Semi-lagrangian Vlasov simulations of trapped ion turbulence

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A 2D phase space Vlasov code has been developed to study trapped ion instabilities in tokamaks. These instabilities belong to the family of ion temperature gradient driven instabilities and are characterised by radial scales of the order of several banana width and frequencies of the order of the trapped particle precession frequency. The wave-particle resonance determines the threshold of the instability, which involves a critical value of the ion temperature gradient. This low frequency kinetic instability has been extensively studied because it is related to two important features of tokamak anomalous transport: the role of low frequency fluctuations and the improved confinement by negative magnetic shear.

I. Model of trapped ion instability in terms of angular and action variables

This problem is expressed in terms of angular and action variables thanks to the particle motion which is integrable and quasi-periodic (cyclotron and guiding center motions). Indeed it is useful to replace the phase space coordinates (**x,p**) by a set of action-angle variables[1] (ϕ,**J**), with $J=(J_1,J_2,J_3)$ and $\phi=(\phi_1,\phi_2,\phi_3)$ where ϕ_1 is the cyclotron phase, ϕ_2 the bounce angle and ϕ_3 is linked to the precession motion. The angle variables are related respectively to the cyclotron frequency $\omega_1=\omega_c$, the bounce frequency $\omega_2=\omega_b$ and the precession frequency $\omega_3=\omega_c$ with $\phi_1=\omega_c(J)t+\phi_{10}$, $\phi_2=\omega_b(J)t+\phi_{20}$, and $\phi_3=\omega_d(J)t+\phi_{30}$. The ion plasma low frequency response is obtained by gyroveraging the Vlasov equation since $(\omega_2, \omega_3) << \omega_1$. J_1 becomes an adiabatic invariant and it remains only 4 independent variables in phase space. Furthermore for trapped particles we have $\omega_3 << \omega_2$ and the bounce motion can be averaged out too. Thus using action and angular variables transform a Vlasov problem of 6 independent variables into a 2 dimensional phase space one. Another advantage of these variables is to allow a very simple and regular expression of the hamiltonian h(J) and consequently the gyrokinetic Vlasov equation. The price to pay is to transfer the topological difficulties to the quasi-neutrality constraint which is now quite complicated. We must pointed out that this reduction does not affect the non-linear aspect of the problem, in contrast to some methods which usually linearize magnetic coordinates. Finally it leads to the following set of relations

$$\frac{\partial}{\partial t} f_E + \left(\omega_d E + \frac{\partial}{\partial \psi}(J_0 U)\right)\frac{\partial}{\partial \phi_3} f_E - \frac{\partial}{\partial \phi_3}(J_0 U)\frac{\partial}{\partial \psi} f_E = 0 \qquad (1)$$

which is the gyroaveraged Vlasov equation parametrized by the particle energy E and

$$CU(\phi_3,\psi,t) = \frac{2}{\sqrt{\pi}}\int_0^\infty \sqrt{E}\, J_0\, f_E(\phi_3,\psi,t)\, dE - 1 \qquad (2)$$

which is the self-consistent quasi neutrality constraint. C is a constant which depends on some parameters of the problem and J_0 is the gyroaveraged operator. ψ is the poloidal flux and plays the role of the radial coordinate, ϕ_3 is the precession phase, U the electrostatical potential and f_E is the distribution function of trapped ions. Linearizing Eq(1), after a little algebra[2], one obtains the dispersion relation

$$\frac{\Delta\omega}{\omega_d}\left[1+\sqrt{\frac{3}{2}+\frac{\Delta\omega}{\omega_d}}\,Z\left(\sqrt{\frac{3}{2}+\frac{\Delta\omega}{\omega_d}}\right)\right]=\frac{\Delta\tau-\Delta\tau_c}{2\,\Delta\tau_c} \quad (3)$$

where ω_d is the trapped precession frequency and $\Delta\tau_c$ the critical value of the ion temperature gradient. More details are given in previous paper[3].

