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Equilibrium Reconstruction of Tokamak Discharges with Anisotropic Pressure

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Theory

Equilibrium reconstruction is the essential tool for determining the field configuration and current density in a Tokamak discharge. These quantities are not measurable directly in a fusion plasma, and must be calculated using MHD theory. Most equilibrium codes use the Grad-Shafranov equation, which relies on the assumption of isotropic pressure. This property is often violated for additionally heated discharges. We therefore allow the pressure to be a tensor. Neglecting plasma flow, we arrive at the magnetostatic equation

$$\mathbf{J}\times\mathbf{B} = \nabla\cdot\mathbf{P} \qquad (1)$$

On the basis of guiding centre theory [1], the pressure tensor is expressed by the parallel and perpendicular pressure component as

$$\mathbf{P} = P_\perp \mathbf{1} + (P_\parallel - P_\perp)\frac{\mathbf{B}\otimes\mathbf{B}}{B^2} \qquad (2)$$

Under the assumption of toroidal symmetry, the magnetic field is expressed with the flux function $\Psi(R,Z)$ and $F=R{*}B_{\mathrm{tor}}$ as $\mathbf{B}=(\nabla\Psi(R,Z)\times\mathbf{e}_\varphi + F(R,Z)\mathbf{e}_\varphi)/R$. Inserting into equation (1) and using the pressure tensor (2), one derives an elliptic equation for Ψ [2]. This equation simplifies considerably for a Tokamak plasma, where the magnetic field is dominated by the toroidal field. Since the toroidal field depends mainly on the major radius as $1/R$, we get a Grad-Shafranov type equation [3]

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\Psi}{\partial R}\right) + \frac{\partial^2\Psi}{\partial Z^2} = -R^2\mu_0\frac{\partial P_\parallel(\Psi,R)}{\partial\Psi} - F(\Psi)\frac{\partial F(\Psi)}{\partial\Psi} \qquad (3)$$

The relation between parallel and perpendicular pressure, which are functions of the flux and the major radius, is given by

$$P_\perp(\Psi,R) = P_\parallel(\Psi,R) + R\frac{\partial P_\parallel(\Psi,R)}{\partial R} \qquad (4)$$

The profile functions $P_\parallel$ and F are difficult to measure directly. For experimental discharges, they can be obtained from measurements with the method of equilibrium reconstruction.

Equilibrium reconstruction with anisotropy

The anisotropy model was implemented in the equilibrium reconstruction code EFIT [4]. The unknown profiles $P_\parallel$ and F are mapped onto an appropriate space of test functions, with a finite number of parameters **C.** The code determines these parameters by minimising a least squares functional

$$\chi^2 = \sum_i \frac{1}{\sigma_i^{\,2}} (F_i^{calc}\{\Psi;\mathbf{C}\} - F_i^{meas})^2 + \Re(\mathbf{C}), \tag{5}$$

with the Grad-Shafranov equation (3) as a constraint. In equation (5), F_i^{meas} is the measured value, σ_i its standard deviation, and F_i^{calc} a functional to recalculate it from the flux function and the coefficients. The code always uses measurements from the magnetic probes located at the vacuum vessel. To enhance the accuracy of the reconstruction, EFIT is able to use internal plasma measurements, such as Faraday rotation, MSE diagnostics, and information about the pressure and safety factor. The term $\Re$ is a regularising term to avoid unphysical oscillations in the current profile. A natural choice for the regularisation term seems to be

$$\Re = \Lambda_1 \int_0^1 \frac{\partial^3 P_\parallel}{\partial \overline{\Psi}^3} d\overline{\Psi} + \Lambda_2 \int_{R_{\min}}^{R_{\max}} \frac{\partial P_\parallel}{\partial R} dR \;, \tag{6}$$

where the first term constrains the pressure profile to have a parabolic shape and the second term forces the pressure to be isotropic. The two constants Λ_1 and Λ_2 are chosen appropriately to avoid oscillations of the current profile.

The R-dependence of $P_\parallel$ makes it more difficult to determine the two profile functions $P_\parallel'$ and FF' separately. However, in heated discharges, the power is absorbed mainly in the central region. ICRH deposits the power in a region, which is limited to 0.4 times minor radius around the resonating surface [5]. Neutral beam heating has a broader deposition region, but less anisotropy is generated due to the geometry of the neutral beam injectors [6]. We therefore assume that the pressure is strongly anisotropic only in the region of maximum power deposition. The ansatz function for $P_\parallel$ is chosen accordingly:

$$P_\parallel' = \sum_k (c_{k0} + \sum_n c_{kn} f_n(\bar{r})) g_k(\overline{\Psi}), \qquad \bar{f}_n(\bar{r}) = (\bar{r}(1-\bar{r}))^{10} * (\bar{r} - 0.5)^{n-1}, \tag{7}$$

with $r=(R-R_{min})/(R_{max}-R_{min})$, and R_{min}, R_{max} being suitable limits outside the plasma. The localisation of the anisotropy makes again FF' and $P_\parallel'$ distinguishable. The function $g_k(\Psi)$ defined in equation (7) are chosen as B-splines [7]. B-splines have a local support and allow for an easy implementation of the regularisation. They proved to give a more robust reconstruction than e.g. polynomials in the anisotropy model.

