Monte Carlo Solution of the Orbit Averaged Fokker-Planck Equation Including Ripple Losses

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1. INTRODUCTION

Fast ions losses caused by the magnetic field ripple in a tokamak are potentially a serious problem. Not least because such losses tend to be fairly localised. The ripple in Tore Supra, for example, is relatively strong (around 7% at the plasma edge) and the resulting losses can be as high as several tens of percent of the input power during Ion Cyclotron Resonance Frequency (ICRF) heating. Moreover, the ripple loss diagnostics in Tore Supra only detect losses caused by ripple well trapping [1]. Direct orbit losses, i.e. particle orbits intersecting the wall near the outer midplane, which are enhanced by ripple induced stochastic diffusion [2] are not measured. Accurate modelling is therefore important for assessing the seriousness of the total ripple induced losses. Especially for future long pulse operation at Tore Supra (the CIEL project).

While the effect of magnetic field ripple on neutral beam injected fast ions and fusion products have been extensively studied, relatively little work on the effects of ripple on ICRF heated fast ions has been carried out. The conventional method for simulating ripple losses is to use an orbit following Monte Carlo code. However, the computation time would be quite long for ICRF heated plasmas. Not least because in the case of ICRF heating, it is necessary to follow low and intermediate energy particles as well as the fast ions. An alternative method is therefore called for. In the present work we use an approach similar to the one presented in Ref. 3. The analysis in Ref. 3 was, however, limited to the small banana width limit. Since in present day experiments the resonating ions are routinely accelerated to the MeV range during ICRF heating [4, 5], it is necessary to account also for finite orbit width and non standard orbit topology. In addition, effects of ICRF induced spatial transport of resonating ions should be included in the analysis, especially for asymmetrically phased ICRF antennas [6]. We have therefore chosen to implement a ripple package in the 3D Code FIDO [7]. This code solves an orbit averaged Fokker-Planck equation for the distribution function of the resonating ions [8], utilising a Monte Carlo technique. Included in the orbit averaged Fokker Planck equation are all aspects of finite (guiding centre) orbit width and ICRF induced spatial transport.

In the modelling presented here, the problem of ripple losses is split in two parts. First there is trapping in local magnetic wells and secondly there is stochastic diffusion. Separate modelling of these two processes has been introduced in the FIDO code. We first briefly discuss the FIDO code and then describe the ripple package implemented in the code. Examples and results from the code are finally presented.

2. THE FIDO CODE

The FIDO code solves the orbit averaged distribution function, f_0, of the resonating ions, the evolution of which can be described by a Fokker-Planck Equation [8],

$$\frac{\partial f_0}{\partial t} = \langle Q(f_0)\rangle + \langle C(f_0)\rangle + \langle L(f_0)\rangle$$

where C is a collision operator, Q is a quasi-linear ICRF operator, L accounts for the ripple and $\langle\cdots\rangle$ denotes averaging over unperturbed guiding centre orbits. The distribution function f_0 is a function of three invariants of the unperturbed motion, e.g. the energy, E, a pitch angle variable, $\Lambda = \mu B_0 / E$ (μ is the magnetic momentum) and the toroidal angular momentum $P_\varphi = mR\mathrm{v}_\varphi + Ze\psi$ (ψ is the poloidal flux). In addition, a label σ is needed to distinguish between orbits in regions where the triplet E, Λ and P_φ corresponds to two orbits. A Monte Carlo technique is employed to solve the orbit averaged Fokker-Planck equation, i.e. a large number of Monte Carlo particles are followed in phase space. The phase space positions of the particles are periodically updated by the application of Monte Carlo operators representing $\langle C\rangle$, $\langle Q\rangle$ and $\langle L\rangle$. It should be noted that each Monte Carlo particle represents many ions uniformly distributed along the bounce phase of an orbit characterised by the triplet E, Λ, P_φ.

2.1 Ripple well trapping. For trapped ions with turning points in the region with local magnetic ripple wells, defined by $\nu = BN\delta / B_R > 1$ (N is the number of coils, δ the ripple amplitude and B_R the magnetic field component along the major radius), a simple algorithm is applied. Since we are merely interested in fast ion losses, only ions with energies above 1.5 times the thermal velocity are considered. For these ions a sub Monte Carlo loop is performed scattering them in and out of the ripple in accordance with the expressions for trapping and de-trapping probabilities given in Ref. [9] for a combination of collisional and non-collisional ripple well trapping. During the time the ions stay ripple well trapped, they are moved along iso-B lines with a rate corresponding to drift velocity, v_D. In practice, however, rather prompt losses take place for most ions which see an effective collision time ($\delta\tau_{ii}$) longer than the ripple bounce time and $d\nu / ds_D > 0$, s_D is a length element along the drift direction. As a consequence, the final result is not too sensitive to the details of the modelling. The presence of a radial electric field, $E_r = -d\Phi / dt$ modifies the picture somewhat. The main point is that the drift will no longer be along iso-B lines, but along lines where the total energy, $W = \mu B + Ze\Phi$, is conserved. As a consequence, in the case the electric field modifies the drift toward smaller major radii, some ions might drift into to the region without ripple wells before they are lost. An algorithm to take this into account has been implemented.

