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Damping of double tearing modes by differential plasma rotation

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Introduction: Advanced tokamak scenarios are often characterized by a reversed shear profile and a large toroidal rotation velocity (e.g. typically advanced scenarios of ASDEX Upgrade: toroidal rotation velocity $v_a = 3 \cdot 10^5$ m/s, which corresponds to a toroidal Mach number of $M_{ta} = 0.30$ and $v_a/v_A = 0.05$, v_A is the Alfvén velocity (subscript a marks quantities given at the magnetic axis)). While a non-monotonic q-profile with $q_{\min} < 2$ leads to the formation of a (2,1) double tearing mode (DTM), differential plasma rotation has a stabilizing effect on this mode [1]. Previous numerical computations [2] were restricted to subsonic toroidal flow, $\rho_0 \vec{v}_0 \cdot \nabla \vec{v}_0 \ll \nabla p_0$ (equilibrium quantities: mass density ρ_0, toroidal velocity $\vec{v}_0$, pressure p_0), that is, static equilibria were considered and the centrifugal force was neglected in the linearized MHD equations for the perturbed quantities.

In this work, the stability of ideal equilibria with purely toroidal flow [3] calculated with the DIVA code is compared with that of static equilibria. Both static and flow equilibria serve as input for the CASTOR stability code [4] which has been generalized for the inclusion of toroidal rotation into the equations of the perturbed quantities. Furthermore, an interface between DIVA and CASTOR has been developed.

Theory: The flow equilibria have been calculated by solving the equilibrium partial differential equation

$$\Delta^*\Psi + \frac{\mu_0^2}{2}\frac{dJ^2}{d\Psi} + 4\pi^2\mu_0 R^2 \left\{ \frac{dp_{\rm s}}{d\Psi} + \frac{p_{\rm s}}{2}\frac{R^2}{R_a^2}\frac{dM_{\rm t}^2}{d\Psi} \right\} \exp\left(\frac{M_{\rm t}^2 R^2}{2R_a^2}\right) = 0 \qquad (1)$$

for the poloidal flux Ψ, where $\Delta^*\Psi = R^2\nabla \cdot ((\nabla\Psi)/R^2)$. R is the distance from the axis of symmetry. Terms additional to the standard Grad-Shafranov equation are given in blue. The DIVA code solves Eq. (1) for given profile functions $J(\Psi)$, $p_{\rm s}(\Psi)$ and $M_{\rm t}(\Psi)$. The pressure $p(R,\Psi)$ is given by

$$p = p_{\rm s} \exp\left(\frac{M_{\rm t}^2 R^2}{2R_a^2}\right) \qquad (2)$$

For $M_{\rm t}(\Psi)$ we use the profile $M_{\rm t} = M_{{\rm t}a}(1-s^2)$ for various Mach numbers $M_{{\rm t}a}$ with s^2 being the normalized poloidal flux coordinate, $s^2 = (\Psi - \Psi_a)/(\Psi_b - \Psi_a)$ (Ψ_a = poloidal flux at the magnetic axis (R_a, Z_a), Ψ_b = poloidal flux at the plasma boundary). Since the CASTOR code needs the equilibrium mass density ρ_0 and temperature T_0, as well as the profile of the toroidal rotational frequency Ω_0, we have also to define a temperature distribution. For these calculations we use $T_0 = T_a(1-(1-\epsilon_{\rm T})s^2)$ with $\epsilon_{\rm T} = 1\cdot 10^{-4}$. $M_{\rm t}$ is related to Ω_0 and T_0 by $M_{\rm t}^2 = m\Omega_0^2 R_a^2/T_0$.

The DIVA-CASTOR interface transforms the DIVA output into a form suitable for CASTOR. It is a DIVA library program called by CASTOR which reads the flow equilibrium data (produced by runs of DIVA) and calculates the information required for carrying through the stability analysis: All fluxes, currents and profiles as functions of s as well as the metric tensor elements of the flux surface geometry used by CASTOR.

The CASTOR code has been extended by adding the terms of toroidal rotation to the equations for the perturbed quantities n (particle density), $\vec{v}$ (velocity), T (temperature in eV) and $\vec{B}$ (magnetic field) (in [2] not n and T but the pressure p is considered):

$$\lambda n = -\nabla n_0\cdot\vec{v} - n_0\nabla\cdot\vec{v} - \nabla n\cdot\vec{v}_0 \tag{3}$$

$$\begin{aligned}\lambda m n_0\vec{v} = &- mn(\vec{v}_0\cdot\nabla)\vec{v}_0 - mn_0(\vec{v}_0\cdot\nabla)\vec{v} - mn_0(\vec{v}\cdot\nabla)\vec{v}_0 - \nabla(n_0T) - \nabla(nT_0)\\ &+\frac{1}{\mu_0}(\nabla\times\vec{B}_0)\times\vec{B} + \frac{1}{\mu_0}(\nabla\times\vec{B})\times\vec{B}_0\end{aligned} \tag{4}$$

