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A Rotating Shell and Stabilisation of the Tokamak Resistive Wall Mode

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1 Introduction

It is well known that when a non-rotating plasma is linearly unstable with no wall present, then placing a finitely conducting wall around it does not alter the fact of instability [1]. Growth rates will be modified, but stability boundaries are unchanged. This is true for any confinement device, and in particular, Advanced Tokamak power plant designs require the plasma β to be well above the no-wall limit. It is essential, then, to find a reliable method of stabilising this instability (the Resistive Wall Mode (RWM)). The RWM preferentially locks to the wall, and one early suggestion for stabilisation was to surround the stationary wall with a rotating wall so that mode locking was impossible [2]. This was examined for the case of the Reversed Field Pinch (RFP, where the RWM was first identified [3]). The RWM in an RFP is typically non-resonant (*i.e.* nowhere is the pitch of the perturbation equal to that of the equilibrium). However, the RWM in a Tokamak (due to the ideal pressure driven external kink) is essentially toroidal in nature and a characteristic of the mode is the presence of poloidal harmonics that *are* resonant. The Tokamak case therefore requires a different analysis.

2 Analysis

It is well known that other than at radial stations where ideal MHD breaks down (*i.e.* any internal resonances, and the two walls themselves), the perturbed radial magnetic field, b_r, is determined by marginal force balance (inertial effects are unimportant as the RWM grows on a much longer time scale). In the framework of the large aspect ratio assumption, marginal force balance is a second order, ordinary differential equation, the Newcomb equation [4]. This equation then connects the non-ideal layers, where b_r undergoes the well known jump in its logarithmic derivative, Δ' [5]. Guided by previous work on the single wall system (see *e.g.* [6],[7]) we choose to expand the RWM eigenmode as a sum of basis functions that have a direct physical interpretation. Figure 1 shows the internal plasma resonance at $r = r_s$, and the first static and secondary rotating walls at r_1 and r_2, respectively. We also illustrate the three basis functions selected. (i) Ψ_1 is the external ideal solution for a tearing mode at r_s, when the first wall is regarded as perfectly conducting. Accordingly, Ψ_1 must be regular at $r = 0$ and zero at r_1. (ii) Ψ_2 is, similarly, the ideal external solution for a tearing mode at r_s when the first wall is absent, and the second wall is taken to be perfect. (iii) Ψ_3 is the ideal kink instability when there are no walls present (identical to Ψ_1 inside r_1 and tending to zero at infinity). To ensure that the system displays the RWM we must have ideal instability in the absence of a wall, and so the region (r_s, ∞) must be ideally unstable. Newcomb [4] analysis shows that in this case Ψ_3 exhibits a zero in (r_s, r_1), as is shown in Fig. 1.

Now, each of the three basis functions will in general have discontinuous derivatives at r_s, and each will be associated with a Δ' there. In fact, Δ'_1 and Δ'_3 have already been shown to be important parameters when treating the single wall case [7]. Δ'_1 represents the tearing mode index when the first wall is perfect, and Ref. [7] wrote $\Delta'_1 = -\delta$, so positive δ signified tearing stability for this case. As mentioned above, Δ'_3 must denote

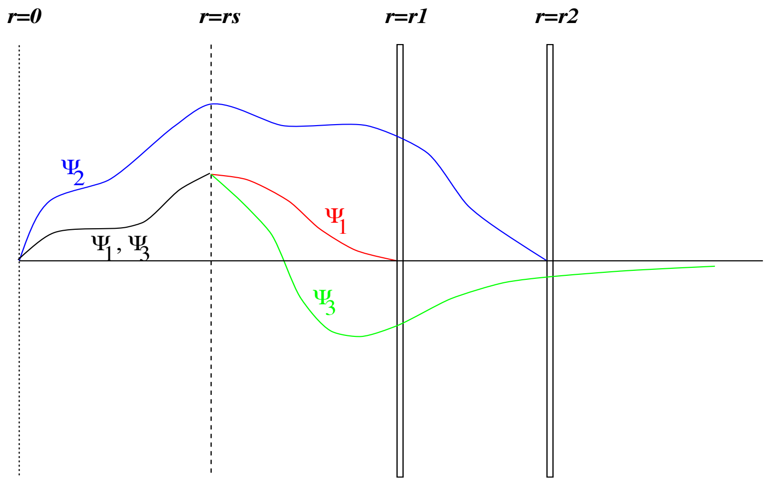

Figure 1: Basis functions used in the RWM analysis.

