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Modeling of X-mode reflectometry : effect of density and magnetic fluctuations on the back-scattered wave amplitude

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Introduction

Reflectometry techniques are widely used to measure fusion plasma density profile. X-mode reflectometry is now a fast-developing technique since it can measure the edge density profile. It can be also used to characterize density and magnetic field fluctuations. A unidimensional study of the effect of localized coherent fluctuations has been performed with the aim of determining their efficiency, with both an analytical model and numerical simulations. The analytical model is based on the transformation of the Helmholtz equation into a Mathieu equation. Although it is in principle valid for a homogeneous plasma, it gives good results for inhomogeneous plasmas as well. We first present the analytical model and then a comparison with numerical results in the simpler case of the O-mode. Then it is applied to tokamak plasmas (here for the Tore Supra parameters) in the case of the X-mode. In particular, a study of the respective weight of the density and magnetic field fluctuations in the Bragg back-scattering process has been performed, together with simulations.

1. Analytical model of the back-scattering

The propagation of a monochromatic wave of wave number k_0 through a plasma can be rewritten as a Helmholtz equation [1]. The plasma is perturbed by radial (in the one-dimensional case) density and/or magnetic field fluctuations. We model the perturbation δQ by a wave train, $\delta Q = a_f \sin[k_f (x-x_f)]$, where a_f represents the normalized amplitude, k_f the wave number and x_f the position of these fluctuations. Their length l_f will be involved in the back-scattering amount. In this case, the Helmholtz equation leads to the Mathieu equation:

$$\frac{d^2E}{d\xi^2} + (p - 2q\cos 2\xi)E = 0 \qquad \text{with} \qquad \xi = \frac{\pi}{4} - \frac{1}{2}k_f(x - x_f) \qquad (1)$$

where the p and q parameters are associated respectively to the properties of the medium and the amplitude of the fluctuations. P and q both depend on the refractive index $N_{O,X}$ (N_O for the O-mode polarization and N_X for the X-mode polarization), on the probing wave pulsation ω and on the wave number k_f of the fluctuations. Moreover, the parameter q depends also on the relative amplitude a_f of the fluctuations. In deriving equation (1), some assumptions must be made, particularly for the X-mode where the refractive index has to be linearized.
According to the values of the p and q parameters, the behavior of the solutions of the equation *(1)* differs (fig.2) [2].

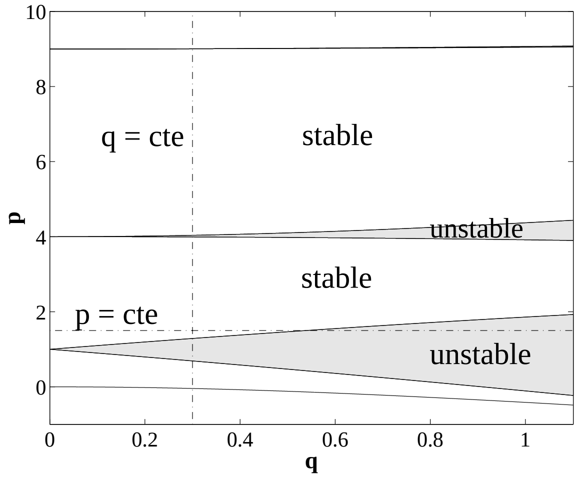

Fig.1 : Domains of stable and unstable solutions of the Mathieu

For some ranges of parameters, the solution becomes “unstable”, with exponentially growing or decreasing solutions. Physically this corresponds to the Bragg scattering. When the amplitude of fluctuations grows with p constant, the wave goes successively across stable and unstable regions, and non-linear harmonic Bragg scattering can occur.
In the unstable domains the electric field is inferred from the Floquet theorem [3] :

$$E(\xi) \propto \cos(\xi) e^{-\mu\xi} \tag{2}$$

with μ the Mathieu characteristic exponent, which has to be real for unstable solution.
It results from the equation *(2)* that the transmitted field is $(1+A_D)e^{-\mu\xi}$ where A_D is the normalized coefficient of the back-scattering. Since the energy flow must be preserved during the propagation, the amplitude of the back-scattered electric field can be inferred from the equation:

$$R + T = A_D{}^2 + (1 + A_D)^2 e^{-2\mu\xi} = 1 \tag{3}$$

where R and T are respectively the reflection (and back-scattering) and transmission energy coefficients. From the computation, it is possible to evaluate the coefficient A_D and then to compare this value to those obtains from equation (3). Furthermore, the expression of the transmission coefficient gives information about the wave phase.

