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Monte Carlo δf simulation of wide-orbit neoclassical transport

A. Bergmann, A. G. Peeters, S. D. Pinches
Max-Planck-Institut für Plasmaphysik, 85740 Garching, Germany

Abstract: Neoclassical ion transport near the axis in a tokamak plasma is studied with guiding centre particle simulations. The Hamiltonian guiding centre code HAGIS was augmented by a Monte Carlo procedure for the pitch angle scattering with momentum conservation. The energy transport in the central part of the plasma is found to be strongly reduced.

Advanced tokamak scenarios with reversed shear and high safety factor q on the axis are candidates for a future fusion reactor, and have been studied recently in several experiments. In discharges with an internal transport barrier (ITB) turbulent transport can be largely suppressed, and ion heat fluxes of the close to or even below the level of conventional neoclassical theory have been observed. Hence, a reliable description of neoclassical transport is needed. Unfortunately, close to the magnetic axis the ordering of neoclassical theory[1] breaks down, the width of the ion orbits is not small compared with the minor radius. The critical radius is of the order of a "potato orbit" width[2-4],

$$r_{\text{potato}} = (2q\rho)^{2/3}\, R_0^{1/3}, \tag{1}$$

the width of a very fat orbit that passes through the magnetic axis (here for a magnetic field with concentric circular flux surfaces, q is the safety factor, ρ the gyro radius and R_0 the major radius at the magnetic axis). In ITB plasmas, owing to the high safety factor and the high ion temperatures that are obtained in the core, this radius can be rather large. Recently, new analytic results for wide-orbit effects on neoclassical transport were presented, based on the assumption that near the magnetic axis these potato orbits would take the role of the banana orbits in the conventional theory. The first theory[3] is based on a modified random walk model; the result is a very strong reduction of the transport down to the level of the Pfirsch-Schlüter transport. The second theory[4] is derived from a kinetic equation and predicts near the axis a transport even higher than the thin-orbit theory.

With view on this status of the theory, and with the aim of calculating the neoclassical transport in real tokamak plasmas, we have augmented the δf guiding centre particle code HAGIS[5] by a Monte Carlo pitch angle collision model. HAGIS is based on a Hamiltonian formulation of the equations of motion for ions in a toroidal magnetic field employing Boozer coordinates. The δf method is used, the distribution function is split into two parts $f = f_0 + \delta f = f_M(\mathcal{E}, \psi) + \delta f$, where, in the original HAGIS, f_0 is a function of the constants of motion. Here, for the calculation of neoclassical transport, it is more convenient to choose f_0 as a Maxwellian with constant density and temperature on a flux surface. The kinetic equation for δf is

$$\frac{\partial \delta f}{\partial t} + (v_{||}\boldsymbol{b} + \boldsymbol{v}_d) \cdot \nabla \delta f + \boldsymbol{a} \cdot \nabla_v \delta f = C(\delta f) - \boldsymbol{v}_d \cdot \nabla f_M. \tag{2}$$

where $\boldsymbol{v}_d$ is the torus drift. Without collisions while a particle moves along it's orbit, the weight δf_i changes according to

$$\frac{d\delta f_i}{dt} = -\boldsymbol{v}_d \cdot \nabla f_M = -\frac{d\psi_i}{dt}\frac{\partial f_M}{\partial \psi}. \tag{3}$$

The equations for the evolution of poloidal flux ψ, toroidal and pooloidal angle, θ, φ and $\rho_{||} = v_{||}/\omega_{ci}$ are[5,6] (with $\boldsymbol{B} = g\nabla\varphi + I\nabla\theta + \delta\nabla\psi$)

$$\begin{aligned}
\dot{\psi} &= -\frac{g}{D}\left(\rho_{||}^2 B + \mu\right)\frac{\partial B}{\partial \theta} && (4)\\
\dot{\theta} &= \frac{\rho_{||}B^2}{D}\left(1 - \rho_{||}\, g'\right) + \frac{g}{D}\left(\rho_{||}^2 B + \mu\right) B' && (5)\\
\dot{\varphi} &= \frac{\rho_{||}B^2}{D}\left(q + \rho_{||}\, I'\right) - \frac{I}{D}\left(\rho_{||}^2 B + \mu\right) B' && (6)\\
\dot{\rho_{||}} &= -\frac{1}{D}\left(\rho_{||}^2 B + \mu\right)\frac{\partial B}{\partial \theta}\left(1 - \rho_{||} g'\right) && (7)\\
D &= \rho_{||}\left(gI' - g'I\right) + \left(I + gq\right) && (8)
\end{aligned}$$

where the prime denotes $\partial/\partial\psi$ and the variables are normalised by major radius and magnetic field at the axis as well as ion charge and mass.

