

Study of the effect of coherent density fluctuations in O-mode pulse reflectometry

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Introduction

Reflectometry is now widely used for density profile measurements in fusion plasmas. In particular, pulse reflectometers are under construction [1], and it is important to understand the propagation of a pulse in the presence of density fluctuations. After comparing different methods to simulate the propagation of a pulse in a plasma, we discuss the effect of various kinds of density fluctuations (Micro-turbulent fluctuations, MHD-like fluctuations) on the reflectometry measurements.

1. Comparison of different methods for the simulation of pulse reflectometry

The cold plasma approximation is generally used to describe the propagation of the probing wave of a reflectometer. In the 1D case, pulse reflectometry experiments for the O-mode can be then simulated by solving the following time-dependent wave equation:

$$\left[\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} + \omega_{pe}^2(x) \right] \vec{E}(x, t) = 0 \quad (1)$$

This method allows to study the time evolution of the wave propagation but it also requires a long time of calculations. An alternative solution consists in a pulse compression method. Such method is based on the fact that the probing pulse is composed from a set of frequencies. The reflected signal can be then obtained from a FFT calculation:

$$E_{ref}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(\omega) \exp(-i\varphi(\omega)) \exp(i\omega t) d\omega \quad (2)$$

where $S(\omega)$ is the frequency-spectrum of the probing pulse and $\varphi(\omega)$ the phase shift induced by the plasma. This method imposes to calculate the phase shift $\varphi(\omega)$ for each component of the frequency-spectrum $S(\omega)$, which can be done analytically (under Born approximation) or by solving the Helmholtz equation:

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\omega^2}{c^2} N^2(x) \right] \vec{E}(x) = 0 \quad (3)$$

The greatest advantage of the pulse compression method is that it is significantly faster than the time-dependent code. However, relation (2) requires that each frequency component is independent from the other ones and that its phase shift is time-independent. As an example, both methods are compared in the presence of a hole of density just behind the cut-off layer (fig. 1 and 2). Such density fluctuations are obtained for a gaussian hole with an amplitude of 50 %. In this case, we can notice significant differences on the amplitude and the shape of the reflected signal. These differences can be explained by the presence of a cavity in the evanescent region of the wave. One part of the wave is trapped in the cavity. As the trapping of the wave is a time-dependent process, the pulse compression method is no longer efficient.

Nevertheless, such cases where the probing wave is trapped appear to be unrealistic. For typical density fluctuations in fusion plasmas, we could have verified that the pulse compression method and the time-dependent code give analogous results.

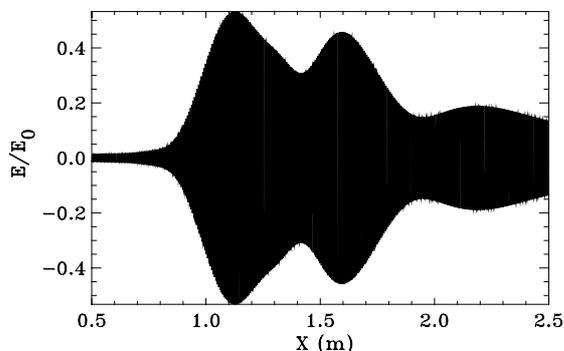


Fig. 1: Reflected signal computed by a pulse compression method (in the presence of a hole of density just behind the cut-off layer)

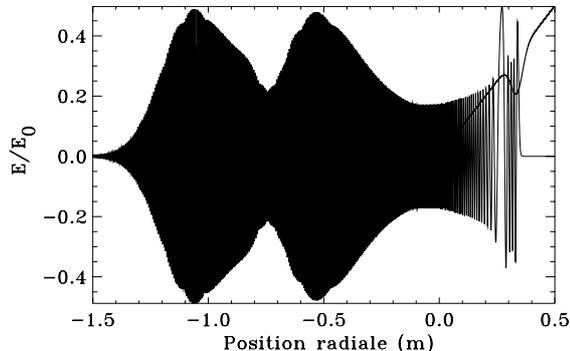


Fig. 2: Reflected signal computed by a time-dependent code (in the presence of a hole of density just behind the cut-off layer)

