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Turbulent transport minimization in stellarator optimization

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The flexibility in the design of stellarator equilibrium configurations allows for optimization of confinement properties such as particle transport and magnetohydrodynamic stability. The helical advanced stellarator (Helias) concept [1] follows the optimization strategy of minimizing the currents flowing in the plasma and reducing Pfirsch-Schlüter currents to a level below diamagnetic currents. This leads to a low Shafranov shift and favourable MHD stability properties, as well as to low neoclassical losses and good alpha particle confinement. The question, if and how turbulent transport may also be reduced by a similar magnetic field optimization, is addressed in this contribution.

I. DRIFT WAVES AND TRANSPORT

Small scale, low frequency fluctuations are generally held responsible for the observed transport losses in fusion plasmas that exceed single particle collisional transport by orders of magnitude. Experimental evidence suggests drift waves as a probable cause of the underlying instabilities [2]. The basic mechanism of instability for drift waves is independent of the confining magnetic field geometry: free energy from a pressure gradient together with a non-adiabatic interaction between ions and parallel electron dynamics leads to a nonlinear growth of wave amplitude. The coupling of different wave numbers results in a homogenous turbulent state on the typical perpendicular scale of an ion gyro radius. Stabilizing and catalyzing factors, like global and local shear damping or curvature of the magnetic field, are on the other hand fundamentally determined by the geometry of magnetic confinement. Different stellarator configurations are thus likely to also show specific anomalous transport behaviour.

A well-known approach for the analysis of magneto-hydrodynamic stability of a plasma utilizes the energy principle. By means of a quadratic form, information on destabilization, for example due to unfavourable curvature, and suitable geometric optimization in a stellarator can be obtained in this case [3].

Linear drift waves, however, do because of non-hermiticity in their governing equations in general not allow for the construction of such a quadratic form. For the special case of a simple, collisionless model of radial drift waves, Antonsen [4] had been able to set up a quadratic stability criterion by extension of the real radial coordinate into the complex plane and rotation of the integration axis. This method has been extended by Chen *et al.* [5] for the case of dissipative drift waves, where it has been shown that those are linearly stable in a sheared slab configuration. In the formulation of the problem as an equation along the field line in order to cover toroidal and helical variations in general three-dimensional geometry, the trick of complex continuation is no longer applicable.

Further, the applicability of such a linear stability criterion on transport prediction would be questionable, as due to the nonlinear character of drift wave turbulence, even for linear stability of all eigenmodes, the formation of nonlinear self-sustained turbulence and thus enhanced transport is possible [6]. Simple analytical criteria for stabilization of drift waves in three-dimensional geometry thus can not be specified. For this reason we have chosen the approach of comparative numerical studies.

II. DIRECT NUMERICAL SIMULATION OF DRIFT-ALFVÉN WAVE TURBULENCE IN ADVANCED STELLARATOR GEOMETRY

For our nonlinear simulations of low frequency plasma edge turbulence, we primarily employ a cold ion model with electromagnetic, isothermal electron dynamics. The resulting system of equations and their implementation in a toroidally consistent flux tube treatment for 2D tokamak geometry in a numerical finite-difference code (DALF3) has been described by B.D. Scott in Ref [7]. Fluid equations are used for both species with the usual assumption of $k_{||} \ll k_{\perp}$ in drift ordering. The dynamics is described by a conservation law for the total current $\mathbf{J}$, a parallel equation of motion for electrons, and a continuity equation for electron density n_e. The perpendicular velocity $\mathbf{u}_{\perp} = \mathbf{v}_E + \mathbf{u}_p$ of the ions is the same $E \times B$ velocity $\mathbf{v}_E$ as for electrons with an additional polarization drift correction $\mathbf{u}_p$ due to finite inertia.

Alignment of the coordinate system to the magnetic field allows use of relatively few grid nodes in the parallel coordinate z: The grid size in our simulations was $64 \times 256 \times 64$ in (x, y, z) space with a computational poloidal domain in (x, y) of one ρ_s per grid node, what corresponds to about 4 cm $\times$ 16 cm for typical edge plasmas with drift scale $\rho_s = \sqrt{T_e M_i} c/eB$. An extension of this flux tube model from an axisymmetric tokamak to a stellarator is in principle straightforward, but not without fundamental obstacles. When in general the rotational transform ι is irrational, the flux tube will not close in on itself. The code DALF3, which we use, solves this problem by aligning the flux tube in a local approximation centrally around a rational field line and corrects deformation in the perpendicular plane caused by finite shear with an appropriate shear shift transformation. In a tokamak, this approach is unproblematic even for a high rational ratio $\iota = m/n$, as the geometric background properties do only depend on the poloidal angle θ, unregarding of the number of toroidal circuits until field line closure. In a stellarator, however, the axisymmetry is broken and helical variations of the metric become significant. The variations of background geometry along the field line scale with the number of toroidal circuits and field periods N_f, and so does the number of necessary parallel grid points. For arbitrary rotational transform, the number of grid points evidently diverges rapidly. Thus, for future quantitative simulations of stellarator turbulence, a 3D hollow flux cylinder model will have to be developed instead of the present flux tube approach. For our current purpose of qualitative identification of the relevant geometrical quantities that determine drift wave turbulence in stellarators, we can take advantage of the fact that the edge rotational transform of advanced stellarators like *Wendelstein 7-X* is near unity: $\iota(\Psi_0) = 0.98 \approx 1$ on the last closed flux surface Ψ_0. Of course, the surface with $\iota \equiv 1$ is governed by generic island formation of the five-period Helias. When we nevertheless assume a rotational transform of unity for the start, we hence do not map the geometrical properties of *Wendelstein 7-X* or any other specific configuration exactly, but still gain a quite accurate model of Helias type geometry.

