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Zonal Flow Excitation by Drift Waves in Toroidal Plasmas

Liu Chen[1], Zhihong Lin[2], Roscoe White[2]

[1]*Department of Physics and Astronomy, University of California, Irvine CA. 92697*
[2]*Plasma Physics Laboratory, Princeton University, P.O.Box 451, Princeton, New Jersey 08543*

Recent 3D gyrokinetic [1, 2] and gyrofluid[3] simulations in toroidal plasmas have demonstrated that zonal flows[4] play a crucial role in regulating the nonlinear evolution of electrostatic drift-wave instabilities such as the ion temperature gradient (ITG) modes and, as a consequence, the level of the anomalous ion thermal transport, and that zonal flows could be spontaneously excited by ITG turbulence[5], suggesting parametric instability processes as the generation mechanism. Diamond et. al.[6] have proposed the modulational instability of drift-wave turbulence ("plasmons") in a slab-geometry treatment.

Consider a large aspect ratio ($\epsilon = a/R \ll 1$) tokamak plasma with the usual radial (r), poloidal (θ) and toroidal (ϕ) coordinates. Here R and a are respectively the major and minor radii. Electrostatic fluctuations are taken to be coherent and composed of a single n ($n \neq 0$) drift wave, $\delta\phi_d$ and a zonal flow mode $\delta\phi_z$; that is, $\delta\phi_d = \phi_0 + \delta\phi_+ + \delta\phi_- + c.c.$,

$$\phi_0(\vec{r},t) = e^{-i(n\phi+\omega_0 t)}\sum_m \Phi_0 e^{im\theta}, \quad \delta\phi_\pm = e^{i(\mp n\phi-(\omega_z\pm\omega_0)t+K_z r)}\sum_m \Phi_\pm e^{im\theta}, \tag{1}$$

and $\Phi_{0\pm}$ are functions of $m - nq$, $\delta\phi_z = \Phi_z e^{i(K_z r - \omega_z t)} + c.c.$. Thus ϕ_0 is the pump drift wave and ω_0 its eigenmode frequency; $\delta\phi_+$ and $\delta\phi_-$ are respectively the upper and lower sidebands produced by the modulation in the radial envelope due to $\delta\phi_z$ at frequency ω_z and radial wavenumber K_z. We have assumed the $n \gg 1$ ballooning mode representation[7] in which $K_z = nq'\theta_0$, $q = rB_\phi/RB_\theta$, and $0 \leq \theta_0 \leq \pi$ is the Bloch phase shift. The pump mode ϕ_0 has $\theta_0 = 0$, (ie. a flat radial envelope) which is, for a given n, usually the linearly most unstable mode. On the other hand $\delta\phi_+$ and $\delta\phi_-$ have $\theta_0 \neq 0$ giving radial envelope modulations. Typically they are linearly stable for moderate values of θ_0 [3]. We are thus dealing with a four-wave coupling process among ϕ_0, $\delta\phi_+$, $\delta\phi_-$, and $\delta\phi_z$.

Since electrons are adiabatic for the $n \neq 0$ drift waves, only ions contribute to the nonlinear physics. $\delta\Phi_z$ is then coupled to Φ_0 and $\delta\phi_\pm$, and the nonlinear coupling coefficient is formally of the Hasegawa-Mima type[8–10], i.e.

$$(-i\omega_z + \nu_z)\chi_{iz}\Phi_z = g\left\langle \sum_m [a_+\Phi_0^*\Phi_+ - a_-\Phi_0\Phi_-]\right\rangle \tag{2}$$