II. Semi-lagrangian Vlasov Codes

All the type of Vlasov equations, depending on the choice of phase space variables, can be written

$$\frac{\partial f}{\partial t}+\mathrm{V}(\mathrm{X},t).\nabla_{\mathrm{X}} f=0 \quad (4)$$

In our Vlasov equation Eq.(1), X stands for the phase space coordinates and V is a divergence free 2-D advection field:

$$\mathrm{X}=(\phi_3,\psi)$$

$$\mathrm{V}(\mathrm{X},t)=\left(\omega_d E+\frac{\partial}{\partial\psi}(J_0U),-\frac{\partial}{\partial\phi_3}(J_0U)\right)$$

where U is given by the quasi-neutrality equation Eq.(2) and depends on X and t.

The idea behind semi-lagrangian advection[4] is to try to get the best of both lagrangian and eulerian approaches: the regular resolution of eulerian schemes and the enhanced stability of lagrangian ones. This is achieved by using a different set of particles at each time step, the set of particles being chosen at the beginning of the time step such that they arrive exactly at the mesh points of the grid at the end of the time step.

More precisely, let us introduce a finite set of mesh points $(\mathbf{x}_i)_{i=1..N}$ where $N=N_{\phi_3}N_\psi$ is the total number of mesh points (N_{ϕ_3} and N_ψ are respectively the number of points for the precession angle and the poloidal flux) and the characteristics of Eq.(1), which are solutions of the dynamical system

$$\frac{d\mathrm{X}}{dt}=\mathrm{V}(\mathrm{X}(t),t) \quad (5)$$

We introduce $\mathbf{X}[t';\mathbf{x},t]$ the solution of eq.(5) at time t' the value of which is x at time t. Then, given the value of the function f at the mesh points at any given time step, say $t_n-\Delta t$ we obtain the new value at mesh point $\mathbf{x}_i$ at time $t_n+\Delta t$ according to

$$f(\mathbf{x}_i,t_n-\Delta t)=f\left(\mathbf{X}[t_n-\Delta t,\mathbf{x}_i,t_n+\Delta t],t_n-\Delta t\right) \quad (6)$$

and because f is a constant along the characteristics.

The next process involves two steps:

(i) we find the starting points of the characteristics ending at $\mathbf{x}_i$, i.e. find $\delta\mathbf{x}_i$ such as $\mathbf{X}[t_n-\Delta t,\mathbf{x}_i,t_n+\Delta t]=\mathbf{x}_i-\delta\mathbf{x}_i$

(ii) Compute $f\left(\mathbf{x}_i - \delta\mathbf{x}_i, t_n - \Delta t\right)$ by interpolation with cubic B-splines, as f is known only at mesh points $\mathbf{x}_i$

δx_i is obtained by solving an implicit equation. The scheme is accurate up to the second order in time. Moreover the semi-lagrangian Vlasov code is very scalable when using parallel computers. More details can be found in ref [4].

III. Simulations results: low turbulent transport and improved confinement

Our aim was first to check the linear theory of trapped ion instability by performing numerical simulations. For example simulations show that the linear growth rate agrees with the analytical estimate. Indeed Figure 1 shows the growth rate γ_{num} of the first fourier mode of the electrostatical potential for $\Delta\tau - \Delta\tau_c = 0.02$. We find $\gamma_{num} = 0.16$ whereas the theorical growth rate is equal to 0.158 according to Eq(3). We focus then on the turbulent transport. Because of their large radial scale, trapped ion modes may be responsible for the anomalous transport in the collisionless regime. Nethertheless some coherent structures observed in phase space, as shown in figure 2, indicate a possible reduction of the turbulent transport. Thus we have computed the turbulent heat flux using the ion pressure

$$\frac{3}{2} P(\psi,t) = \int_0^{2\pi} \frac{d\phi_3}{2\pi} \frac{2}{\sqrt{\pi}} \int_0^{\infty} dE \sqrt{E}\; E\; e^{-E}\; f_E\left(\phi_3, \psi, t\right) \qquad (7)$$

and averaging the gyrokinetic equation eq.(1) to obtain the heat equation

$$\frac{3}{2}\frac{\partial P}{\partial t} + \frac{\partial Q}{\partial \psi} = 0 \qquad (8)$$

where the heat flux Q is given by

$$Q(\psi,t) = \int_0^{2\pi} \frac{d\phi_3}{2\pi} \frac{2}{\sqrt{\pi}} \int_0^{\infty} dE\; \sqrt{E}\; E\; e^{-E}\; \frac{\partial\left(-J_0 U\right)}{\partial \phi_3}\, f_E\left(\phi_3, \psi, t\right) \qquad (9)$$

It appears that the time average heat flux given by simulations is much lower than the quasilinear estimate. It indicates a low turbulent transport for simulations close to the threshold.

