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Nonlinear Waves in Dusty Plasmas and the Effect of Electromagnetic Radiation

S.I. Popel[(a)], A.A. Gisko[(a)], A.P. Golub'[(a)], T.V. Losseva[(a)],
R. Bingham[(b)], P.K. Shukla[(c)]
(a) Institute for Dynamics of Geospheres, Leninsky pr. 38, Building 6,
117979 Moscow, Russia
(b) Rutherford Appleton Laboratory, Chilton, Didcot,
Oxfordshire, OX 11 OQX, U.K.
(c) Institut für Theoretische Physik IV, Ruhr–Universität Bochum,
D–44780 Bochum, Germany

1. Introduction

Here we study the formation of nonlinear electrostatic wave structures in dusty plasmas in the presence of external electromagnetic radiation. Calculations are performed for the case of Solar radiation spectrum in the vicinity of the Earth. The most important nonlinear wave structures in dusty plasmas are shocks. Such shocks can be observed, for example, in active rocket experiments which involve the release of some gaseous substance in near–earth space.

2. Assumptions and equations

We use the following properties of the dusty plasma and the assumptions. The average radius a of the grains is much smaller than the electron Debye length λ_D, the spatial scale of the perturbations, and the distance between the plasma particles; the dust can be considered as stationary and its density n_d constant in the ion acoustic time scale. Furthermore, in the absence of perturbations the quasineutrality condition $n_{i0} = n_{e0} + Z_{d0} n_d$ holds. Here $q_d = -Z_d e$ is the dust particle average charge, $-e$ is the electron charge, $n_{i,e}$ are the ion and electron densities, and the subscript 0 denotes unperturbed quantities.

The charge variation of the dust grain is both due to the microscopic electron (I_e) and ion (I_i) grain currents (originating from the potential difference between the plasma and the grain surface) and the photoelectric electron current (I_{ph}). Here we restrict ourselves only to the case when the dust grain charges are positive.

According to the orbit–limited probe model, we have

$$I_e \approx -\pi a^2 e \left(\frac{8T_e}{\pi m_e}\right)^{1/2} n_e \left(1 + \frac{e q_d}{a T_e}\right), \tag{1}$$

and

$$I_i = \sqrt{\frac{\pi}{2}} a^2 v_{Ti} e n_i \left\{ 2 \exp\left(-\frac{v_i^2 + v_{\min,i}^2(q_d)}{2 v_{Ti}^2}\right) \cosh\left(\frac{v_i v_{\min,i}(q_d)}{v_{Ti}^2}\right) \right.$$
$$\left. + \sqrt{\frac{\pi}{2}} \frac{v_{Ti}}{v_i} \left(1 + \frac{v_i^2}{v_{Ti}^2} - \frac{2 e q_d}{a m_i v_{Ti}^2}\right) \left[\mathrm{erf}\left(\frac{v_{\min,i}(q_d) + v_i}{\sqrt{2} v_{Ti}}\right) - \mathrm{erf}\left(\frac{v_{\min,i}(q_d) - v_i}{\sqrt{2} v_{Ti}}\right) \right] \right\}, \tag{2}$$

where m_e is the electron mass, T_j and v_{Tj} are the temperature, and thermal speed of the species j $(= i, e)$, $v_i \mathbf{e}_x$ is the ion fluid velocity, $v_{\min,i}(q_d) = (2 e q_d / a m_i)^{1/2}$, and $\mathrm{erf}(x)$

is the error function. For electromagnetic radiation with the spectrum $\Phi(\omega)$, where $\Phi = \int \Phi(\omega)d\omega$ is the luminous flux, we find

$$I_{ph} = \frac{\pi\beta e a^2}{\hbar} \int\limits_{\omega_R-(e^2 Z_d/a\hbar)}^{\infty} \frac{\Phi(\omega)}{\omega} d\omega. \tag{3}$$

Here β is the probability of emission of the electron due to the action of one photon on dust particle surface, $\hbar$ is the Planck's constant, $\hbar\omega_R$ is the photoelectric workfunction. We approximate the Solar radiation spectrum as the spectrum of perfectly black body (with the effective temperature $T_s = 6000$ K and the luminous flux $\int \Phi(\omega)d\omega = 1.4 \cdot 10^6$ erg/(cm^2·s)) in the vicinity of the Earth.

The average charge of the stationary dusts is governed by the equation

$$\partial_t q_d = I_e(q_d) + I_i(q_d) + I_{ph}(q_d) \tag{4}$$

for charge conservation. The electron density is taken to be Boltzmann with constant T_e.

For the situation considered when the photoelectric effect leads to the positive charges of dust particles the ion current I_i is several order of magnitude less than I_e and I_{ph}. Thus for the time scales characteristic for ion acoustic wave propagation we can neglect the effect of change in number of ions due to the charging processes and can use the ion conservation equations. Finally, we use the Poisson equation for the electrostatic electric field.