2. Effect of high wave-number fluctuations

We first study the effect of density fluctuations with high wave-number. For such fluctuations, one part of the wave can be scattered if their wave-number k_f satisfied the following Bragg resonance condition:

$$2k_A < k_f < 2k_0$$

We consider two cases according to the fact that the back-scattering occurs far away from the cut-off layer or near the cut-off layer. Consider first the case of a linear density profile ($n_0 = 6.10^{19} \text{ m}^{-3}$, $R = 0.5 \text{ m}$) and a density fluctuation with a wave-number $k_f = 18 \text{ cm}^{-1}$, an amplitude $a_f = 3 \%$ and a width at half-amplitude $d_f = 8 \text{ cm}$. For a probing frequency equal to 60 GHz, the scattering of the wave occurs at the position $x_B = 17.93 \text{ cm}$ far away from the cut-off layer $x_c = 37 \text{ cm}$. The back-scattered and the reflected pulses are in this case clearly separated (fig. 3) and it is in principle possible to define a time of flight for each pulse. A reflectometer which permits to detect similarly the back-scattered and the reflected pulses could thus provide an information on both the fluctuation position and the cut-off layer position.

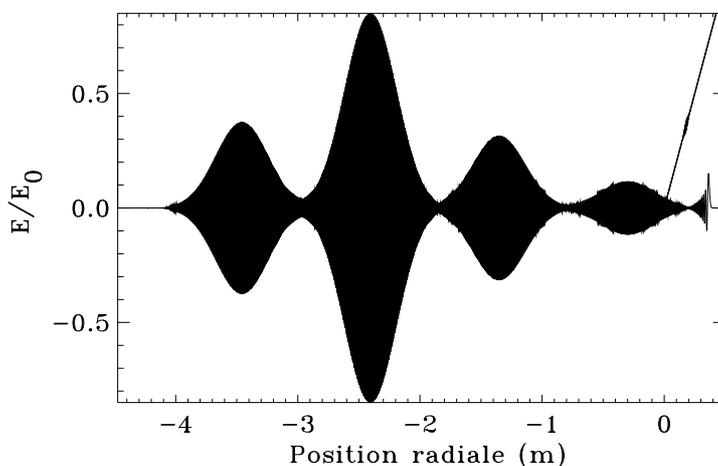


Fig. 3: Reflected signal in the presence of Bragg back-scattering far away from the cut-off layer

We now study the case of a density fluctuation with a wave-number $k_f = 4 \text{ cm}^{-1}$, an amplitude $a_f = 3 \%$ and a width at half-amplitude $d_f = 4 \text{ cm}$. For a probing frequency equal to 50 GHz, the Bragg resonant position $x_B = 19 \text{ cm}$ is just in front of the cut-off layer $x_c = 19.5 \text{ cm}$. On the reflected signal (fig. 4), we can notice an overlap between the back-scattered and the reflected pulses. It is impossible here to distinguish these two pulses, which can lead to a strong error on the time of flight measurement. The theoretical position of the reflected pulse is represented by the dotted-line at $x = -22 \text{ cm}$. Thus, we can expect an time of flight error about 0.7 ns.

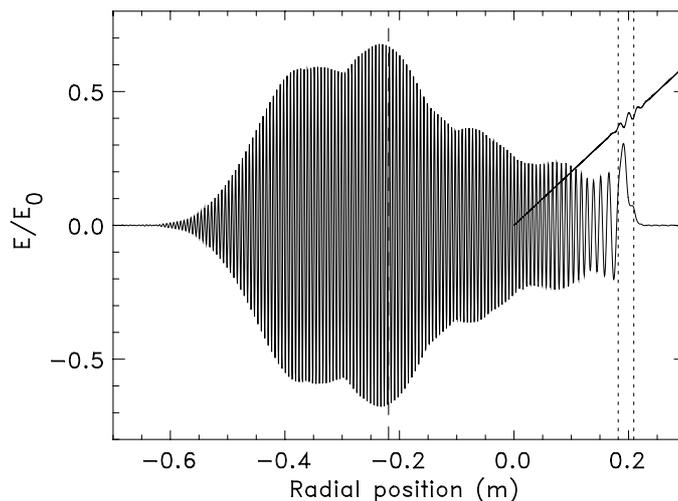


Fig. 4: Reflected signal in the presence of Bragg back-scattering in the vicinity of the cut-off layer

3. Effect of MHD-like fluctuations

We now treat the case of MHD fluctuations induced by the presence of magnetic islands in tokamaks. Such density fluctuations present very small wave-numbers and are usually represented by a gaussian envelop. When MHD fluctuations are located far away from the cut-off region, their effect on the reflected pulse is insignificant. On the contrary, if they are positioned near the cut-off layer, strong deformations of the signal can be induced. This is exemplified for a fluctuation with an amplitude $a_f = 10 \%$, a width at half-amplitude $d_f = 1 \text{ cm}$ (fig 5).

