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An exact solution strategy for regular shock refraction at a density discontinuity

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Abstract

We consider the problem of regular refraction of a planar shock at an inclined planar density discontinuity, separating two gases at rest. When the shock impinges on the inclined density discontinuity, it refracts and in the hydrodynamical case 3 signals arise. Regular refraction means that these signals meet at a single point, called the *triple point*.

After reflection from the top wall, the contact discontinuity becomes unstable due to local Kelvin-Helmholtz instability, causing it to roll up and form a Richtmyer-Meshkov instability. We quantify the growth rate of the vorticity deposited on the contact interface and investigate the effect of a perpendicular magnetic field. A numerical solution strategy is presented, and compared to simulations performed by AMRVAC [2, 3]. We predict wave pattern transitions, in agreement with experiments [1].

Introduction The problem of regular shock refraction is parametrised by 6 independent initial parameters: the angle α between the shock normal and the initial density discontinuity *CD*, the sonic Mach number M of the shock, the density ratio η across the *CD* and the ratios of specific heat γ_l and γ_r on both sides of the *CD*. Adding a perpendicular magnetic field, B, also introduces a plasma-β in the pre-shock region (Fig. 1). The shock refracts in 3 signals. By solving for the primitive variables in the linear phase of the process, we can quantify the vorticity deposited on the *CD*, ω_{CD}, which is a measure for its instability.

Solution strategy During refraction the triple point moves along the contact. The streamlines in this frame are shown in Fig.1. We look for self-similar solutions in the frame where the triple point is stationary. The stationary *MHD*-equations now become $\mathbf{F}_x + \mathbf{G}_y = \mathbf{0}$, where $\mathbf{F} = (\rho v_x, \rho v_x^2 + p, \rho v_x v_y, v_x(\frac{\gamma}{\gamma-1}p + \rho\frac{v_x^2+v_y^2}{2}), v_x B, v_x\gamma\rho)^t$ and $\mathbf{G} = (\rho v_y, \rho v_x v_y, \rho v_y^2 + p, v_y(\frac{\gamma}{\gamma-1}p + \rho\frac{v_x^2+v_y^2}{2}), v_y B, v_y\gamma\rho)^t$. Note that γ is treated as a variable rather than as a parameter. The latter equation of the system expresses that $\nabla\cdot(\gamma\rho\vec{v}) = 0$. Written in *quasilinear form*: $\mathbf{u}_x +$

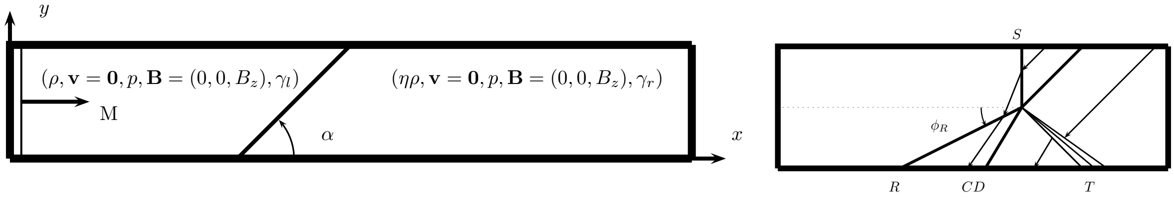

Figure 1: *Left*: Initial configuration: a shock moves with shock speed M to an inclined density discontinuity. *Right*: The wave pattern during interaction of the shock with the *CD*. Also the streamlines in the frame of the stationary triple point are shown. Notice that the direction of the velocity remains constant across the *CD*.

$\left(\mathbf{F_u}^{-1} \cdot \mathbf{G_u}\right) \mathbf{u}_y = \mathbf{0}$, where $\mathbf{u} = (\rho, v_x, v_y, p_{tot}, B, \gamma)$. Since we are searching for self-similar solutions, we can eliminate x and y by introducing $\xi = \frac{y}{x} = tan\phi$, so that $\mathbf{u} = \mathbf{u}(\xi)$. Assuming that $\xi \mapsto \mathbf{u}(\xi)$ is differentiable, we know that $\mathbf{A}\mathbf{u}_\xi = \xi \mathbf{u}_\xi$. So the eigenvalues of $\mathbf{F_u}^{-1} \cdot \mathbf{G_u}$ represent $tan\phi$, where ϕ is the angle between the refracted signals and the negative x-axis. Note that $\nabla \cdot \mathbf{B} = 0$ is trivially satisfied.