3. Shock waves

We consider a quasi–stationary structure moving with a speed V in the x direction, satisfying the condition $v_{Ti} < V \ll v_{Te}$ for ion acoustic wave propagation. Thus, all parameters depend only on $\xi = x - Vt$. We shall also assume that in the moving frame $v_i \gg v_{Ti}$, which imposes a lower limit on the magnitude of the electrostatic potential φ of the nonlinear waves. We use the normalization $e\varphi/T_e \to \varphi$, $V/c_s \to M$, $\xi/\lambda_D \to \xi$, $z = -eq_d/aT_e$, and $\delta z = -e\delta q_d/aT_e$. Here $c_s = (T_e/m_i)^{1/2}$ is the ion acoustic speed and $\delta q_d = q_d - q_{d0}$.

The set of equations described in the previous section can be numerically integrated provided that the existence conditions for nonlinear wave solutions are known. The conditions $\varphi \to 0$ if ξ tends to both $+\infty$ and $-\infty$ cannot be satisfied simultaneously. This shows that the nonlinear waves like solitons cannot exist in the situation considered. For the shock wave solutions to exist, there must be two distinct asymptotic values for φ (say at $\xi \to \pm\infty$), and the derivatives of the perturbed quantities must vanish there. Two different asymptotic solutions $\varphi = 0$ and $\varphi = \varphi_A$ exist only if the following condition is fulfilled:

$$M^2 > M_0^2 \equiv \left(1 + \frac{1 + Z_{d0} d}{Z_{d0} d} z_0 G\right) \left(\frac{z_0 G}{Z_{d0} d} + E\right)^{-1}, \tag{5}$$

where

$$G = \frac{\exp\left(z_0 T_e/T_i\right)\left(T_e/T_i\right)\left(2 + z_0 T_e/T_i\right) + \sqrt{m_i T_e/m_e T_i} + F_{ph}}{\exp\left(z_0 T_e/T_i\right)\left(1 + z_0 T_e/T_i\right)},$$

$$E = \frac{\sqrt{m_i T_e/m_e T_i}(1 - z_0)}{\exp{(z_0 T_e/T_i)}\,(1 + z_0 T_e/T_i)},$$

and $F_{ph} = \sqrt{\pi/8}(J_{ph}/A)(w_{R0} - z_0)^2/(\exp{[T_e(w_{R0} - z_0)/T_s]} - 1)$, $d = n_{d0}/n_{e0}$, $z_0 = Z_{d0}e^2/aT_e$, $J_{ph} = \pi\beta e^2 a^2 T_e^{3/2} m_i^{1/2} \Phi_0(\lambda_D/a)/\hbar^4$, $A = a[(1 + Z_{d0}d)/4\lambda_D](T_i/T_e)^{1/2}$, $\Phi_0 = 5.5 \cdot 10^{-55}$ g·s, $w_{R0} = \hbar\omega_R/T_e$.

The requirement of the existence of the solution (with the asymptotic values $\varphi = 0$ and $\varphi = \varphi_A$ at $\xi \to \pm\infty$) for the entire range $-\infty < \xi < +\infty$ results in the inequality

$$M^2 \leq M_1^2 \equiv 1 + Z_{d0}d. \tag{6}$$

A solution bridging these two asymptotic ones can now be obtained. Here we consider an example of a dusty plasma with the parameters typical for the earth's ionosphere at the altitudes 500–600 km [1] $n_{e0} = 10^3$ cm^{-3}, $n_{i0} = 8 \cdot 10^2$ cm^{-3}, $T_e = 2$ eV, $T_i = 0.5$ eV, and the parameters of dust [2] $a = 10^{-4}$ cm, $\lambda_R \equiv 2\pi c/\omega_R = 2 \cdot 10^{-5}$ cm (which is typical for most materials), $\beta = 0.1$.

In Figs. 1(a)–1(b) the profiles of the potential $\varphi(\xi)$ and the electric field $E = -d_\xi \varphi$ are shown. A similar profile for the normalized charge perturbation δz is given in Fig. 1(c). In Fig. 1(d) the profile of the ion density n_i normalized to the unperturbed electron density n_{e0} is presented.

The solution given in Figs. 1(a)–1(d) can be treated as the steady–state shock–like wave solution. The dissipativity is related to the variation in dust particle charges which is due to the microscopic electron and ion grain currents and the photoelectric electron current.

The observation of shock waves related to the dust charging process in the presence of electromagnetic radiation is possible in active rocket experiments which use the scheme of the experiments Fluxus–1 and –2 [3, 4], involve the release of some gaseous substance in near–Earth space in the form of high–speed plasma jet, and are carried out at the altitudes of 500–600 km. For the effective manifestation of the dust charging effect and observation of shock waves related to this effect the velocities of the jet should not exceed the value of 10 km/s [5]. Furthermore, because of their unique conditions of existence, dust shock waves in the presence of electromagnetic radiation can be useful in the diagnostics of astrophysical dusty plasmas.