$$\lambda n_0 T = -n_0\vec{v}\cdot\nabla T_0 - n_0\vec{v}_0\cdot\nabla T - n\vec{v}_0\cdot\nabla T_0 - (\gamma-1)n_0T_0\nabla\cdot\vec{v} \tag{5}$$

$$\lambda\vec{B} = \nabla\times(\vec{v}_0\times\vec{B} + \vec{v}\times\vec{B}_0 - \frac{1}{\mu_0}\eta_0\nabla\times\vec{B}) \tag{6}$$

where the subscript 0 marks the unperturbed quantities. The quantity m is the particle mass and $\gamma = \frac{5}{3}$ is the adiabatic constant. In the coloured terms the equilibrium flow is a toroidal plasma rotation of the form $\vec{v}_0 = R^2\Omega_0(\psi)\nabla\phi$, where $\Omega_0(\psi)$ is the rotational frequency and ϕ is the angle around the axis of symmetry. The first coloured term of Eq. (4) describes the centrifugal force (not included in [2]), while the next two terms are Coriolis-type forces. When using flow equilibria as input we have also to take into account that the pressure and therefore the density is not constant on flux surfaces, that is $\vec{B}_0\cdot\nabla n_0 \neq 0$.

Results: At the beginning, a set of static equilibria ($M_{\rm ta} = 0$) with equal $p_{\rm s}(\Psi)$ but varying $J_{\rm s}^2(\Psi) = J^2(\Psi) + C_{\rm s}^2$ has been computed by changing $C_{\rm s}$. The constant current $C_{\rm s}$ does not change the solution of equation (1) but shifts the q-profile $q_{\rm s} = q(1+(C_{\rm s}/J)^2)^{1/2}$ as shown in Fig. 1. As marked by the thin, straight, black line the distance of the $q = -2$ surfaces, Δs, grows with decreasing minimum q-value in between. The growth rate of the DTM as function of Δs is plotted in Fig. 2 (coupled TM: black line, decoupled TM: orange line). There the coloured dots belong to the q-profiles shown in Fig. 1 (compare equal colours). With growing distance the coupling between the two TM decreases. For $\Delta s > 0.37$ the modes are decoupled but not stable because of the used pressure and current profiles, while for $\Delta s > 0.45$ also the decoupled modes become stable. Here the DTM is stable for $\Delta s < 0.15$.

Using these static equilibria and taking into account differential rotation, the growth rates, marked by the coloured diamonds, are obtained. With rising differential rotation (that is with growing Mach number because the shape of the Ω-profile has been kept fixed) the growth rate decreases and keeps then constant or even increases for larger Mach numbers (compare Fig. 5, violet curve). Only strongly coupled tearing modes are significantly damped by toroidal rotation (left part of the curve in Fig. 2), while it has almost no influence on decoupled modes (orange curve in Fig. 2). Furthermore, the Mach number necessary for maximum damping increases with growing Δs (e.g. ◇: M_{ta}=0.02, ◇: M_{ta}=0.08, ...). The DTM with Δs = 0.159 (red dot on the left-hand side in Fig. 2) is stabilized for M_{ta}=0.005.

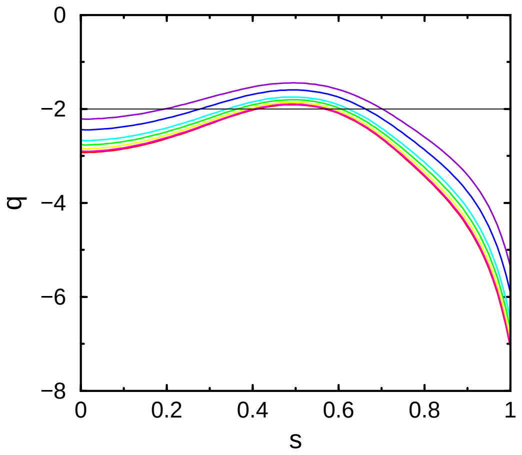

Fig. 1: Various q-profiles of static equilibria.

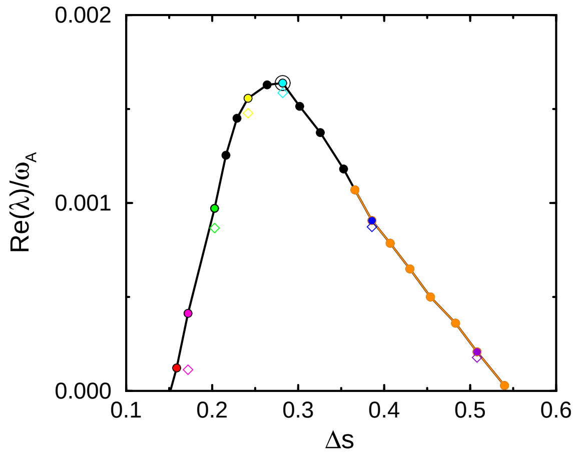

Fig. 2: Growth rate of DTM as function of Δs without (coupled TM: black line, decoupled TM: orange line) and with (diamonds) differential plasma rotation.

