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Stability of Revisited Neoclassical Rotation in Tokamaks

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1. Introduction

Poloidal plasma rotation in toroidal systems is a source of various mechanisms. It is usually believed that the neoclassical transport should be ambipolar and independent of the radial electric field. However, this requirement is strictly valid only in an equilibrium plasma, in which there is no other source or damping of toroidal momentum. Stringer [1] was the first to notice that the resistive diffusion rate in a toroidal plasma can not only be non-ambipolar, but can also be negative for some values of the poloidal rotation velocity. Rosenbluth and Taylor [2], considered the stability of toroidal diffusion using a fluid model and proved that if the resistivity were the only dissipative mechanism, then even if all plasma deformations were excluded, there could be no stable poloidal rotation velocity. Within the approximation they used, they could show, that there are two stable rotating states, one corresponding to positive plasma potential and the other to negative. The otherwise unstable poloidal or toroidal rotations, can also be observed as spontaneous spin-up phenomena in tokamaks. An interaction between a spontaneous poloidal or toroidal spin-up and the turbulence driven anomalous transport is also believed to be a likely reason for the L-H mode transition in tokamaks. Plasma rotation is always accompanied by a radial electric field, whose origin appears to be complicated due to various competing effects. A further consequence of an unstable rotation is that, a poloidally asymmetric particle transport may also render the radial electric field unstable [3]. To analyse such poloidal or toroidal rotation related phenomena in a collision dominated toroidal plasma within the framework of revisited neoclassical theory [4-6], we use the fluid equations including mass and momentum sources. The revisited theory introduces additional terms into the parallel momentum equation, when the parameter $\Lambda_1 \equiv (\nu_i / \Omega_i)(q^2 R^2 / r L_\psi) \geq 1/3$.

2. Governing equations

At the plasma layer just inside the separatrix, plasma is collisional enough to be treated by the fluid equations including the parallel , perpendicular and gyro stress tensor expressions given by Braginskii (extended and completed by Michailovsky and Tsypin [7]). Equations for a two-component plasma, describing the continuity of species J with sources $S_J{}^N(\vec{x},t)$, the momentum balance with friction $\vec{R}_J{}^M(\vec{x},t)$ and momentum input $\vec{S}_J{}^M(\vec{x},t)$, and the energy balance with similar terms and energy input $S_J{}^E(\vec{x},t)$ are

$$\frac{\partial N}{\partial t} + \nabla \cdot N\vec{U}_{J\perp} + \vec{B} \cdot \nabla (N\vec{B} \cdot \vec{U}_J / B^2) = S_J{}^N \tag{1}$$

$$\frac{\partial (m_J N\vec{U}_J)}{\partial t} + \nabla \cdot (m_J N\vec{U}_J\vec{U}_J + P_J\vec{I} + \vec{\Pi}_J) = e_J N(\vec{E} + \vec{U}_j \times \vec{B}) + \vec{R}_J{}^M + \vec{S}_J{}^M \tag{2}$$

$$\frac{3}{2}\left(\frac{\partial P_J}{\partial t}+\nabla\cdot P_J\vec{U}_J\right)+(P_J\vec{I}+\vec{\Pi}_J)\cdot\nabla\cdot\vec{U}_J=-\nabla\cdot\vec{q}_J+(R_J{}^E+S_J{}^E)-\vec{U}_J\cdot(\vec{R}_J{}^M+\vec{S}_J{}^M)+\frac{m_J}{2}U_J{}^2S_J{}^N \tag{3}$$

The terms S_J on the r.h.s. of (3) indicate heating due to energy sources and losses, whereas R_J indicate terms relating to collisional energy transfer and frictional heating. The correct forms of these terms require a kinetic and atomic approach. However, here we shall assume them, as given functions. The radial electric field satisfies Ohm's law, $E_r+(\vec{U}_i\times\vec{B})_r=(1/eN)\partial P_i/dr$. In the revisited neoclassical theory [4], a plausible ordering just inside the separatrix is introduced by a small parameter μ (~0.1) as

$$\left(\frac{qR\nu_J}{c_J}\right)^{-1}\sim\frac{L_\psi}{r}\sim\frac{r}{qR}\sim\frac{B_\vartheta}{B_\phi}\sim\mu \qquad \text{and} \qquad \frac{e_J\upsilon}{T_J}\sim\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}}\sim\frac{a_i}{L_\psi}\sim\mu^2 \tag{4}$$

