Bootstrap Current in Low Aspect Ratio Reversed Field Pinch

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Abstract

We are concerning with the low aspect ratio reversed field pinch with large bootstrap current fraction and high stability beta limit. The equilibrium is investigated by solving Grad-Shafranov equation under partially relaxed state model condition reasonably close to a stable minimum energy state at finite beta. A considerable amount of bootstrap current can flow even at the moderate aspect ratio of 2\textendash}3 where we can expect the less neutron wall loading and enhance the engineering power gain (Q) of steady state RFP reactor. The strong paramagnetism observed enhances the mass power density (MPD). The compatibility of high Q and high MPD would lead to the low cost of electricity.

1. Introduction

In our previous study, the benefit of radiofrequency wave current drive had been theoretically demonstrated by the significant reduction of nonlinearly turbulent level associated with the dynamo activity in RFPs, suggesting significant improvement of energy confinement time because of the much smaller rf-driven current than the plasma current in equilibrium\([1-4]\). For the dynamo-free stable, steady state RFP reactor, the circulating power must be minimized if the engineering power gain Q is to be maxmized; this goal is achieved by minimizing the noninductive current drive power requirements with plasma self-inductive seed current due to bootstrap current effect.

2. Low aspect ratio PRSM-RFP equilibrium

The enhancement of bootstrap current is predicted by a divergent tendency of neoclassical viscosity as aspect ratio approaches to unity. The low aspect ratio RFP configuration is investigated by solving Grad-Shafranov equation under the partially relaxed state model (PRSM) condition \(j_\psi = \lambda B_\psi/\mu_0\) or \(\mu_0 dI/d\psi = \lambda\), which is reasonably close to a stable minimum energy state at finite beta\([5]\). Here is assumed the \(\lambda\) profile to be uniform (\(\lambda = \lambda_0\)) and the pressure profile \(p(\psi) = P_0(1-\psi)^2\). For convenience, \(l(\psi)\) and \(p(\psi)\) are represented
by \( I(\psi) = I_{ic} + I_{0}(1-\hat{\psi}) \), \( p(\psi) = P_{a}(1-\hat{\psi})^{2} \), \( \hat{\psi} \equiv (\psi - \psi_{0}) / (\psi_{lim} - \psi_{0}) \), where \( I_{ic} \) is the current flowing in toroidal field coil (center conductor), \( I_{0} \) is the total poloidal current in plasma, \( \psi_{0} \) and \( P_{a} \) are the poloidal flux and plasma pressure at magnetic axis, respectively, and \( \psi_{lim} \) is the poloidal flux at plasma boundary. The constant value of \( \lambda_{0} \) is represented by \( \lambda_{0} = - \mu_{0} I_{0} / \left( (\psi_{lim} - \psi_{0}) \right) \). The geometry of plasma boundary is assumed to be fixed in the form using cylindrical coordinates \((R, Z, \phi)\) with \( Z \) the vertical axis and \( \phi \) the toroidal angle, \( R(\theta) = R_{0} + a \cos(\theta + \delta \sin \theta) \), \( Z(\theta) = \kappa a \sin \theta \). Here \( R_{0} \) and \( a \) are major and minor radius, \( \delta \) and \( \kappa \) are triangularity and ellipticity, respectively.

3. Expression for bootstrap current

The bootstrap current arises owing to anisotropy in the electron pressure tensor, and is defined as a parallel current density, by the sum over all species of the product of density \( n \), charge \( Z \), and the parallel fluid flow \( <u_{i} B> \), \( \langle j \cdot B \rangle_{bs} = \sum \frac{Z n_{i} Z_{i} < u_{i} B >}{1} \). The angular brackets refer to flux surface averages. It can be seen that the parallel fluid flow depends on the pressure \( p(\psi) \) and temperature \( T(\psi) \) profiles, collisionality \( \nu_{i} \), ion charges \( Z \), aspect ratio \( A \) and trapped particle fraction \( f_{i} \), \( \langle u_{i} B \rangle = f_{i} \left( p(\psi), T(\psi), \nu_{i}, Z, A, f_{i} \right) \). An useful expression that is often used is the single ion model in the collisionless limit(\( \nu_{i} \rightarrow 0 \), \( \nu_{i} \) is the ratio of the effective collision frequency for detrapping to the bounce frequency of a trapped particle)[7]. The model used for \( f_{i} \) has been shown to be more accurate and is faster to evaluate than the full integral trapped particle fraction[8]. The bootstrap current density for a single ion plasma is

\[
\frac{\langle j \cdot B \rangle_{bs}}{\langle B \cdot \nabla \phi \rangle} = - \frac{p_{e}}{1/R^{2}} \left\{ A_{i} \left[ \frac{1}{p_{e}} \frac{dp_{e}}{d\psi} + \frac{1}{p_{i}} \frac{dp_{i}}{d\psi} - \alpha_{i} \frac{1}{T_{i}} \frac{dT_{i}}{d\psi} \right] - A_{e} \frac{1}{T_{e}} \frac{dT_{e}}{d\psi} \right\}
\]

with \( A_{i} = x(0.754+2.21Z_{i}+Z_{i}^{2})+x^{2}(0.347+1.24 Z_{i} + Z_{i}^{2})/D_{e} \), \( A_{e} = x(0.885+2.08 Z_{i})/D_{e} \), \( D_{e} = 1.41 Z_{i} + Z_{i}^{2} + x(0.754+2.65 Z_{i} + 2.00 Z_{i}^{2})+x^{2}(0.347+1.24 Z_{i} + Z_{i}^{2}) \), \( \alpha_{i} = 1.17/(1.0+0.46x) \), where \( x \) is the ratio of trapped to circulating particles, \( f_{i}(1- f_{i}) \), and \( p_{e} = p_{i} = p/2 \), \( T_{e} = T_{i} = T/2 \), \( T = T_{i}(1- \psi) \). are assumed. It should be emphasized that the collisionless model used is valid for arbitrary flux surface shape and aspect ratio, but noted that the effects of collisionality do not only influence the total bootstrap current but also its profile since the collisionality scales as \( p/T^{3/2} \). The decrease of bootstrap current due to collisionality is evident particularly at the plasma edge where the plasma becomes more collisional (temperature is lowered). On contrary, the single ion approximation underpredicts the bootstrap current at higher temperature.
4. Up-to-date results