2.2 Stochastic diffusion. In regions without local magnetic wells, the particle motion is still perturbed, which can lead to the turning points of banana trapped ions diffusing along iso-B lines [2]. In this region the operator $\langle L\rangle$ can simply be represented by

$$\langle L(f_0)\rangle = g^{-1/2}\frac{\partial}{\partial P_\varphi}\left[g^{1/2} D_{Rip} \frac{\partial f_0}{\partial P_\varphi}\right],$$

$$D_{Rip} = D_{Rip}^{\max}(I_W^{Stoch} + I_W^{Coll}),\quad D_{Rip}^{\max} = (\Delta P_\varphi^{\max})/\tau_b,\quad I_W^{Stoch.} = 1.0/\left(1+\exp(6.9-5.5\gamma)\right)$$

where $g^{1/2}$ is the Jacobian of the transformation; $\gamma = N\left|\Delta P_\varphi \partial\varphi_b / \partial P_\varphi\right|$ is a Chirikov parameter, φ_b is the toroidal angle at the bounce point of a trapped ion; τ_b is the bounce time; the factors I_W^{Stoch} and I_W^{Coll}, taken from Ref [10] (I_W^{Coll} is not given here due to lack of space), account for the non-linear and collisional decorrelation of the toroidal bounce phase φ_b, respectively; and the maximum perturbation of P_φ can be written as [11]

$$\Delta P_{\varphi}^{\max} = \Lambda E\sqrt{2\pi N / \ddot{\varphi}_b} \ .$$

The Monte Carlo operator corresponding to the diffusion operator above

$$\Delta P_{\varphi} = g^{-1/2} \frac{\partial}{\partial P_{\varphi}} \left[g^{1/2} D_{Rip} \right] \Delta t + \xi \sqrt{2 D_{Rip} \Delta t}$$

where ξ is a random number with zero mean and unit variance, has been added to the FIDO code. The main effect of a radial electric field is to add a term E_r / B_{θ} to the expression for the bounce phase, thus modifying the Chirikov parameter. It also introduces a change in Λ. However, since it is mainly high energy particles that are affected by stochastic diffusion, the effect of the electric field on this process is normally rather modest.

3. NUMERICAL EXAMPLES

Results from the FIDO code including the ripple modelling have been compared to experimental results from Tore Supra. For a review of the experimental results and techniques see Ref. [1]. An example of a comparison is shown in Fig. 1, where the measured and calculated ripple well loss power fractions are plotted as a function of the plasma density. The discharges were characterised by I_p=1.4MA B=3.7T, f_{ICRF}=57MHz, i.e. R_{Res}≈2.37m, P_{ICRF} = 4MW, hydrogen minority heating in deuterium, n_H/n_D≈10%. As can be seen, the code calculations are in good agreement with the experimental results, with the decrease in losses with density being well reproduced. The error bars on the hydrogen concentration, which is estimated from charge exchange measurements, are fairly large. We have therefore varied the hydrogen concentration in the simulations for the case n_e=6·10^{19} m^3 in order to study the sensitivity. The result is shown in Fig. 2, where the total losses are shown together with a simulation where the stochastic diffusion has been suppressed .

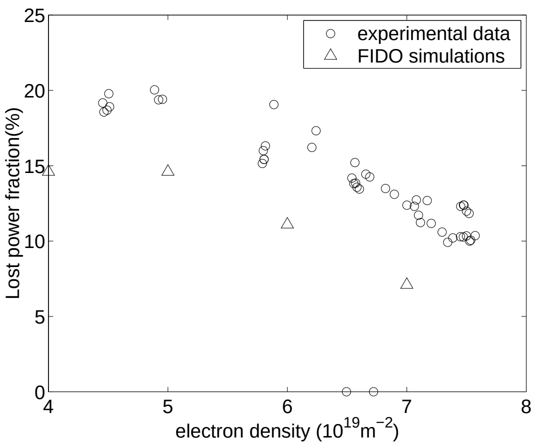

Fig. 1 Power loss fraction as a function of the plasma density.