an ideal, inertial response at the resonance. In Ref. [7], therefore, we wrote $\Delta_3' = -1/\epsilon$. Positive ϵ is then a measure of the ideal MHD growth rate when no wall is present.
The analysis proceeds by representing the RWM eigenfunction as a sum of the three basis functions and applying the boundary conditions at $r = 0, r_s, r_1, r_2$ and infinity. The details will be given in a later paper [8] and we simply quote the result here

$$\Delta_s'(p) = \frac{X^2Y\left(\delta+\Delta_2'\right) - \delta\left(1+\epsilon\Delta_2'\right)(X+p\tau_2)\,p\tau_1 + XY\Delta_2'\left(1-\epsilon\delta\right)p\tau_2}{-\epsilon X^2Y\left(\delta+\Delta_2'\right) + \left(1+\epsilon\Delta_2'\right)(X+p\tau_2)\,p\tau_1 + XY\left(1-\epsilon\delta\right)p\tau_2}, \quad (1)$$

where $X = 2m/(1-Y), Y = (r_1/r_2)^{2m}$, m is the poloidal mode number, $\tau_{1,2}$ the two wall times, and we have assumed the mode $\sim \exp(pt)$.
In Eqn.(1) we now choose the resistive layer response to be 'visco-resistive' [9], $\Delta_s' = p\tau_{VR}$, where $\tau_{VR} \sim \tau_A^{1/3}\tau_R^{5/6}/\tau_V^{1/6}$ and τ_A, τ_R, τ_V are, respectively, characteristic layer Alfvén, resistive and viscous times. This response is appropriate to most tokamak plasmas [9]. Next we Doppler shift $p\tau_{VR} \to (p - i\Omega_{pl})\tau_{VR}$ and $p\tau_2 \to (p - i\Omega_2)\tau_2$ to simulate plasma and second wall rotation. The resulting complex cubic (1) was solved, and contours of equal growth rate plotted in Ω_2, Ω_{pl} space. Now, Δ_2' and δ are important parameters because, between them, they implicitly determine where the two walls are positioned radially with respect to two naturally occurring radii, r_I and r_R. These are, respectively, the radii that a perfect wall has to be placed to make the ideal and tearing modes marginally stable. We now construct examples of the various possibilities.

3 Results

(1) Consider the case where the second wall is *outside* r_I. This means that the ideal mode is unstable even if the second wall is perfect. This in turn means that Δ_2' will be large and negative (as r_2 goes from just inside r_I to just outside, then Δ_2' goes from large positive to large negative). For the case $\Delta_2' = -100$, and 'typical' values for the rest of the parameters ($\epsilon = 0.1, \delta = 1, \tau_1 = \tau_2 = \tau_{VR} = 1, r_1/r_2 = 1.2/1.4, m = 2$) we find that growth rates are positive for all Ω_{pl}, Ω_2. This is not surprising as the ideal mode is not really a RWM, but an ideal mode 'in its own right' as $r_2 > r_I$.
(2) Now let us move r_2 inside r_I. Δ_2' will now be generically large and positive. Figure 2 shows the results for this case ($\Delta_2' = 5$, all other parameters the same as given in (1)), and we see that stable regions have appeared (solid contours have negative growth

rates). However, access to the stable regions requires a sufficient amount of plasma rotation. As the second wall is moved further towards the plasma then Δ_2' drops and so do the amounts of Ω_2, Ω_{pl} required for stabilisation. The symmetry evident in the figure is a straightforward consequence of the model geometry.