where $g = \frac{c}{2B}\alpha_i\rho_i^2 k_\theta K_z$, $a_+ = k_{0\perp}^2 - k_{+\perp}^2$, $a_- = k_{0\perp}^2 - k_{-\perp}^2$, $\chi_{iz} \simeq 1.6\epsilon^{3/2}K_z^2\rho_i^2 B_\phi^2/B_\theta^2$[11], $\nu_z = (1.5\epsilon\tau_{ii})^{-1}$[12], $\vec{k}_{0\perp} = \hat{\theta}k_\theta + inq'\hat{r}\partial_\zeta$, $k_\theta = nq/r_0$, r_0 refers to one reference mode surface, $\zeta = m - nq$ corresponds to the fast radial variable, $\vec{k}_{\pm\perp} = \hat{r}K_z \pm \vec{k}_{0\perp}$, and $\langle A\rangle = \int_{-1/2}^{1/2} A d\zeta$ is an averaging with respect to r_0, $\alpha_i \simeq \delta P_{0\perp}/(Ne\Phi_0) + 1$ and $\delta P_{0\perp}$ is the perturbed perpendicular pressure due to Φ_0. The detailed expression for α_i depends on the specific drift wave mode and plasma parameters; e.g. $\alpha_i \simeq 1+\tau+\eta_i\tau$, $\eta_i = dlnT_i/dlnN$ for the electron drift wave, and $\alpha_i \simeq \tau(1+\eta_i)/[(3\tau-1)L_n/R + 1/2] + 1$ for ITG in the

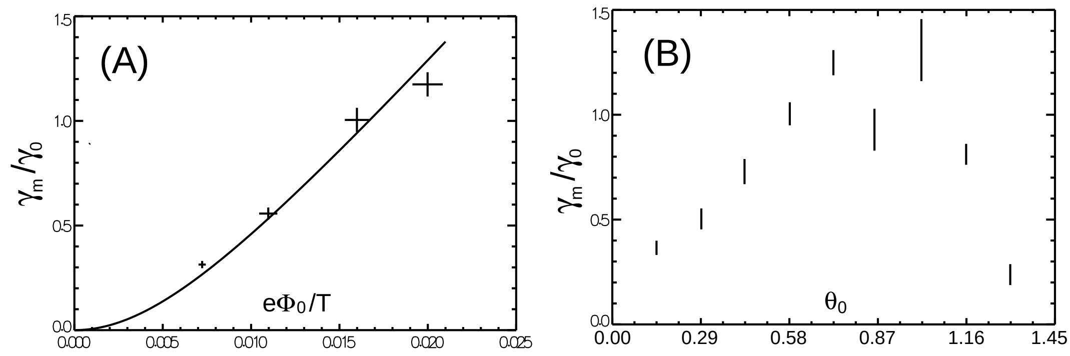

Figure 1: Gyrokinetic simulation results of zonal flow growth rate (A) vs. ITG mode amplitude for fixed θ_0, and (B) vs. θ_0 for fixed ITG amplitude, normalized to ITG growth,γ_0. The line in (A) is the solution of Eq. 3.

fluid ion local approximation[3]. Here $L_n^{-1} = dlnN/dr$ and $\tau = T_i/T_e$. In deriving Eq. 2 we have assumed $|k_\perp \rho_i| < 1$ with ρ_i the ion gyro radius, $|\omega_z| < \omega_{GAM}$, and averaged over the Geodesic Acoustic Mode[13].

The coupling of $\delta\phi_\pm$ to Φ_0 and $\delta\Phi_z$ can be calculated using the nonlinear gyrokinetic equation[9, 10] and the quasi neutrality condition. The details are presented in a previous publication[17]. We finally obtain the desired linear dispersion relation for the modulational instability

$$\Gamma_z + \nu_z = \gamma_M^2(\Gamma_z + \gamma_d)/[\Delta^2 + (\Gamma_z + \gamma_d)^2] \tag{3}$$

where we have let $-i\omega_z = \Gamma_z$ and $\gamma_M^2 = (\alpha_i/1.6\epsilon^{3/2})(B_\theta k_\theta c_s K_z \rho_s/B_\phi)^2 \left\langle\left\langle |e\hat{\Phi}_0/T_e|^2 \right\rangle\right\rangle$. With appropriate α_i Eq. 3 is valid for various branches of drift waves such as the electron drift wave or ITG.