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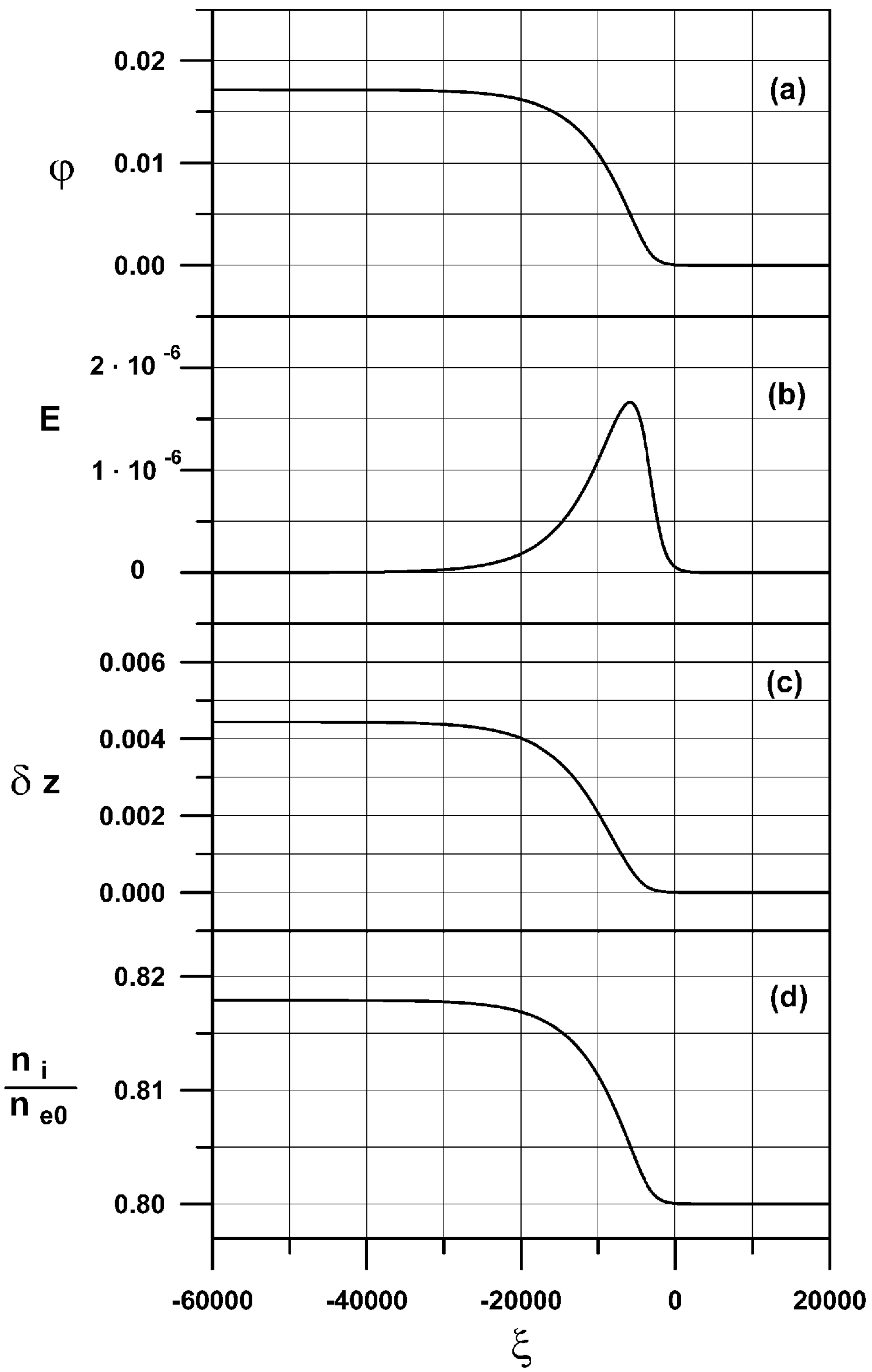

Figure 1: The profiles of $\varphi(\xi)$ (a), $E(\xi) = -d_\xi \varphi(\xi)$ (b), $\delta z(\xi)$ (c), and n_i/n_{e0} (d) in the nonlinear wave structure for the parameters $n_{e0} = 10^3$ cm^{-3}, $n_{i0} = 8 \cdot 10^2$ cm^{-3}, $T_e = 2$ eV, $T_i = 0.5$ eV, $a = 10^{-4}$ cm, $\lambda_R = 2 \cdot 10^{-5}$ cm, $\beta = 0.1$, $\Phi = \Phi_s = 1.4 \cdot 10^6$ erg/(cm^2·s), and $M = 0.89$. The asymptotic solutions are $\varphi = 0$ and $\varphi_A \approx 0.018$.