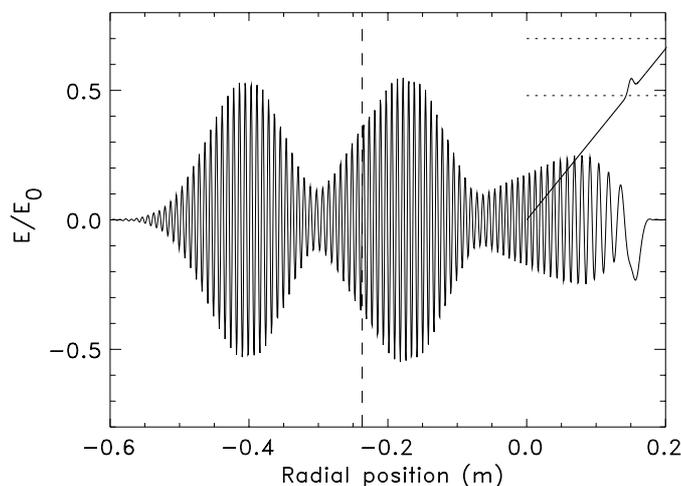


Fig. 5: Reflected signal in the presence of a MHD fluctuation near the cut-off layer

This fluctuation is located at $x_f = 15$ cm in the vicinity of the cut-off layer (delimited by two horizontal dotted lines). As one part of the pulse is reflected in front of the density bump induced by the fluctuation and the other one is reflected behind this bump, the reflected signal is split into two parts. It becomes then difficult to detect precisely the theoretical position of the reflected pulse (indicated by the vertical dotted line) and a significant error on the time of flight is expected (about 0.7 ns).

Time of flight variations induced by the propagation of MHD-like fluctuations have been illustrated experimentally [2]. In order to explain these variations a 1D model have been developed, expressing the temporal variations of the fluctuation amplitude as follows:

$$\delta n_e(x,t) = A_f \cos(\omega_f t) \exp\left[-\frac{(x-x_f)^2}{D_f^2}\right] \quad (4)$$

where a_f is the maximal amplitude of the fluctuation and ω_f its pulsation. The time of flight variations computed for a fluctuation with a width $d_f = 3$ cm, an amplitude $a_f = 10\%$ and a pulsation $\omega_f = 1256$ rad/s have been presented on fig. 6. The jumps in the time of flight measurements are due to the fact that the pulse is alternatively reflected in front and behind the density bump. These results are qualitatively conform to those obtained experimentally.

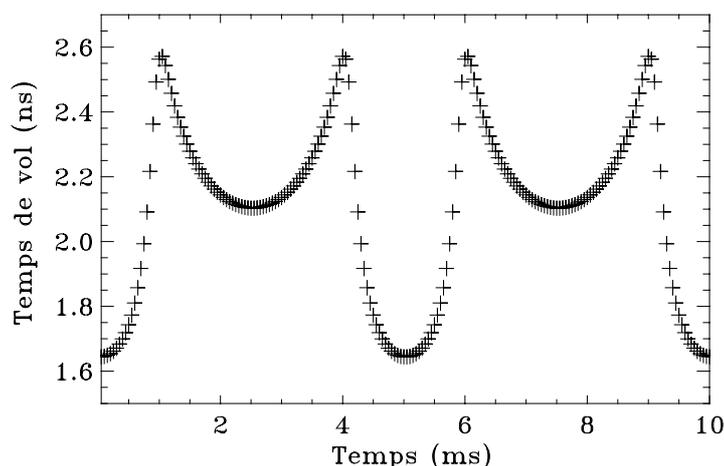


Fig. 6: Time of flight variations induced by the poloidal propagation of MHD fluctuations

Conclusion

Different methods for the simulation of the propagation of an electromagnetic wave in a plasma have been presented. In particular we have shown the interest of the pulse compression method, which permits a fast calculation for the propagation of a pulse. This method has also been developed for the X-mode and has been validated by comparison with a time-dependent code. We show in particular its validity for typical density fluctuations in fusion plasmas. The effect of coherent density fluctuations have been then investigated. It is shown that both small wave-number fluctuations and MHD-like fluctuations can lead to a significant change on the reflected pulse shape when they are in the vicinity of the cut-off layer. In particular, a good agreement is obtained with experiments in the case of poloidal propagation of MHD fluctuations.

[1] C.A.J Hugenoltz, A.J.H. Donn e, B.S.Q. Elzendoorn *et al*, Rev.Sci.Instrum. **70** 1034 (1999)

[2] J.C. Van Gorkom, C.A.J Hugenoltz, A.J.H. Donn e *et al*, 25th EPS Conf. CFPP Europhys. Conf. **22C** 1526 (1998)