Moreover, our kinetic code allows to study improved confinement scenario according to the well-known dependence of ω_d on the magnetic shear[5]. The negative shear results in a reversal of the precession frequency for some particles. Figure 3 show the growth rate of the first fourier mode of the electrostatical potential for $\omega_d = 1$ and $\omega_d = -1$. Thus the reversal process clearly inhibits the trapped ion instability.

Figure 1

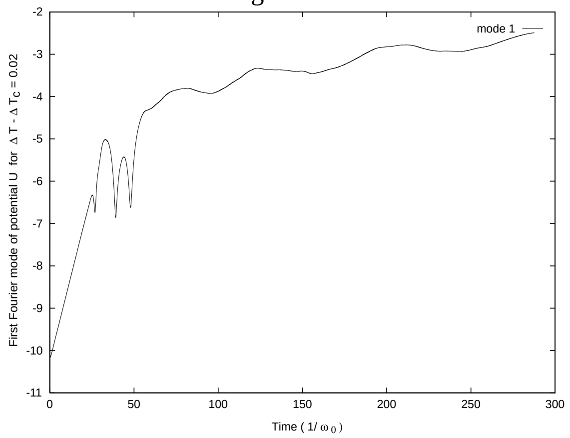


Figure 3

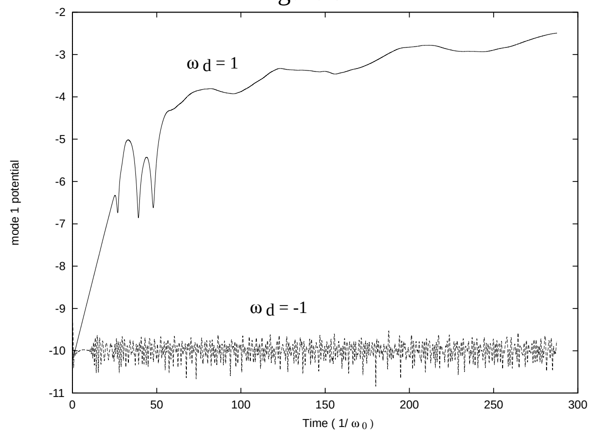
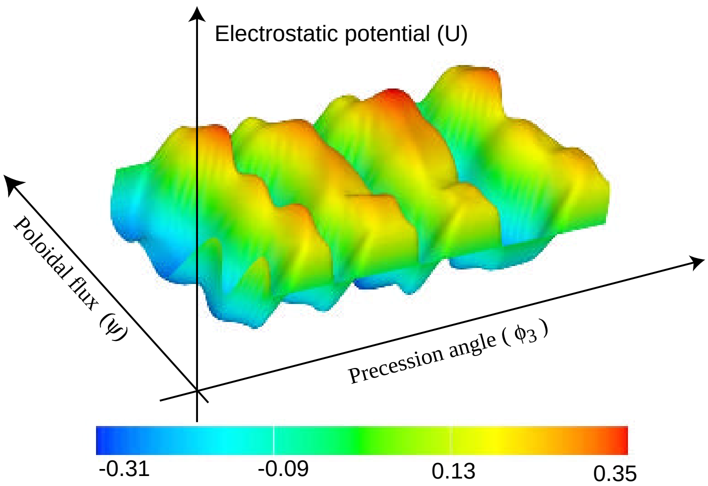


Figure 2



IV. Conclusion

The trapped ion driven mode has been studied by solving a 2D gyroaveraged Vlasov equation using a semi-lagrangian method. The accuracy of this numerical scheme has been verified with the good agreement between the linear growth rate and the analytical estimate. We have computed the turbulent heat flux and found it lower than its quasi-linear estimate close to the threshold, which indicates a low turbulent transport. We have shown the effects of the negative magnetic shear on the growth rate. Now work is in progress to study the turbulent transport far from the threshold.

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