2. Comparison between the theory and the simulations for the O-mode

The O-mode propagation equation needs only one equation, and the physical processes involved are well known. Thus a 1D code solving the time dependent wave equation has been first implemented for the O-mode to test the analytical model. In one dimension, this wave equation is:

$$\left[\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} + \omega_{pe}{}^2(x)\right] E(x,t) = 0 \tag{4}$$

The properties of the numerical code are written in [4]. This code has been improved by a new injection method with non perturbative wave injection conditions. Thus we can reach the asymptotic value and work with monochromatic wave. This results in a reduced numerical domain and an important time saving. Furthermore, with such a method, we can simulate frequency sweep reflectometry.
For the O-mode, the p and q parameters of the Mathieu for the O-mode are the following:

$$p = 4\omega^2 N_O{}^2(x_f)/c^2 k_f{}^2 \quad \text{and} \quad q = 2a_f \omega_{pe}{}^2(x_f)/c^2 k_f{}^2 \tag{5}$$

On figure 2 we compare the theoretical back-scattering coefficient to the simulation results as the relative amplitude of the fluctuation increases. The results are in an excellent agreement even at very high amplitude. The figure 3 represents the back-scattered signal during a frequency sweep (rather than the wave frequency, the fluctuation spectrum wave number is on the x-axis, since for back-scattering there is the Bragg relation between them). It shows that the validity of the model breaks down when the scattering process occurs in the vicinity of the cut-off. It shows also that during the sweep the whole fluctuation spectrum leads to Bragg back-scattering.

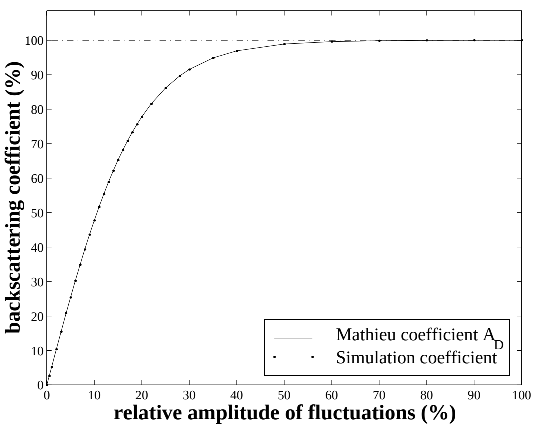

Fig.2 Back-scattering of a wave train according to the relative amplitude of the density fluctuations

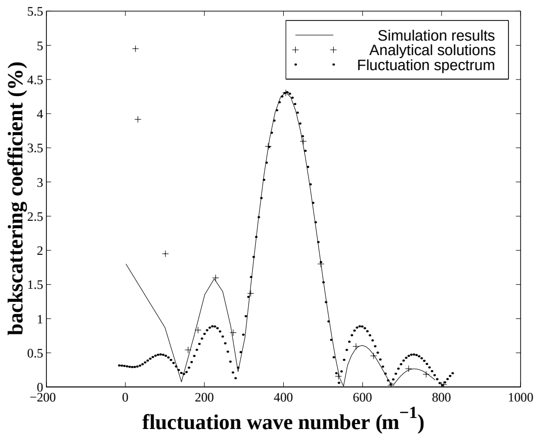

Fig.3 Fluctuation spectral effects on the back-scattered wave during frequency sweep

The analytical model can be applied to the study of inhomogeneous plasmas, provided the density gradient length is large enough. In this case, the back-scattering coefficient is still well calculated.

3. Comparison between theory and simulations for the X-mode

The propagation equation for the X-mode needs the resolution of four simultaneous equations:

$$\begin{cases} \left[\frac{\partial^2}{\partial t^2} + \omega_{pe}{}^2\right] E_x = -\omega_{pe}{}^2 B_0 v_y, & \left[\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} + \omega_{pe}{}^2\right] E_y = \omega_{pe}{}^2 B_0 v_x \\ \frac{\partial v_x}{\partial t} = -\frac{e}{m_e} E_x - \omega_{ce} v_y, & \frac{\partial v_y}{\partial t} = -\frac{e}{m_e} E_y + \omega_{ce} v_x \end{cases} \qquad (6)$$