Starting from the static equilibrium with maximum growth rate (see Fig. 2, encircled cyan dot) flow equilibria are computed for increasing Mach numbers.

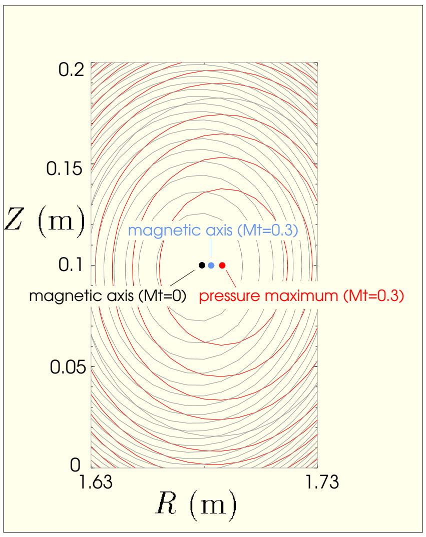

Fig. 3: Flux surfaces (grey) and surfaces of constant pressure (red).

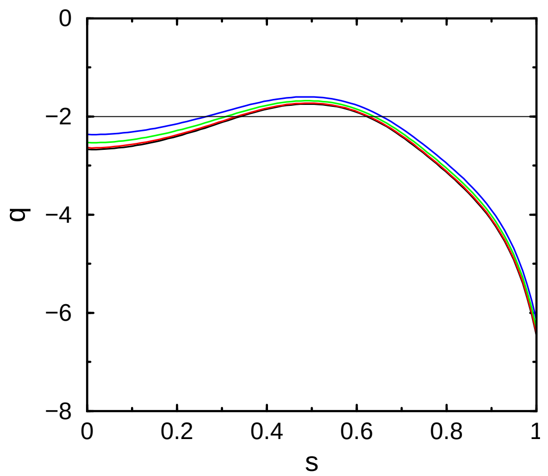

Fig. 4: q-profiles for flow equilibria with M_{ta} = 0.0, 0.1, 0.2 and 0.3.

With rising Mach number the magnetic axis moves in outward direction (see Fig. 3). Furthermore, surfaces of constant pressure and flux surfaces do not coincide for Mach numbers unequal zero. The pressure surfaces are located in outward direction with respect to the flux surfaces. The corresponding q-profiles are also shifted, that is, $|q|$ decreases and Δs increases (see Fig. 4). Because of the shift of the q-profile we already obtain a considerable change in the growth rate of the DTM without taking into account differential rotation in the stability computations (cyan curve in Fig. 5). Computing equilibrium and stability with toroidal flow the orange curve in Fig. 5 is obtained for the growth rate. Because of the small coupling between the modes (compare Fig. 2), differential rotation only slightly influences the growth rate.

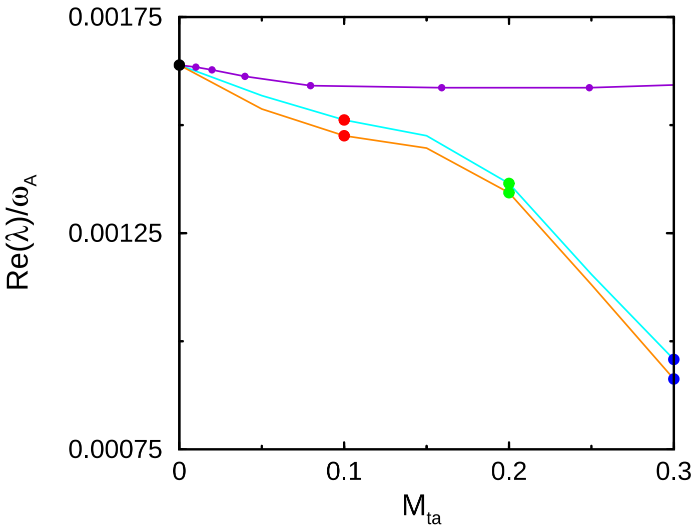

Fig. 5: Normalized growth rate of DTM as function of M_{ta} for: • static equilibria and stability with flow (violet curve); • flow equilibria and stability without flow (cyan curve); • flow equilibria and stability with flow (orange curve). The coloured dots correspond to the q-profiles plotted in Fig. 4.

Summary: We have generalized the theory of ideal, static and stationary MHD equilibria by inclusion of — not necessarily small — differential plasma rotation into the stability equations. After development of a suitable interface between the DIVA equilibrium and CASTOR stability codes corresponding flow equilibria were calculated and analysed for the presence and growth rates of double tearing modes. For $M_{ta} > 0.1$ we obtain significantly shifted q-profiles with respect to the corresponding static equilibrium. Since the growth rates of DTMs depend strongly on the distance between the rational surfaces, future equilibrium calculations (instead of using the standard approach we have chosen to solve equation (1)) should be preferably done on the basis of given q- and M_t-profiles, and the free function $p_s(\Psi)$ determined for a best fit between experimentally measured pressure profiles and the function $p(R, \Psi)$ defined by equation (2).

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