{v}_0 \omega_{ci})^{1/2}$.

Although growth is not limited to the ion cyclotron frequency in the perpendicular orientation, it is still an important frequency when the MHD and Weibel growth rates are compared. The Weibel (1959) growth rate is $\gamma = k\mathrm{v}_\mathrm{d}$ where v_d is the root mean square drift velocity averaged over all electrons. According to Ren et al (2004), Weibel growth is driven by the thermal current, so we can approximate v_d to v_0. Hence, for short wavelengths, $k>\omega_\mathrm{ci}/\mathrm{v}_0$, the Weibel growth rate is greater, but at longer wavelengths, $k>\omega_\mathrm{ci}/\mathrm{v}_0$, the MHD instability described here grows more quickly, although the nature of the two instabilities is quite different since the MHD instability requires a zeroth order magnetic field. The MHD instability grows more quickly than the Weibel instability on scalelengths greater than

$$L_{crit} = \frac{\mathrm{v}_0}{\omega_{ci}} = \frac{A^2 I_{19}}{Z^2 \rho_{cgs} T_{MeV} B_{100}} \ \mu\mathrm{m}$$

This expression indicates that the MHD instability is relatively stronger at high density and large magnetic field, but both instabilities grow more rapidly at low density. In experiments related to Fast Ignition, Baton et al (2008) have found fast electron transport inhibition in a low density plasma inside a cone target. Their simulations indicate the presence of magnetic field of the order of 100MG. The MHD instability dominates over times greater than $\omega_\mathrm{ci}^{-1} = (A/Z)B_{100}^{-1}$psec but it may also be important over shorter times because it grows by expanding small scale perturbations to a larger scale.

References

Baton SD et al 2008, Phys Plasmas 15, 042706

Bell AR 2004, MNRAS 353, 550

Lagage PO, & Cesarsky CJ 1983 A&A 125, 249

Lucek SG & Bell AR 2000, MNRAS 314, 65

Ren C et al 2004 Phys Rev Lett 93, 185004

Weibel 1959 Phys Rev Lett 2, 83