The collisions are modelled by a Monte Carlo procedure. The scattering operator is composed of the pitch angle part of the Landau-Fokker-Planck collision operator and a correction term for ensuring momentum conservation[7],

$$C(f) = \frac{3\sqrt{2\pi}}{4\tau_i}\, G(v/v_T)\,\frac{1}{2}\,\frac{\partial}{\partial\xi}\left(1-\xi^2\right)\frac{\partial}{\partial\xi} - \boldsymbol{w}\cdot\boldsymbol{v}\; G(v/v_T) f_M(v/v_T) \tag{9}$$

with

$$G(x) = \left(\left(x^2 - 1/2\right)\mathrm{erf}(x) + x e^{-x^2}/\sqrt{\pi}\right)/x^3, \tag{10}$$

$\xi = v_{||}/v$, and $v_T = \sqrt{2T/m}$. The Monte Carlo procedure amounts to change $v_{||}$ and $v_\perp$ according to

$$\delta v_{||} = -v_{||}\nu\Delta t + \eta v_\perp \sqrt{\nu\Delta t}, \qquad \delta v_\perp^2 = -(2v_{||} + \delta v_{||})\delta v_{||}, \tag{11}$$

employing random numbers η with $\langle\eta\rangle = 0$, $\langle\eta^2\rangle = 1$. The parameter $\boldsymbol{w}$ in Eq.(9) is determined such that the momentum is conserved. This is achieved by a change of the particle weights,

$$\Delta\delta f_i = -R v_{it}\Delta p_{it}\, G(v_i/v_T) f_M(v_i/v_T, \psi_i) \Big/ \sum_i (R v_{it})^2 G(v_i/v_T) f_0(v_i/v_T, \psi_i)\Delta\Omega_i \tag{12}$$

where, in order to avoid the binning of the particles in the poloidal angle, the toroidal angular momentum averaged over the flux surface is taken instead of the parallel momentum. This is justified in the long mean free path regime. $\Delta p_{i||}$ is the momentum change due to pitch angle scattering, and $\Delta\Omega_i$ is the phase space volume element of the ith particle.

The radial fluxes are calculated from the radial particle drift,

$$\Gamma_\psi = \langle \int \boldsymbol{v}_\mathrm{d} \cdot \nabla\psi \, \delta f \, d^3v \rangle, \qquad Q_\psi = \langle \int \boldsymbol{v}_\mathrm{d} \cdot \nabla\psi \frac{mv^2}{2} \, \delta f \, d^3v \rangle \tag{13}$$

where the flux surface average is replaced by volume integration over a thin shell

$$\frac{1}{n} \langle \int A \, \delta f \, d^3v \rangle \approx \int_{\psi-\delta\psi}^{\psi+\delta\psi} A \, \delta f \, d\Omega / \int_{\psi-\delta\psi}^{\psi+\delta\psi} f_0 d\Omega. \tag{14}$$

Owing to the momentum conservation, the particle flux decays on a time scale of order the collision time τ_i. We study the ion energy transport, and assume a diffusive transport law for calculating a transport coefficient χ for the purpose of comparison with the theory. In reality, the transport near the axis might well depend nonlocally on the temperature gradient if wide-orbits are important.

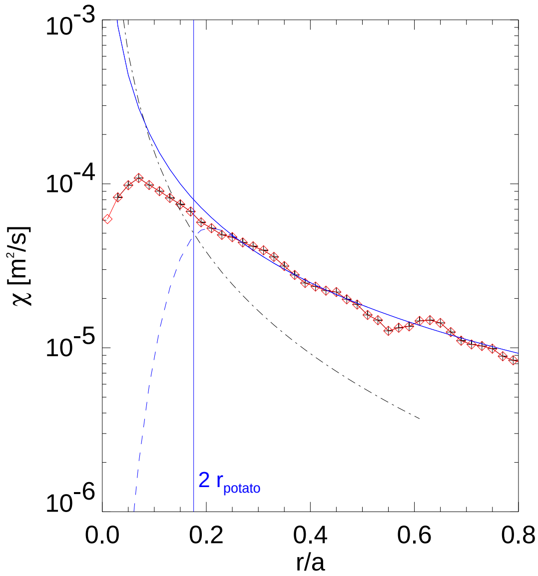

Fig. 1: χ_i for a plasma with concentric circular magnetic surfaces. Comparison of simulation (symbols) and thin-orbit theory (solid line), theory I[3] (dashed) and theory II[4] (dash-dotted), $r_\mathrm{potato} = (2q\rho)^{2/3} R_0^{1/3}$.

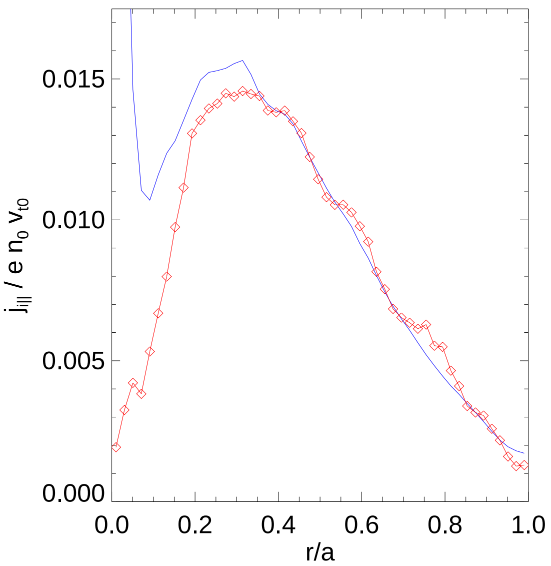

Fig. 2: Parallel ion current (symbols) compared with conventional neoclassical theory (solid line).