The predicted modulational instability features have been observed in 3D global gyrokinetic simulations of ITG modes using the gyrokinetic toroidal code [2]. These nonlinear simulations keep only a single toroidal mode $n \neq 0$ initially. The starting fluctuation level is very low to allow linear ITG eigenmode structure to be formed before nonlinear saturation. When the ITG mode grows to a desired amplitude, an external damping is applied so that the mode amplitude stays constant. Zonal flow with a single radial mode number is now self-consistently included. We observe exponential growth of zonal flow until it reaches a high level where the ITG mode is suppressed. The radial envelope modulation of the ITG mode correlates with the zonal flow radial structure. As shown in Fig. 1 (A), the growth rate of zonal flow with a fixed radial mode number linearly depends on the ITG mode amplitude. Analytical prediction of zonal flow growth rate from the solution of Eq. 3, is shown by the solid line in Fig. 1 (A). For a fixed ITG mode amplitude, measured zonal flow growth rate increases linearly with radial mode number (or θ_0) for small θ_0 and decreases for large θ_0, as shown in Fig. 1 (B), consistent with theory.

We now consider the nonlinear evolution of this modulation instability. As $\delta\phi_z$ and $\delta\phi_\pm$ exponentiate in amplitude, they will nonlinearly couple and induce damping in the pump wave amplitude. Replacing ω_0 by $\omega_0 + i\partial_t$, letting $\left\langle\left\langle |e\hat{\Phi}_0/T_e|^2 \right\rangle\right\rangle = A_0^2$ and including the

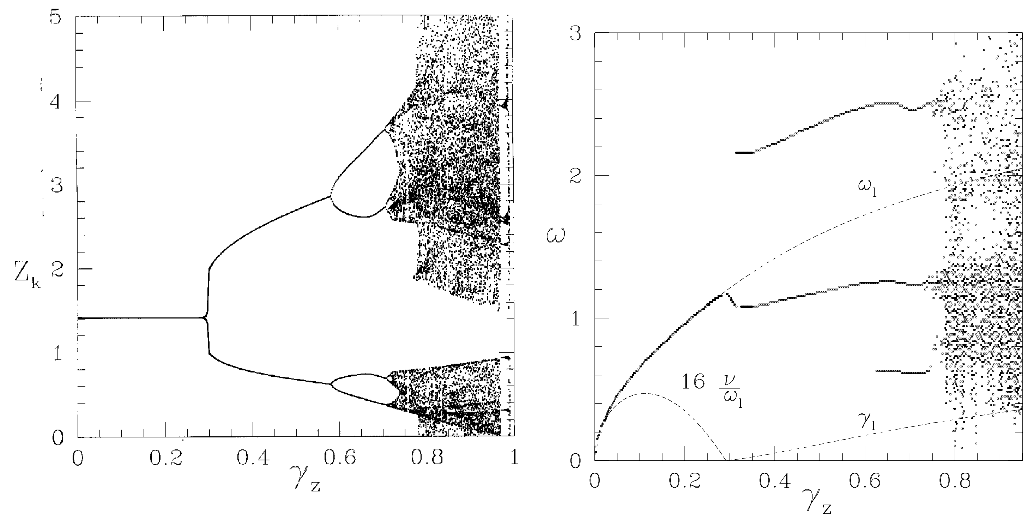

Figure 2: Values of Z_k (left), and Numerical frequencies, linear values ω_l, γ_l, and damping ν near the fixed point (right), $\delta = 2$, $\Gamma_d = 2$

linear growth rate γ_0 the equation for $A_0(t)$ becomes

$$\left(\frac{d}{dt} - \gamma_0\right) A_0 = -\frac{cT_e}{eB}\frac{\tau k_\theta K_z}{\partial D_{0r}/\partial\omega_0}(A_-\Phi_z + A_+\Phi_z^*) \tag{4}$$