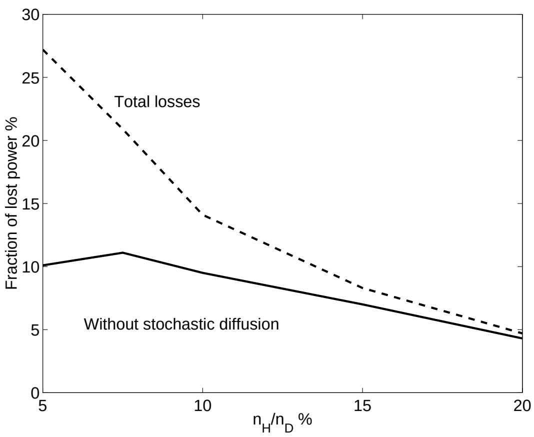

Fig. 2 Losses as a function of the hydrogen concentration.

As can be seen, the losses are fairly sensitive to the minority concentration. In particular, it is the effect of stochastic diffusion that increases with decreasing minority concentration. The reason is of course that a lower minority concentration leads to more energetic minority ions and these are the ones affected by the stochastic diffusion. In the case with the lowest minority concentration, direct orbit losses contribute with about 6% to the total losses of around 27%. In view of the fairly significant losses found in Tore Supra, it is of interest to optimise the discharges so as to keep the losses at a manageable level. One of the most direct ways of reducing the losses is to move the cyclotron layer towards smaller major radii. Since the turning points of resonating trapped fast ions tend to pile up around the cyclotron resonance layer, the losses are reduced as the cyclotron layer is moved away from the ripple well region on the low field side. The drawback is of course that the power is not deposited in the centre, which might be preferable, when the resonance is moved far to the high field side. In order to asses how much can be gained from changing the resonance layer position, we have carried out a resonance scan for a case with the same parameters as the discharge with n_e=6·10^{19} m^{-3}, Fig. 3. The losses are reduced from about 20% for a resonance 10cm on the low field side to about 5% for a resonance 20cm on the high field side. Another possibility for reducing the losses, which also demonstrates the importance of including ICRF induced spatial transport in the calculations, is to use asymmetrically phased antennas. If waves are launched predominately in the toroidal direction of the plasma current, the turning points of the resonating trapped ions will drift towards the mid-plane as a result of the interaction with the waves [6, 12]. Thus the effect of the stochastic diffusion can be counteracted. We have tested this idea by simulating the losses for a symmetric (toroidal mode numbers N=±28) and an asymmetric spectrum (toroidal mode number N=28), the result is shown in Fig. 4. The parameters for the discharge with n_e=6·10^{19} m^{-3} was used in this simulation. As can be seen it is possible to reduce the losses to some extent, especially at high power. The effect is, however, not so strong for moderate power levels.

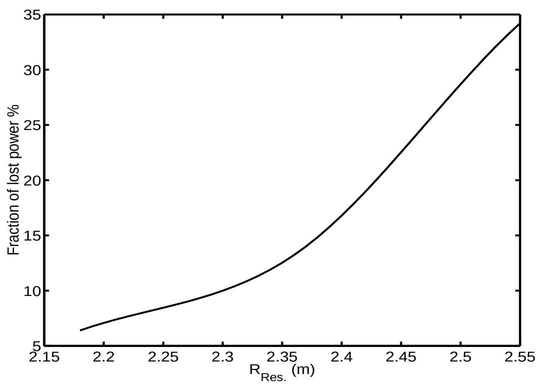

Fig. 3 Loss fraction as a function of the major radius of the cyclotron resonance layer.

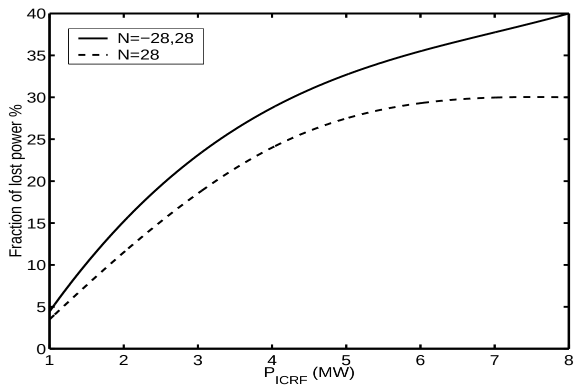

Fig. 4 Fraction of loss power as a function of the ICRF power for a symmetric and an asymmetric wave spectrum.

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