When $\mathbf{u}_\xi$ exists and $\mathbf{u}_\xi \neq \mathbf{0}$, i.e. inside of the expansion fans, $\mathbf{u}_\xi$ is proportional to a right eigenvector $\mathbf{r}_i$ of $\mathbf{A}$. Derivation of $\xi = \lambda_i$ with respect to λ gives $(\nabla_\mathbf{u} \lambda_i) \cdot \mathbf{u}_\lambda = 1$ and thus $\mathbf{u}_\lambda = \frac{\mathbf{r_i}}{\nabla_\mathbf{u} \lambda_i \cdot \mathbf{r_i}}$. Denoting $\frac{d\mathbf{u}_i}{\mathbf{r_i}} = \kappa$, it follows that $[\mathbf{l}_i \cdot d\mathbf{u}]_{dx = \lambda_j dy} = (\mathbf{l}_i \cdot d\mathbf{r}_j)\kappa = \kappa \delta_{i,j}$, where $\mathbf{l_i}$ and $\mathbf{r_i}$ are respectively left and right eigenvectors corresponding to λ_i. Therefore, if $i \neq j$, $[\mathbf{l}_i \cdot d\mathbf{u}]_{dx=\lambda_j dy} = 0$. These 5 independent equations allow for numerical Runge-Kutta integration across expansion fans. Inside of the expansion fan we can derive $\frac{dp_{tot}}{d\phi}$, inspired by [5]. Across the *CD*, we have that p_{tot} and $\frac{v_y}{v_x}$ are invariant.

Since the system is nonlinear and thus allows for large-amplitude waves, which in the limit case can become discontinuous, the analysis given is not sufficient. We will include the possibility of *weak solutions*, which are only solutions of the *stationary Rankine-Hugoniot conditions*: $[[\mathbf{F}]] = \xi [[\mathbf{G}]]$, where $\xi = tan\phi$ and ϕ is the angle between the negative x-axis and the shock.

Solving the problem, we first guess the total pressure p^* across the *CD*. R is a shock, only if $p^* > p_{post}$ and T is a shock, only if $p^* > p_{pre}$. Note that the jump $[[\frac{v_y}{v_x}]](p^*)$, across the *CD*, as a result of numerical integration through both R and T, should vanish. A simple Newton-Raphson iteration on in this function, finds the correct p^*, with vanishinh jump. We can then quantify ϕ_R, ϕ_T, ϕ_{CD} and thus $\mathbf{u}(x,y,t)$.

Results In 1978, a *fast/slow HD* shock tube experiment, where a very weak shock ($M = 1.12$) refracts at a CO_2/CH_4 interface was studied [1], under a varying range of incident angles α.

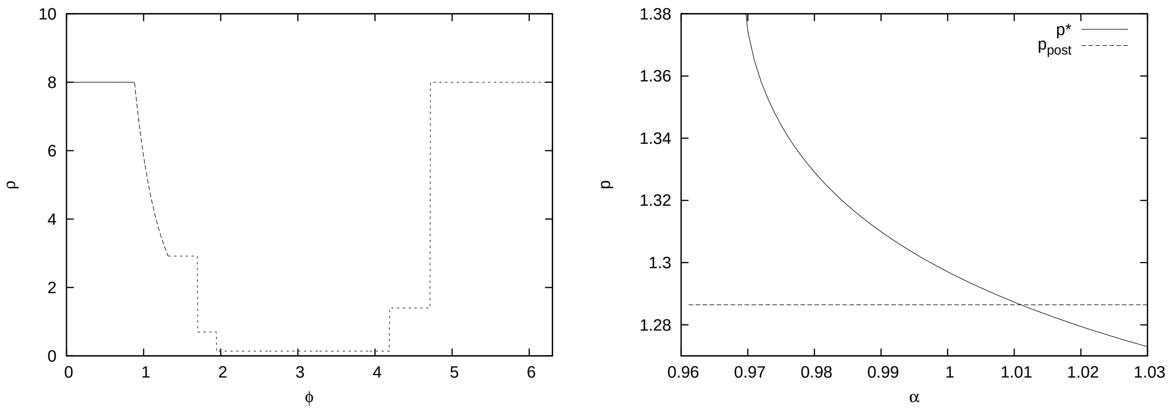

Figure 2: *Left*: $\rho(\phi)$, for $\left(M,\alpha,\beta^{-1},\gamma_l,\gamma_r,\eta\right)=\left(10,\frac{\pi}{4},0,\frac{7}{5},\frac{7}{5},\frac{1}{10}\right)$; *Right*: $p^*(\alpha)$ and $p_{post}(\alpha)$ for the Abd-El-Fattah experiment: $\alpha_{crit}=0.97$ and $\alpha_{trans}=1.01$.