Using dimensionless time $\tau = \gamma_0 t$ and performing straightforward normalizations such that $A_0 \propto P$, $A_+ \propto Se^{i\Psi(t)}$, and $\Phi_z \propto Z$, we find[17]

$$\frac{dP}{d\tau} = P - 2ZScos(\Psi), \quad \frac{dS}{d\tau} = -\Gamma_d S + ZPcos(\Psi) \tag{5}$$

$$\frac{dZ}{d\tau} = -\gamma_z Z + 2PScos(\Psi), \quad \frac{d\Psi}{d\tau} = \delta - \frac{PZ}{S}sin(\Psi) \tag{6}$$

with $\Gamma_d = \gamma_d/\gamma_0$, $\gamma_z = \nu_z/\gamma_0$, and $\delta = \Delta/\gamma_0$. These equations are similar to those for three-wave coupling[16], hence we anticipate similar behaviour, such as the existence of a stable attractor and a period doubling route to chaos.

Introduce an associated one dimensional map by defining times τ_k at successive zeros of $dZ/d\tau$. Numerical plot of the values of Z_k in steady state (after transients have died away) is shown in Fig. 2 for $\delta = 2$, $\Gamma_d = 2$. Eqs. 5-6 have a fixed point attractor for $\gamma_z < 0.3$. For $0.3 < \gamma_z < 0.58$ the attractor is a stable limit cycle with the bounding values of Z given by the two branches in Fig. 2. The initial bifurcation of the stable fixed point into the limit cycle corresponds also to period doubling, as can be seen in the plot of associated frequencies, Fig. 2. Frequency, damping and growth about the fixed point ω_l, γ_l, are also shown. Apparent chaos sets in for $\gamma_z > 0.75$.

Present turbulence simulations have $\Gamma_d \sim 1$, with values of γ_z and δ placing them in the stable fixed point domain. The oscillations observed are thus probably nonlinear transient decay to the fixed point, with the decay time much longer than the simulation time. The drift wave intensity is $I_d = P_0^2 + 2S_0^2$. Assuming weak turbulence scaling of $\chi_i \propto I_d$, where

χ_i is the anomalous ion thermal transport coefficient, we find that in the stable domain $\chi_i \propto \gamma_z \propto \nu_{ii}$ consistent with the trend observed in simulations [5].

These results can be readily generalized to the case of a single zonal flow radial envelope mode coupled to a multi-n drift wave ITG spectrum. Let A_{n0} be the drift mode pump wave on toroidal mode number n and the sidebands be $A_{n\pm}$ with $A_{n-} = A^*_{n+}$. We then have

$$\left(\frac{d}{dt} - \gamma_{n0}\right) A_{n0} = -\gamma_{1n}(A^*_{n+}\Phi_z + A_{n+}\Phi^*_z), \tag{7}$$

$$\left(\frac{d}{dt} - i\Delta_n + \gamma_{nd}\right) A_+ = \gamma_{1n} A_{n0} \Phi_z, \tag{8}$$

$$\left(\frac{d}{dt} + \nu_z\right) \Phi_z = \sum_n \gamma_{2n}(A^*_{n+}A_{n0} + A_{n+}A_{n0}). \tag{9}$$

We have carried out numerical integrations of this set of coupled equations for $k \gg 1$ drift modes with different n values using random values of the correct order of magnitude for the frequency mismatch Δ_n and ITG growth γ_{n0}, but with $\gamma_{nd} = 2\gamma_{n0}$. Remarkably, in all of the simulations, in a very short time one drift wave mode with large γ_{n0} and Δ_n is singled out and all the others are driven to very small amplitude. The results thus revert to the single-n case discussed above, the $3k+1$ dimensional phase space always contains an attractor defined by one value of n, with dimension ranging from zero to two. Other multi-n models will be considered in future publications.

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