Application to Tokamak discharges

As additional internal plasma measurement for fitting in EFIT, we use the parallel pressure component found by the transport analysis code TRANSP. The pressure is the sum of kinetic background pressure, assumed to be isotropic, and the parallel component generated by neutral beam heating and ion cyclotron heating. As first application, we treat the reversed shear discharge 40847, at 45.7 sec, e.g. shortly after the onset of full heating (18 MW neutral beam, 6 MW ICRH). Earlier attempts to analyse this discharge with the isotropic EFIT and using TRANSP pressure data were unsatisfactory. In particular, the central safety factor was well below unity (see figure 2). Given the absence of sawteeth, this seems to be unrealistic. Relaxing the isotropy condition decouples the position of magnetic axis and peak of the pressure distribution, and gives more freedom for the reconstructed equilibrium. Figure 1 shows field lines and isobars of the reconstructed equilibrium with a magnetic axis at 3.07 m. The q profile, which is flat and slightly inverted, with a central q of 1.66 (see figure 2). The q profile and the magnetic axis position agree well with soft X-ray data for similar reversed shear discharges, e.g. 40572 [8]. From equation (4), we calculate the perpendicular pressure. Profiles of parallel and perpendicular pressure as a function of the major radius and on the height of the magnetic axis are shown in figure 3. The anisotropy is limited to the centre, in agreement with the ansatz for $P_{||}$ defined by equation (7). The perpendicular pressure agrees with the pressure found by TRANSP.

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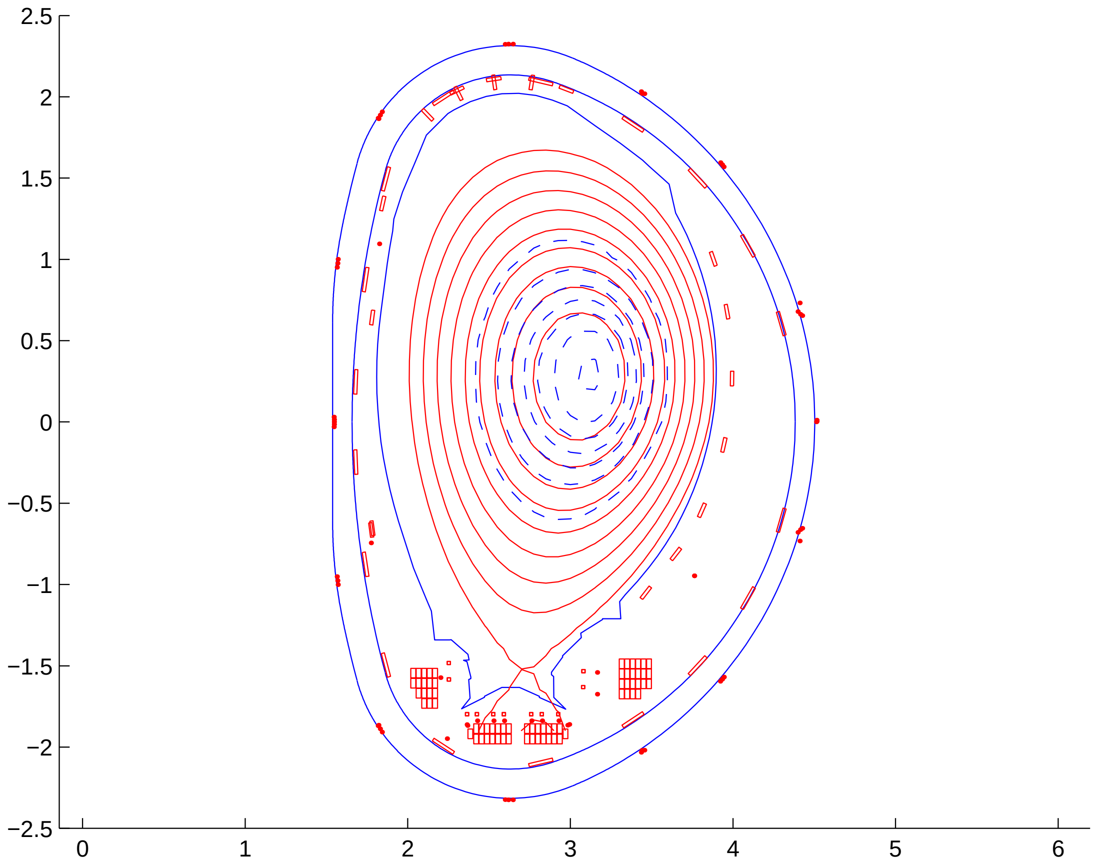

Figure 1: Equilibrium reconstruction of shot 40847 with anisotropic pressure. The field lines (solid) do no longer coincide with the isobars (dashed).