Up-to-date results obtained where \( I_{tc} = -15\, \text{kA}, I_\theta = 420\, \text{kA} \) for PRSM-RFP and \( I_{tc} = 210\, \text{kA}, I_\theta = 100\, \text{kA} \) for PRSM tokamak at the fixed parameters of \( a = 0.24\, \text{m}, \kappa = 1.4, \delta = 0.4, \psi_{im} = 0 \) and profile constants and at different \( A \) and \( P_0 \) are summarized as follows: 1) the paramagnetism defined as the ratio of \( B_\phi \) (in plasma) to \( B_\phi \) (in vacuum) on axis is ~27 for the RFP and ~1.5 for the tokamak, being nearly independent of \( A \) and \( P_0 \). The much stronger paramagnetism means that superconducting toroidal field (TF) coil is not necessarily used and would enhance the mass power density of reactor (MPD, [9]). Furthermore, externally supplied toroidal current to generate RFPs is the much less, which would enhance the engineering reactor power gain \( Q \) because of the much smaller resistive dissipation in the TF coil. 2) The bootstrap current fraction \( F_{bs} \) defined as the ratio of total toroidal bootstrap current \( (I_\phi^{bs}) \) to equilibrium plasma current \( (I_\phi^{eq}) \) for \( A = 1.25, P_0 = 4.2\, \text{kPa} \) is ~0.5 for the RFP (with \( F/\Theta = -0.16/3.63 \)) and ~1.77 for the tokamak. This large difference of \( F_{bs} \) between RFP and tokamak indicates that the variations of \( F_{bs} = C_{bs} \beta_p / A^{1/2} \) for the same \( \beta_p \), owing to the hidden dependences in \( C_{bs} \), is quite large. The most important dependences are the kinetic profile, \( n \) and \( f \). The kinetic profile dependence can be understood by introducing the parameter \( \eta_p = n(dT/d\psi)/T(dn/d\psi) \) in the expression of bootstrap current density. It is noteworthy that the observed fraction \( F_{bs} \) is strongly sensitive to the \( f \) term determined by magnetic field structure since the other terms are assumed to the same for both configurations. 3) The fraction \( F_{bs} \) increases with increasing beta value, which is explained by the beta dependence of \( F_{bs} \propto \beta_p / A^{1/2} \approx A^{1/2} \beta_p / A \propto A^{1/2} \beta \), but decreases beyond a critical beta value which depends on \( A \) so that it becomes the lower at the higher \( A \), which can be understood by the observed dependence of \( C_{bs} \) on \( A \) and \( \beta \). As the result, the dependence of \( F_{bs} \) on \( A \) and \( \beta \) is observed to be the larger at the higher \( A \) in low beta case, but at the lower \( A \) in high beta case, within a range of volume averaged beta \( \langle \beta \rangle = 0.2 \sim 1.0 \) (Fig. 1). Fortunately, even at the moderate aspect ratio of 2~3 where we can expect the less neutron wall loading, the fraction is significantly large because of the moderate value of \( \beta_p \) and \( C_{bs} \), leading to the enhancement of \( Q \) and the low cost of electricity by the compatibility of high \( Q \) and high MPD. 4) The observed parallel bootstrap current density \( \langle j \cdot B \rangle_{bs} \) is relatively small to the desired equilibrium parallel current density \( \langle j \cdot B \rangle_{eq} \) near the plasma center and edge regions because of the small pressure gradient there (Fig. 2). Therefore, the pressure profile must be relatively peaked to place the bootstrap current near the plasma center and avoid large currents near the plasma edge, and \( \beta_p \) will be limited to lie below a critical value to keep \( \langle j \cdot B \rangle_{bs} \leq \langle j \cdot B \rangle_{eq} \), then a rf power spectrum be selected in order to that rf current drive should create a current profile \( \langle j \cdot B \rangle_{eq} - \langle j \cdot B \rangle_{bs} \) to generate steady state RFPs.
5. Conclusion

A considerable amount of bootstrap current can flow even at the moderate aspect ratio of 2~3. An important difference from a low aspect ratio tokamak is that the plasma $\beta$, the paramagnetism and the resistive losses in the copper TF coil do not strongly on aspect ratio, as the result, there exists the compatibility of high $\beta$ and high Q even at the moderate aspect ratio where we can expect the less neutron wall loading. An important difference from steady state tokamak with conventional aspect ratio is that there exists the compatibility of high Q and high MPD which would lead to the low cost of electricity.

![Fig.1](image1.png)

*Fig.1 Dependence of bootstrap current fraction on aspect ratio $A$ and beta value ($P_0$) in PRSM-RFPs.*

![Fig.2](image2.png)

*Fig.2 Profiles of flux surface averaged parallel bootstrap and equilibrium current density, $\langle \mathbf{j} \cdot \mathbf{B} \rangle_{bs}$ and $\langle \mathbf{j} \cdot \mathbf{B} \rangle_{eq}$ in the PRSM-RFP with aspect ratio of $A = 2.0$ and central plasma pressure of $P_0 = 8.4$[kPa].*

References