As a benchmark we compare the numerical results with the conventional expression for neoclassical transport. For this purpose calculations with a model equilibrium with concentric circular surfaces were performed. For such an equilibrium, in the conventional theory the heat transport coefficient is[1] $\chi_i = \nu_i \, (q^2 \rho_i^2 / \varepsilon^{3/2}) \, K_2$, where K_2 depends on ν_* and ε. In Fig. 1 the match between theory and simulation outside the central region is shown. Towards the magnetic axis the numerical results deviate from the thin orbit theory. The critical radius is about twice the potato orbit width [Eq. (1)]. Neither of the two theories referred to above matches the numerical result in this region. Although the transport is strongly reduced it remains much higher than the Pfirsch-Schlüter transport.

The ion contribution to the bootstrap current, $\langle en_i v_{i||} B \rangle$, is shown in (Fig. 2). It matches the neoclassical prediction[8], except near the axis, where it is strongly reduced.

The code was applied to a discharge of ASDEX Upgrade with an internal transport barrier. The magnetic field as well as the density and temperature variation of f_0 have been taken from experimental data. The high $Z_{eff} \approx 3$ is modelled approximately by increasing the collision frequency. Owing to the q profile (almost 4 at the axis, 1.5 at half radius and 4 at the edge) and the high ion temperature in the centre (14 keV), the potato orbit width is very large. In Fig. 3 the numerical result is compared with the thin-orbit theory[9] (with $\langle B^2 \rangle$, $\langle B^{-2} \rangle$ and $dr/d\psi$ calculated numerically for the experimental flux surfaces, r is the radius of a circular flux surface with the same volume). Here, χ continues to increase towards the axis, but much less than in the thin-orbit theory; the transport level is about 5 times the Pfirsch Schlüter transport. The parallel ion current is smaller than predicted by the conventional theory in the central part of the plasma (Fig. 4).

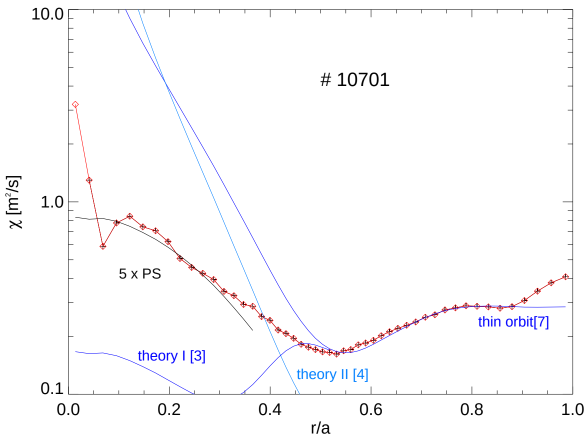

Fig. 3: χ_i in a plasma with reversed shear. Comparison of simulation result (symbols), thin-orbit theory[9] (solid line), theory I[3] and theory II[4]. Also shown: Pfirsch-Schlüter contribution multiplied by a factor 5.

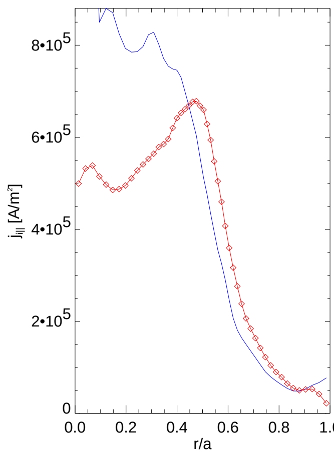

Fig. 4: Parallel ion current (symbols) in the same plasma compared with neoclassical theory (solid line).

Summary: Guiding centre simulations of the collisional ion energy transport in a tokamak show that near the magnetic axis the transport is smaller than in the conventional small orbit neoclassical theory; neither of two recent theoretical predictions is confirmed.

References:

1. F.L. Hinton, R.D. Hazeltine, Rev. Mod. Phys. 48, 239 (1976).
2. T.E. Stringer, Plasma Phys. 16, 651 (1974).
3. Z. Lin, W.M. Tang, W.W. Lee, Phys. Plasmas 4, 1707 (1997).
4. K.C. Shaing, R.D. Hazeltine, M.C. Zarnstorff, Phys. Plasmas 4, 771 (1997).
5. S.D. Pinches et al., Comp. Phys. Comm. 111, 133 (1998).
6. R.B. White, M.S. Chance, Phys. Fluids 27, 2455 (1984).
7. Z. Lin, W.M. Tang, W.W. Lee, Phys. of Plasmas 2, 2975 (1995).
8. O. Sauter, C. Angioni, Phys. of Plasmas 6, 2834 (1999).
9. C.S. Chang, F.L. Hinton, Phys. of Fluids 25, 1493 (1982).