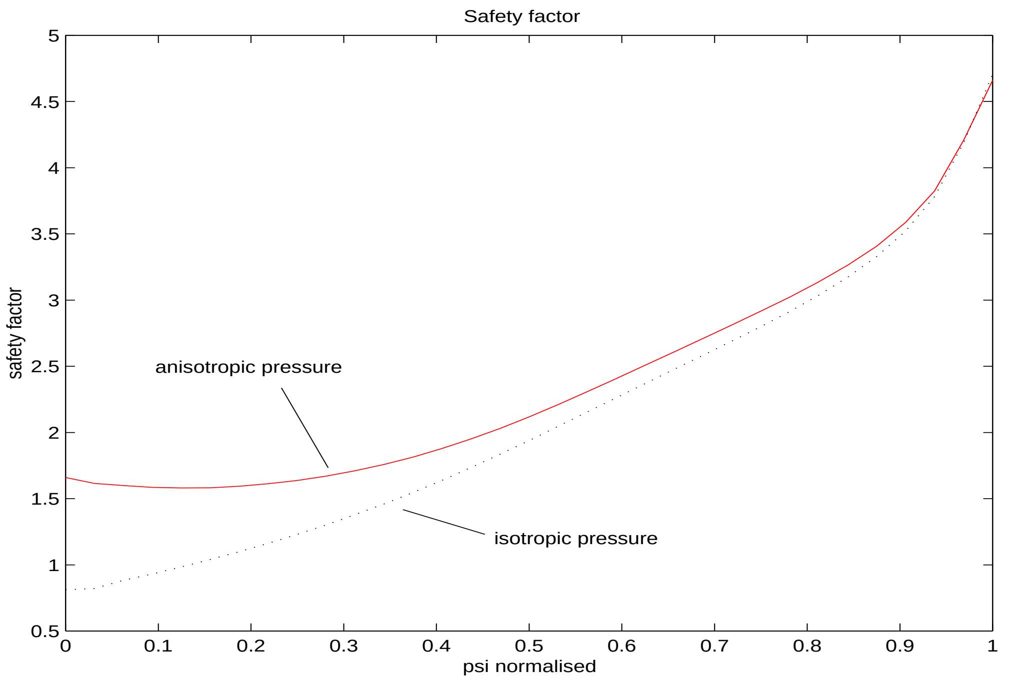

Figure 2: Safety factor of the equilibrium reconstruction of shot 40847. The dotted line shows the q profile when trying to fit to isotropic pressure, giving a central q which is too low. The assumption of isotropic pressure is therefore inconsistent with the other data.

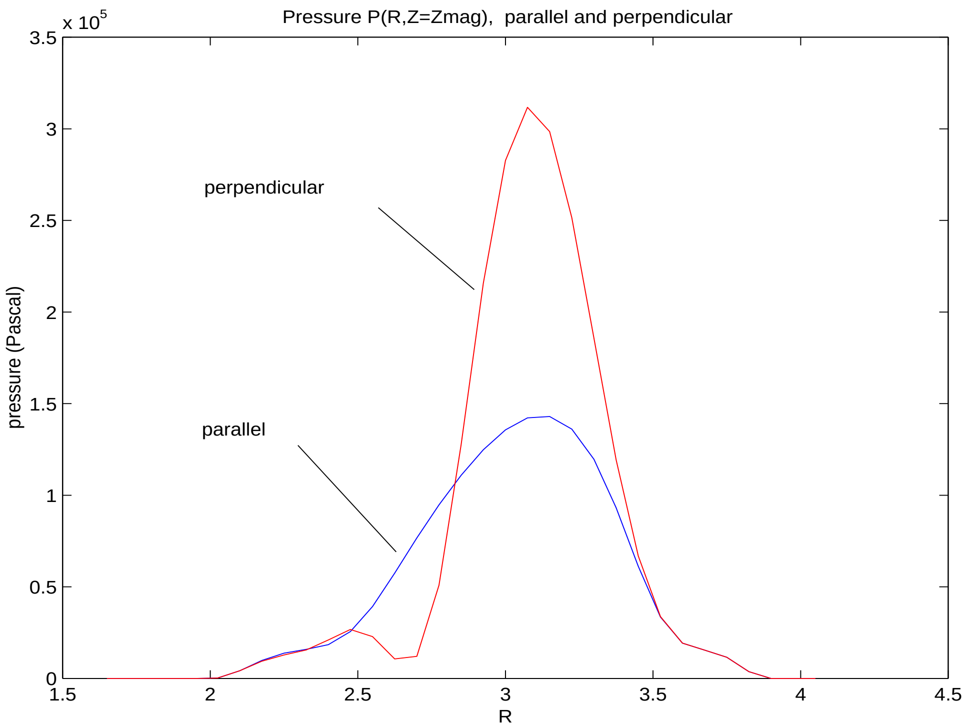

Figure 3. Pressure profiles $P_{||}$ and $P_{\perp}$ as a function of major radius