where L_ψ is the radial gradient scale, c_J and a_J are the thermal speed and the gyro radius of the species J, respectively; υ is the loop voltage and ν_J is the collision frequency between like particles J. Using the magnetic field aligned orthonormal unit vectors $(\hat{p},\hat{b},\hat{n})$ in radial, binormal and parallel directions, and the small parameter μ, the velocity of species J can be assumed as,

$$\hat{p}\cdot\vec{U}_J\equiv\mu^6U^{(6)}{}_{\psi,J}(\psi,\chi)+\cdots;\quad \hat{b}\cdot\vec{U}_J\equiv\mu^2U^{(2)}{}_{\beta,J}(\psi)+\mu^3U^{(3)}{}_{\beta,J}(\psi,\chi)+\cdots;\quad\text{and}$$
$$\hat{n}\cdot\vec{U}_J\equiv\mu U^{(1)}{}_{||,J}(\psi)+\mu^2U^{(2)}{}_{||,J}(\psi,\chi)+\cdots. \tag{5}$$

Assuming that the magnetic field, density, temperature, potential, etc., are independent of the poloidal angle in dominant order, these are also expanded in perturbation series. For example, the density and the magnetic field are written as, $N(\psi,\chi)\approx N^{(0)}(\psi)[1+\mu n^{(1)}(\psi,\chi)+...]$ and $B(\psi,\chi)\approx B^{(0)}(\psi)[1+\mu b^{(1)}(\psi,\chi)+...]$, respectively. For a tokamak plasma with circular cross section, also the use is made of the toroidal unit vectors $(\vec{e}_r,\vec{e}_\vartheta,\vec{e}_\varphi)$. Taking toroidal and parallel projections of the momentum equation and averaging them over the magnetic flux surfaces, and imposing the ambipolarity condition, one obtains a pair of coupled nonlinear equations for the toroidal and poloidal ion velocities in terms of other plasma variables, such as temperature, density, and electric field [4]. Main results describing the radial transport of toroidal momentum in a collisional subsonic plasma with steep gradients, are [5,6],

$$m_iN_i\frac{\partial U^{(1)}{}_{\varphi,i}}{\partial t}=\frac{\partial}{\partial r}\left[\eta_{2,i}\left(\frac{\partial U^{(1)}{}_{\varphi,i}}{\partial r}-\frac{0.107\,q^2}{1+Q^2/S^2}\frac{\partial\ln T_i}{\partial r}\frac{B_\varphi}{B_\theta}U^{(2)}{}_{\theta,i}\right)\right]$$
$$+J_rB_\theta-m_i\oint\frac{d\vartheta}{2\pi}h^2S_i{}^NU_{i\varphi}+\oint\frac{d\vartheta}{2\pi}h^2\vec{S}_i{}^M\cdot\vec{e}_\varphi \tag{6}$$

where, $h=1+(r/R_0)\cos\vartheta$, $Q=[4B_\varphi U^{(2)}{}_{\theta,i}-2.5(T_i/e_i)\partial\ln N_i^2T_i/\partial r]B^{-1}$, $S=(2r\chi_{\|,i}N_i^{-1})/q^2R^2$; J_r is radial polarization current, and parallel heat diffusion coeff. is $\chi_{||,i}=3.9\ P_i/m_i\nu_i$. The poloidal rotation driven by the temperature gradient seen inside the paranthesis on the right hand side of (6) results from the gyro-stress tensor and acts like another source term, i.e., as a toroidal momentum source or sink, depending on the sign of its radial gradient. External momentum sources can be direct, such as fast ions provided by the neutral beam injection, collisions by alpha particles, or indirect and due to particle sources such as charge exchange with cold recycling neutrals. Important modifications of the toroidal momentum equation are manifested by its nonlinear coupling to the equation for poloidal rotation.

In the parallel momentum equation using the ambipolarity condition and the extended forms of the stress tensors, one can cancel the time derivative and the source terms. The result in lowest order is a nonlinear equation between the radial derivatives of the poloidal and toroidal plasma velocities:

$$U^{(2)}{}_{\theta,i} + 1.833(e_i B_\varphi)^{-1}\frac{\partial T_i}{\partial r} = 0.36\frac{\eta_{2,i}/\eta_{0,i}}{1+Q^2/S^2} q^2 R^2 \frac{e_i B_\varphi}{T_i}\frac{\partial \ln T_i}{\partial r}\left[\frac{T_i}{e_i B_\theta}\frac{\partial U^{(1)}{}_{\varphi,i}}{\partial r} + \frac{1}{2}U^{(1)2}_{\varphi,i}\right.$$
$$\left. - U^{(1)}{}_{\varphi,i}\frac{B_\varphi}{B_\theta}\left(U^{(2)}{}_{\theta,i} - \frac{T_i}{e_i B_\varphi}\frac{\partial \ln N_i^2 T_i}{\partial r}\right) + 1.90\frac{B_\varphi^2}{B_\theta^2}\left(U^{(2)}{}_{\theta,i} - 0.8\frac{T_i}{e_i B_\varphi}\frac{\partial \ln N_i^{1.6} T_i}{\partial r}\right)^2\right]$$
$$-\frac{2R^2}{3\eta_{0,i}} J_r B_\varphi \qquad (7)$$

Returning to Eq. (6), we look into the particle source and momentum input terms on the r.h.s. Using the symbolism, $\langle A\rangle \equiv \overline{A} \equiv \oint (d\vartheta/2\pi) hA$, and $\tilde{A} \equiv A - \overline{A}$, for surface averages and the poloidal-angle- dependent parts, these terms can be written as,

$$\langle S_i{}^N U_{i\varphi} h\rangle = \langle (\overline{S}_i{}^N + \tilde{S}_i{}^N)[U_{i\varphi}{}^{(1)}(r) + U_{i\varphi}{}^{(2)}(r,\vartheta)](1+\varepsilon\cos\vartheta)\rangle \quad \text{where } (\varepsilon \sim q\mu) \qquad (8)$$
$$\langle S_{i\varphi}{}^M h\rangle = \langle (\overline{S}_{i\varphi}{}^M + \tilde{S}_{i\varphi}{}^M)(1+\varepsilon\cos\vartheta)\rangle,$$

To calculate (8), we need $U_{i\varphi}{}^{(2)}$. According to the revised neoclassical theory [4], we find that

$$U_{i\varphi}{}^{(2)}(r,\vartheta) \approx -\mu U_{i\varphi}{}^{(1)}(r) A(r,\vartheta) + \mu C(r,\vartheta), \quad \text{where } A \equiv n^{(1)} - b^{(1)},$$
$$C \equiv -\frac{cB_0}{eN^{(0)}} RB_\varphi \frac{P^{(0)}}{B_0{}^2}\left(\frac{e}{T^{(0)}}\frac{\partial V}{\partial r} - \frac{\partial \ln N^{(0)}}{\partial r}\right) n^{(1)} - 2\left[\frac{e}{T^{(0)}}\frac{\partial V^{(0)}}{\partial r} + \left(1 + \frac{N^{(0)}}{T^{(0)}}\frac{\partial T^{(0)}/\partial r}{\partial N^{(0)}/\partial r}\right)\frac{\partial \ln N^{(0)}}{\partial r}\right] b^{(1)} \qquad (9)$$

The perturbed density $N^{(1)}(r,\theta) \equiv n^{(1)}(r,\theta) N^{(0)}(r)$ in Eq.(8) can be found from the expanded and averaged forms of Eqs.(1-3), reflecting the symmetry behaviour of magnetic field $B^{(1)}(r,\theta)$ and the sources. For example, from the energy equations, omitting the sources [6]

$$2\chi_{\|i}\frac{B_\chi}{rB}\frac{\partial^2 n^{(1)}}{\partial\vartheta^2} = N_i\left[RB_\varphi\frac{T_i}{e_i B}\left(4e_i\frac{\partial V}{T_i\partial r} + \frac{\partial \ln T_i^{3/2} N_i^{-1}}{\partial r}\right) + 4U_{\|i}\right]\frac{\partial n^{(1)}}{\partial\vartheta} + 5RB_\varphi\frac{P_i}{e_i B}\frac{\partial \ln T_i}{\partial r}\frac{\partial b^{(1)}}{\partial\vartheta}$$