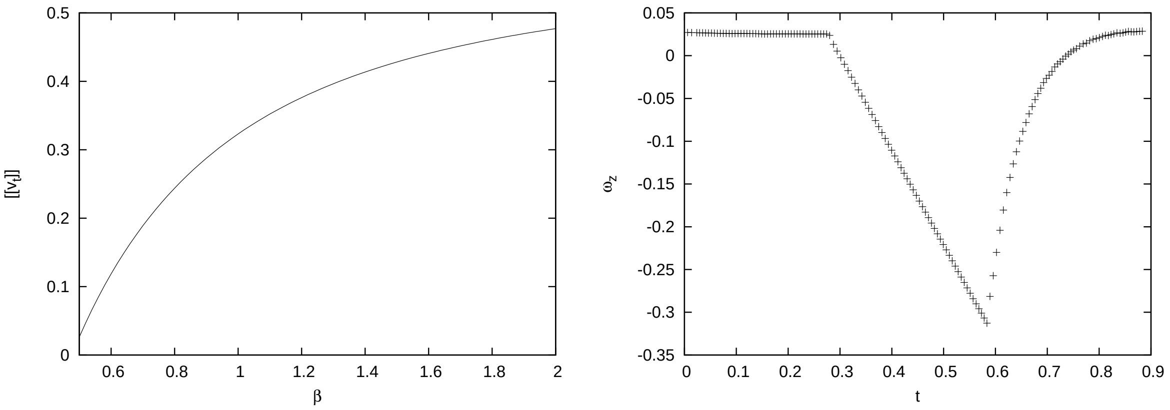

Figure 3: *Left*: $[[v_t]](\beta)$ is increasing, with finite $\lim_{\beta\to\infty}[[v_t]]$. *Right*: The evolution of the vorticity $\omega(t)$ in time in the whole domain.

The general von Neumann theory [4] predicts $\alpha_{crit}=0.97$ and $\alpha_{trans}=1.01$, where the reflected signal is irregular if $\alpha<\alpha_{crit}$, a shock if $\alpha_{crit}<\alpha<\alpha_{trans}$ and an expansion fan for $\alpha_{trans}<\alpha$. This is in agreement with our exact solution (see Fig.2) and AMRVAC simulations (see below).

For $\gamma_l=\gamma_r$, our strategy predicts a reflected rarefaction fan for $\eta<1$, and a reflected shock for $\eta>1$, which is in agreement with our AMRVAC-simulations.

Adding a perpendicular field causes the angles of refraction to broaden, and the growth rate $\frac{d\omega_{CD}}{dt}$ to increase (see Fig.2).

AMRVAC simulations Numerical experiments were performed by AMRVAC [2, 3], exploiting *adaptive mesh refinement*. We used the TVDLF-scheme, with Woodward limiter. The effective resolution of our simulations performed so far is 1920×384. Our numerical simulations fit with our theoretical predictions, where we only predict the self-similar regime: we can quantify the growth rate of ω_{CD}, as a measure of instability. The deposition of vorticity on the shocked

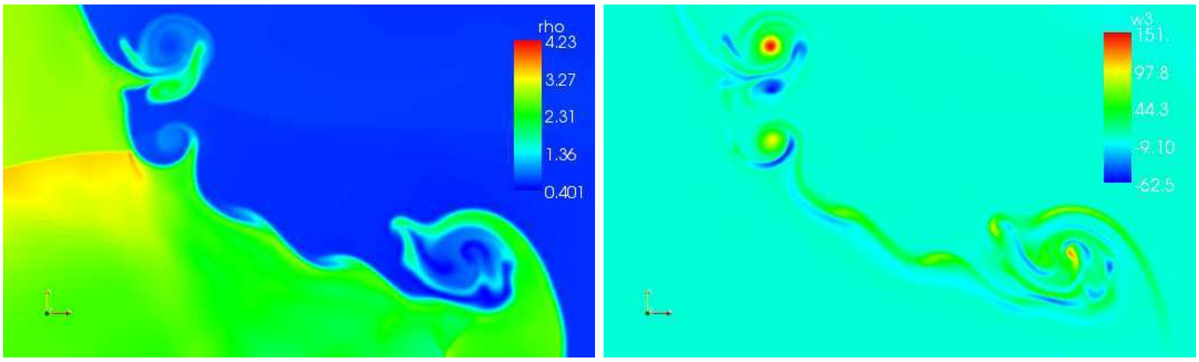

Figure 4: Simulation for $(M, \alpha, \beta, \gamma_l, \gamma_r, \eta) = \left(10, \frac{\pi}{4}, \frac{2}{9}, \frac{7}{5}, \frac{7}{5}, \frac{1}{10}\right)$: *Left*: Density; *Right*: Vorticity.

contact is linear, as predicted by Richtmyer. After reflection from the top wall, ω changes sign and the *RMI* appears: local Kelvin-Helmholtz instability causes the interface to roll up (see Fig.3). Finally, Fig.4 shows a density and a vorticity plot of the interface after reflection from the top wall.

Conclusions We developed an exact Riemann solver-based solution strategy for shock refraction at a density discontinuity. Our self-similar results agree with AMRVAC simulations. We predict α_{crit} for regular refraction, and the results fit with AMRVAC- results, experimental results and the von Neumann shock refraction theory. The stability of the contact increases with decreasing β. We will generalise our results for arbitrary uniform magnetic fields, where upto 7 signals arise.

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