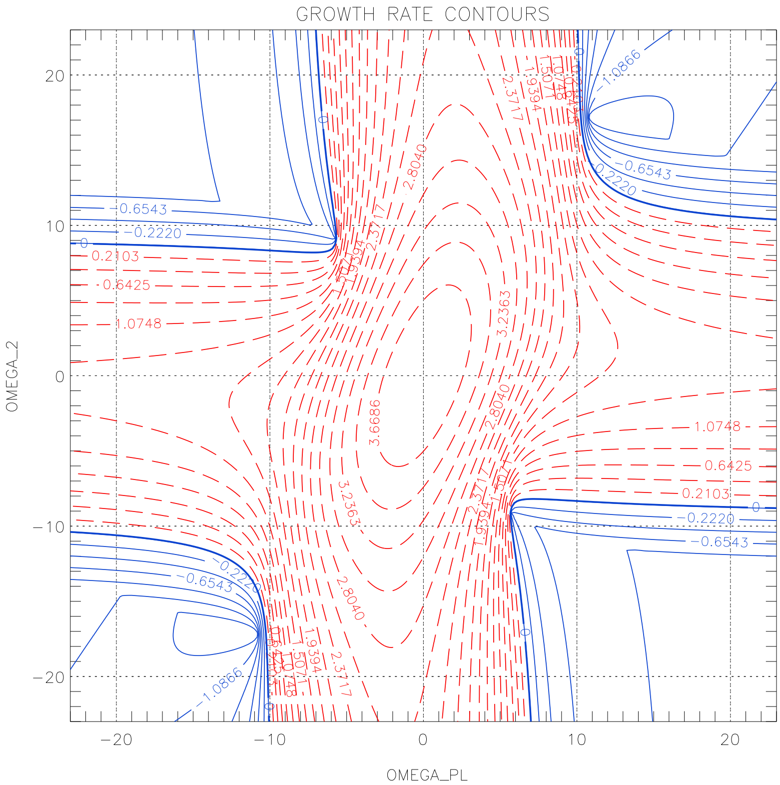

Figure 2: The second rotating wall is inside the ideal marginal point and stabilisation is possible providing there is sufficient plasma rotation.

(3) As the wall is moved further in then the next radius of importance it encounters is r_R, the marginal radius of the resistive mode. At this point, of course, $\Delta_2' = 0$. Figure 3 shows the case where r_2 has just moved inside r_R and $\Delta_2' = -0.3$ (all other parameters the same as given in (1)). We see there has been a topology change due to the change of sign of Δ_2', and now stability is possible with *no* plasma rotation present.

(4) As the second wall is now moved towards r_1, although RWM stabilisation is still possible with $\Omega_{pl} = 0$ it becomes increasingly more difficult in terms of the amplitude of Ω_2 required. In fact, as $r_2 \to r_1$ inspection of Eqn.(1) shows that in the limit, the two walls merge and are 'seen' by the plasma as a single wall. This is then the single wall problem [7], and in the thin shell limit stabilisation is impossible.

4 Conclusions and Discussion

We have investigated the use of a secondary rotating wall to stabilise the Tokamak RWM. A model that simulates the toroidal nature of the Tokamak RWM (generated by the ideal pressure driven external kink mode) has been used. Results depend strongly on the position of the second wall (r_2) with respect to the ideal and resistive marginal radii r_I and r_R (these are, respectively, the radii at which a perfect wall must be placed to make the ideal and resistive modes marginally stable). RWM stabilisation is impossible if $r_2 > r_I$, but possible with finite plasma rotation if $r_R < r_2 < r_I$. Further, the rotation rates required are 'slow' in the sense that they are of order the inverse wall time of the least conducting wall. If $r_2 < r_R$ then stabilisation is possible even in the absence of

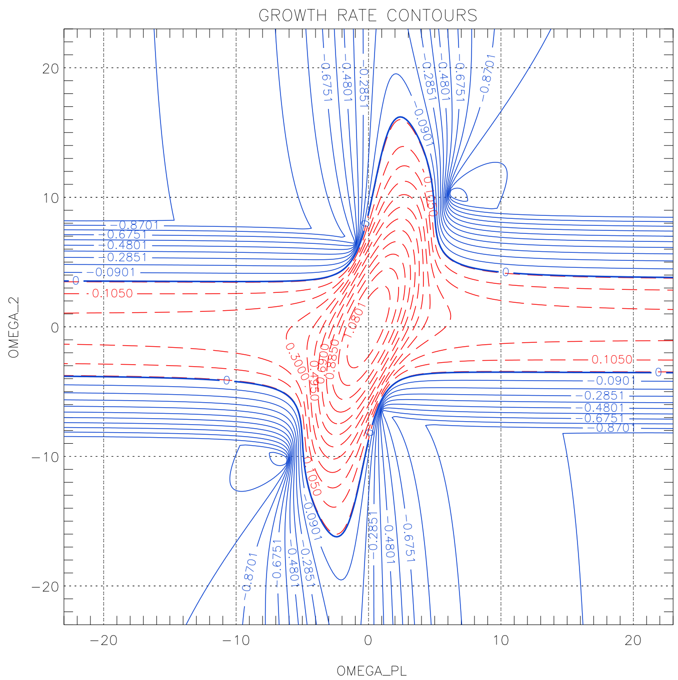

Figure 3: The second rotating wall is inside the resistive marginal point and RWM stabilisation is possible with no plasma rotation.

plasma rotation. (However, as r_2 approaches r_1 stabilisation becomes increasingly more difficult, and there is an optimisation problem.)
This scheme was first considered for the RFP [2]. However, it was later realised that a secondary rotating wall could be 'faked' by a suitable array of external sensors and active coils [10]. What is more, such a wall can be 'projected' and need not reside at the actual location of the coils, a property which may be required in power plant designs [11]. Reference [11] also showed that the gain, bandwidth, current and total power requirements are within the scope of present technology. This scheme, together with that of the 'intelligent' shell [12] (which seeks to directly simulate an ideal wall with external sensors and coils), may well form a useful basis for the essential task of stabilising the RWM in Tokamak power plants.

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