Three-dimensional emission tomography of tokamak plasmas with a single tangentially viewing camera

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Mainly because of the limited set of projections poloidally viewing plasma emission diagnostics together with subsequent tomographic inversion do sometimes lead to non-unique solutions. Usually, we can reliably restore two-dimensional images of three-dimensional objects only, if we use a priori information, i.e. constraints. Additional data from a toroidally viewing system can provide such information. A tangentially viewing camera that gives two-dimensional soft X-ray images with a resolution of 100x100 pixels has been installed on TEXTOR-94 in 1999 \cite{1} and will be improved in 2000.

A numerical simulator for the tangentially viewing camera has been constructed, it can look at toroidal plasma emission from different points outside the plasma, using different values for angle and aperture. This allows us to simulate the pictures from different types of plasmas e.g. such ones with MHD activity of different mode structure. As an example of the output image from such Tokamak Viewer, a modelled emission projection, is shown in the Figure 1.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Model cone projection viewed from a position outside the tokamak.}
\end{figure}

An inverse problem solver has also been developed. Our 3D tomography problem with one projection can be solved only, if additional a priory information is available. We assume that the emission were constant along the magnetic flux tubes, which are to be inferred from the rotational transform and the Shafranov shift both deduced from other diagnostics. Thus, the two-dimensional emission distribution $g_0(x,y)$ in the poloidal cross-section at the toroidal angle $\theta$ is connected by the rotation matrix $T_\varphi$ with a distribution $g_0(x,y)$ at toroidal position $\theta=0$; the angle $\varphi$ depends on toroidal position of the plane. In our numerical simulation we began with simple dependency $q = \frac{\partial \theta}{\partial \varphi}$. Here $q$ is the inverse rotational transform or safety factor.

In this way the 3-dimensional inversion problem is reduced to a 2-dimensional one with curved beams. This our problem is then posed as an inversion of an integral equation of the first kind:
\[ f(u,v) = \int_L g(x,y,z) dl_{uv} \equiv D_{uv} g , \quad (1) \]

where \( f(u,v) \) is measured two-dimensional projection of emission from an unknown three-dimensional distribution \( g(x,y,z) \) along straight lines \( L \), which originate at the detector \((u,v)\). In computerized tomography the operator \( D \) is called the D-transform [2]. The rotation of the poloidal cross-section along the beams has been introduced by another transform:

\[ g(x,y,z) = \tilde{g}_\theta(x,y) = T_\theta g_0 (x,y) . \quad (2) \]

Tomographic inversion procedures tend to be unstable and prone to noise in the experimental data. To this behalf we have introduced appropriate regularization procedures into our inversion algorithm. It uses a modification of what is called the ART method and we employ an additional damping parameter \( \lambda \), which is changing between subsequent iterations. The local plasma emission distribution for the poloidal cross-section \( g_\theta(x,y) \) for \( \theta=0 \), was calculated by the following iterative procedure:

\[ g^{n+1} = g^n + \lambda^n (f_i - \tilde{f}_i^n) / L_i , \quad (3) \]

Here \( L_i \) is the number of points along the ray trajectory, which is projected on the plane \( \theta=0 \), and the function \( \tilde{f}_i^n \) is the forward problem solution along the \( i \)-th ray (e.g., pseudo-projection) when using the solution \( g^n \) from the \( n \)-th iteration:

\[ \tilde{f}_i^n = A_f D_i A_g g^n . \quad (4) \]

\( D_i \) is the D-transform along ray \( i \), \( A_f \) and \( A_g \) are operators of a priori information about the projections and the tomogram, respectively.

In the Fig.2 both, a model emission distribution and its two-dimensional projection are depicted. In Fig.3 there are examples of ray traces for different values of \( q \) of the rotational transform. Figure 4 shows results of reconstruction for the model used together with its pseudo-projection, the errors norms are 17.7 and 0.96 percents, respectively.

References

Figure 2. Model two-dimensional cross-section of three-dimensional plasma emission (left) and its conic projection (right); safety factor \( q=1.5 \).

Figure 3. Projections of all tangential camera rays onto central poloidal plane, \( \theta=0^\circ \). Safety factor \( q=1.0 \) (left), 1.5 (middle), 2.0 (right).
Figure 4. Reconstructed tomogram (left) and its pseudo-projection (right) for the model distribution from Figure 2.