We represent all relevant geometrical quantities for a given equilibrium in terms of metric elements $g^{ij} = \nabla x_i \cdot \nabla x_j$, using three-dimensional numerical equilibrium code data. The metric elements g^{xx}, g^{xy} and g^{yy} are expressed in local flux tube coordinates $x_i \in (x, y, z)$, derived from Hamada coordinates (s, θ, ζ). Components g^{iz} are small compared to the present terms and are neglected. All geometric quantities are assumed to be only dependent on z and constant on the x-y domain. The perpendicular Laplacian now for example becomes $\nabla_\perp^2 = \nabla \cdot \nabla_\perp = g^{xx}\partial_x^2 + g^{xy}\partial_{x,y}^2 + g^{yy}\partial_y^2$. The curvature operator $\mathcal{K} = \mathcal{K}^x\partial_x + \mathcal{K}^y\partial_y$ is expressed in terms of geodesic curvature κ_G, normal curvature κ_N and integrated local shear $\Lambda = g^{s\alpha}/g^{ss}$. All of the effects of poloidal and toroidal asymmetry are thus retained. The assumption of $\iota = 1$ now allows to make use of the five-fold toroidal symmetry of a Helias configuration, in that field lines with label $\alpha_0 = \theta - \iota\zeta$ and $\alpha_1 = \alpha_0 + 2\pi/5$ are identical in their geometric properties. As has been shown by the authors in detail in Ref [8], the most pronounced dependence of calculated transport $\Gamma_n(\alpha)$ is on local shear properties of the specific field lines. In the nearly "shearless" Helias stellarator, where global shear is minimized in order to avoid major resonances within closed flux surfaces, now the average of absolute local shear $\langle |S| \rangle$ with $S = -\mathbf{B} \cdot \nabla\Lambda$ takes over the stabilizing role for drift waves. For the same reasons, a reduction of Γ_n has been found with increasing ellipticity in an l=2 stellarator model metric.

Local shear in a Helias configuration has its maxima in the "corners" of the flux surfaces, whose location winds along the connection of areas of strongest poloidal curvature from the lower tip of the bean shaped poloidal cross section helically across the outmost tip of the triangular cross section on to the upper tip of the bean shaped ploidal plane. The strong deformation observed at this regions of the flux surface results from the neoclassical requirement to locate trapped particles primarily in regions of low field curvature [9]. Thus, a demand of turbulent optimization to allow field lines to frequently cross regions of high local shear, is already implicitly present in former strategies on advanced stellarator optimization. It seems consequent to allow for an explicit maximization of absolute local shear as an additional optimization criterion in future configurations, provided it is not in opposition to MHD and neoclassical demands. From this point of view, a five period configuration seems more suitable for transport reduction than recently (for the purpose of low aspect ratio ignition experiments) studied configurations with less field periods but similar edge rotational transform [10].

III. COMPLEMENTARY LINEAR MODELS

The relevant role of local shear in drift wave stabilization is already present in linear fluid models, as we have shown in Ref [11]. Even for a series of similar stellarator configurations, local variations in the geometric quantities g^{ss}, $|B|$, κ_G, κ_N and Λ can influence the linear instability significantly. For linear drift modes driven by dissipative trapped electrons (DTEM) a favourable effect has been observed due to the neoclassical optimization strategy of localizing trapping regions in regions of low curvature.

The electromagnetic gyrokinetic ion-temperature-gradient (ITG) instability at finite plasma beta has been studied for core plasma parameters of a Helias reactor configuration (HSR), and a predominant influence of curvature properties has been found [12]. Both localization of eigenmodes along the field line in an eikonal model and variation of growth rate $\gamma(\alpha)$ with field line label suggested quantitative dependence on local unfavourable normal curvature. Our present ITG mode studies do not yet include effects of trapped ions, which can be expected to be of importance, and are radially local. Global mode calculations in Helias geometry are currently being prepared by R. Hatzky [13].

We find that electrostatic ITG modes are slightly stabilized in calculations of a series of HSR equilibria for rising plasma beta. Finite beta leads to an increase of the magnetic well from $V'' = -0.68\%$ in the vacuum case to $V'' = -9.14\%$ for $\langle\beta\rangle = 5\%$ [14]. Local shear appears in ITG mode description only in connection with the geodesic curvature, which is minimized in a Helias stellarator. A destabilization through unfavourable curvature also occurs in ideal MHD ballooning theory. In both cases, the normal curvature enters as a product κ_N/g^{ss} with the distance between neighbouring flux surfaces. The magneto-hydrodynamic optimization strategy to compress magnetic surfaces in regions of unfavourable curvature thus also has a stabilizing effect on ITG modes.

IV. CONCLUSIONS

By means of direct numerical simulation of drift wave turbulence and complementary linear modelling in advanced stellarator geometry we were able to obtain tentative optimization criteria for a possible systematic minimization of turbulent transport in Helias configurations. Absolute local shear has been identified to take an important stabilizing role for drift waves and edge turbulence. Previous optimization strategies with respect to neoclassical transport and MHD stability proved favourable also for DTEM and ITG mode stability, respectively. The quality of a microstability optimized stellarator configuration will again have to be evaluated with the help of numerical simulations. Experience from MHD stability analysis shows, that for this purpose a global treatment of the whole stellarator flux surface might prove necessary, once the numerical realization will be practicable with growing available computing capacities.

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