The algorithms, based on a finite difference scheme, are described in [4]. Nevertheless, the parameters chosen in this paper can lead to numerical instabilities. With some improvements, we have obtained a perfectly stable code, with very low numerical dispersion and dissipation (error on the time of flight less than 1% for a 30000 λ_0 plasma length and 2.5 10^6 time steps).
The model Mathieu equation of section 1 has now the following parameters. p is the same as for the O-mode, with N_X instead of N_O. Assuming that the relative amplitude of density or magnetic fluctuations remains small, the q parameter can be expressed as:

$$q = 2a_f \frac{\omega^2}{k_f{}^2(x_f)c^2}\left[2(1-N_X{}^2(x_f)) - \frac{\omega_{pe}{}^2(x_f)N_X{}^2(x_f)}{\omega^2-\omega_{uh}{}^2(x_f)}\right] \qquad \textit{for density fluctuations} \qquad (7)$$

$$q = 2a_f \frac{\omega^2}{k_f{}^2(x_f)c^2}\frac{2\omega_{ce}{}^2(x_f)[1-N_X{}^2(x_f)]}{\omega^2-\omega_{uh}{}^2(x_f)} \qquad \textit{for magnetic field fluctuations} \qquad (8)$$

The evolution of the back-scattering coefficient with the amplitude of magnetic fluctuations is shown on figure 4 (for a homogeneous plasma). The theoretical predictions and the numerical results are in a good agreement over the whole amplitude range (negligible error up to 20%, and less than 5% at higher amplitudes).

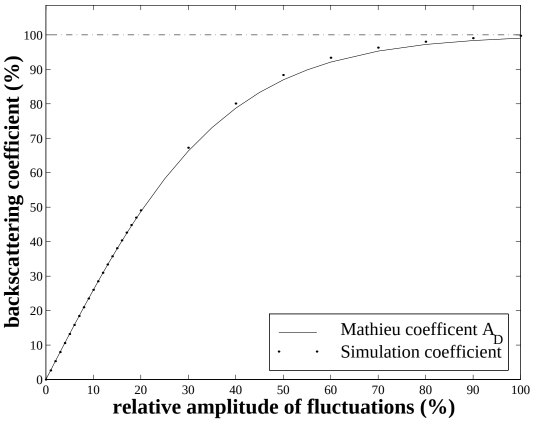

Fig.4 : Back-scattering in a homogeneous plasma perturbed by magnetic field fluctuations

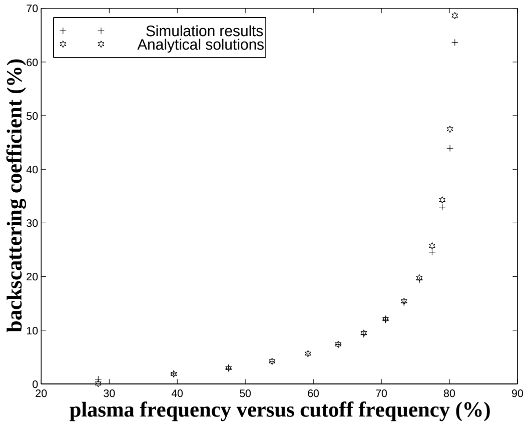

Fig.5: Back-scattering in an inhomogeneous plasma perturbed by density fluctuations

The case of an inhomogeneous plasma has also been performed in the presence of density or magnetic fluctuations (fig.5). As already mentioned for the O-mode, the analytical model fails when the fluctuations are localized in the vicinity of the cut-off.

Thus in most cases, with the analytical model based on the Mathieu equation we are able to evaluate the respective weight of density and magnetic fluctuations. On the upper branch of the X-mode, the density fluctuations are usually dominant in the back-scattering process.

However, on the lower branch, the magnetic field fluctuation effects may sometimes become dominant. For instance, we have studied the case of a plasma with tokamak typical density and magnetic field profiles (ω_{ce0} = 66GHz and ω_{pe0} = 70GHz) perturbed by fluctuations of wave length λ_f = 6mm. The model predicts that the efficiency ratio of density fluctuations versus magnetic field fluctuations is about 30. It implies that in this case, density fluctuations with an amplitude of 1% have the same effect on the back-scattered wave as magnetic field fluctuations with an amplitude of 3.10^{-4}.

Conclusion

An analytical model based on the Mathieu equation describes the back-scattering process for the O-mode and the X-mode. For the O-mode, this model can predict the evolution of the back-scattered wave for any amplitude of the fluctuations with a high accuracy. For the X-mode, which needs additional restrictions, the error is always less than 5%. Provided the gradient lengths are large enough, the model can be used for inhomogeneous plasma. In this case, the theory breaks down when the fluctuations are in the vicinity of the cut-off. The model has been applied to the effect of fluctuations on the back-scattered wave in plasmas with tokamak parameters. It predicts that magnetic field fluctuations can be seen in the back-scattered signal when the X-mode is operated on the lower branch.

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