We note that in-out or up-down symmetry behaviour of A and C imposed by the magnetic field, may be further modified by the influence of the sources, Hence, returning to the averages, we find

$$\langle S_i{}^N U_{i\varphi} h\rangle$$
$$\approx U_{i\varphi}{}^{(1)}[\langle S_i{}^N\rangle(1 - q\mu^2\langle\cos\vartheta\tilde{A}\rangle) + q\mu^2\langle\cos\vartheta\tilde{S}_i{}^N\rangle - \mu^2\langle\tilde{A}\tilde{S}_i{}^N\rangle - q\mu^3\langle\cos\vartheta\tilde{A}\tilde{S}_i{}^N\rangle]$$
$$+\mu\langle C\rangle\langle S_i{}^N\rangle + q\mu^2\langle\cos\vartheta\tilde{C}\rangle + q\mu^2\langle\cos\vartheta\tilde{S}\rangle + \mu^2\langle\tilde{S}\tilde{C}\rangle + q\mu^3\langle\cos\vartheta\tilde{S}\tilde{C}\rangle \qquad (10)$$

and

$$\langle S_\varphi{}^M h\rangle \approx \langle \overline{S}_\varphi{}^M\rangle + 2q\mu\langle\cos\vartheta\tilde{S}_\varphi{}^M\rangle \qquad (11)$$

If we look for a qualitative understanding for the solutions of the partial differential equation (6), introducing the first radial derivative from (7) it can be written within a radial neighborhood of the separatrix approximately as

$$\frac{\partial U_\varphi}{\partial t} + [U_\varphi + f(t,\xi)]\frac{\partial U_\varphi}{\partial \xi} = K(t,\xi)U_\varphi + L(t,\xi) \tag{12}$$

where ξ is a boundary layer coordinate,.i.e., $\xi \equiv (r\text{-}a)/L_\psi$. Here, a is the radial position of separatrix. Functions f, K and L involve U_ϑ, which we consider known. The sign of K depends on the sources. In the simplest case with f=L=0 and K=const. solutions involve shocks and, for K>0 are unstable.

If we look into the equilibrium solutions of (6) and (7) by numerical means, these solutions go to the neoclassical values well inside the separatrix. For this purpose, we use some in radial direction exponentially decaying temperature and density profiles, as shown in Fig.1.

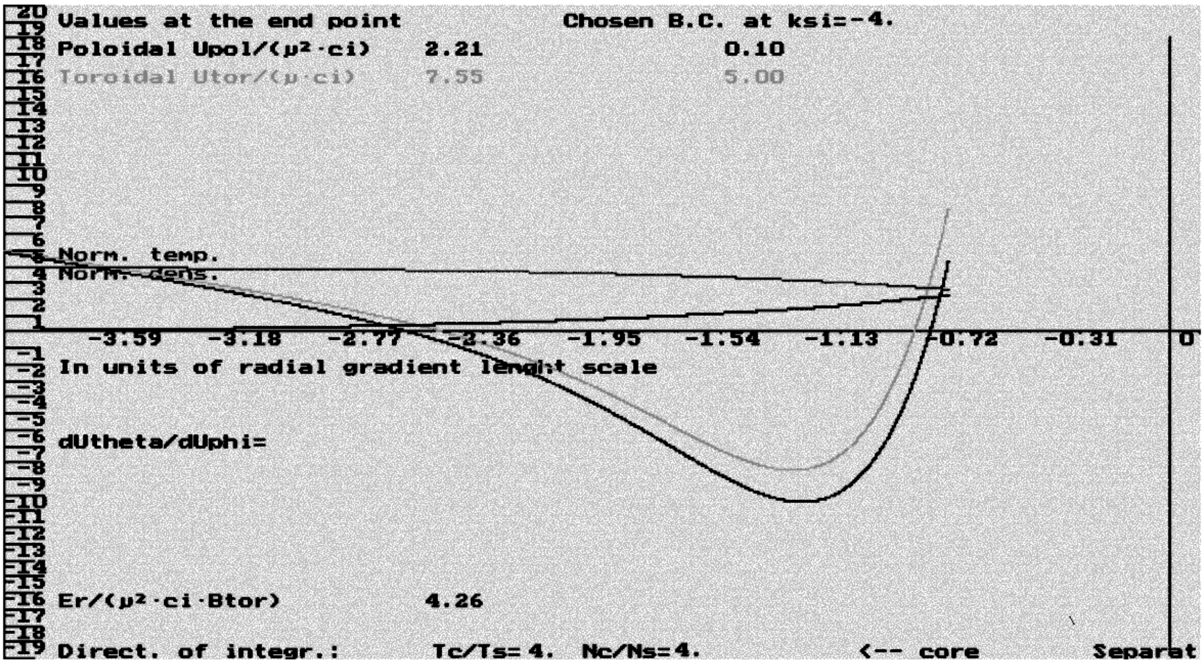

Fig.1 *Normalized rotation velocities and the radial electric field over ξ. Monoton decreasing curve represents the temperature and density profiles. On the left border U_θ starts from the neoclassical value. The lowest curve is the radial electric field. Light gray is U_φ.*

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