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Temperature gradients in magnetic reconnection

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Introduction. Magnetic reconnection in its nonlinear stage merges flux tubes which extend through regions of space that can be far apart and, in the presence of temperature gradients, have different temperatures. Of particular interest are collisionless, Hamiltonian reconnection processes, which are presently believed to provide a plausible explanation for fast magnetic topology changes in hot plasmas, where the time scale of the magnetic activity is found to be smaller than the collision time.
In this paper the problem of Hamiltonian reconnection in the presence of a temperature gradient is studied introducing a model that is a kinetic extension of the reduced 2–fluid model in [1]. The model retains electron inertia and includes the Hall term in the generalized Ohm law, a term which arises due to the coupling between the electron parallel compressibility and the ion motion. The effect of a temperature difference in the specific case of a stationarily reconnecting solution is treated. This time–independent problem avoids the effects of Landau resonances that usually occur in time–dependent problems. In general, far from the resonant layer the phase velocity of the reconnecting mode is much smaller than the thermal velocity. In this zone the temperature is assumed to be a flux function. Close to the resonant layer the phase velocity exceeds the thermal velocity and the equation of state approaches the adiabatic law. Between these two zones Landau resonances are important, and kinetic effects have to be considered. For infinitesimal island widths, linear kinetic models have been extensively studied, see *e.g.* [2], in particular for ω^* effects on the reconnection rate.

The model. We consider a strongly magnetized, low-β plasma, where the perturbation scale-length perpendicular to $\boldsymbol{B}$ is much smaller than the perturbation scale-length in the direction parallel to $\boldsymbol{B}$. Excluding compressional Alfvén waves, a simple representation of the magnetic field can be adopted. Introducing the magnetic flux ψ and electric potential ϕ, the electromagnetic fields can be described by $\boldsymbol{B} = B_0(\boldsymbol{e}_z + \boldsymbol{e}_z \times \nabla\psi)$, and $\boldsymbol{E} = (B_0 c^{-1})(-\nabla\phi + \boldsymbol{e}_z \partial_t \psi)$, where $\boldsymbol{e}_z$ is the unit vector in the z– direction. Curvature and $\nabla\boldsymbol{B}$ effects are negligible, hence parallel and perpendicular dynamics are decoupled and and the drift kinetic equation for the electron distribution function can be integrated over the magnetic moment, leading to the closed system of equations

$$D_t f + v_\parallel \nabla_\parallel f = \Omega_e (D_t\psi - \partial_z\phi)\frac{\partial f}{\partial v_\parallel}, \qquad \Delta\psi = -e\int dv_\parallel v_\parallel f, \qquad \Delta\phi = \Omega_i \log n/n_0, \quad (1)$$

where $f = f(\boldsymbol{x}, v_\parallel, t)$ is the electron distribution function, Ω_e, Ω_i are the electron and ion gyrofrequencies, $D_t = \partial_t + [\phi, \ldots]$, $\nabla_\parallel = \partial_z + [\psi, \ldots]$, and $[g, h] = \nabla g \times \nabla h \cdot \boldsymbol{e}_z$. The Poisson equation was simplified as in Ref. [1] by taking the ion temperature much smaller than the electron temperature so that ions are passively advected by the $E \times B$ flow. The adopted ordering retains electron inertia which provides the mechanism for Hamiltonian reconnection on the scale of the electron inertia skin depth, d_e. The first two moments of Eq. (1), closed by the energy equation $\nabla_\| T = 0$, recover the fluid model of Ref. [1], which has been extensively used to study Hamiltonian reconnection (see Ref. [3, 4] and references therein).

In this paper, we assume z, the toroidal coordinate, to be ignorable. The generalized momentum $p = v_\| + \Omega_e \psi$ is then conserved. Moreover, density and temperature perturbations are small with respect to a static background described by a distribution function $f_0(v_\|)$ with density n_0 and thermal velocity v_0. It is then possible to introduce a reduced electron distribution function $F(\boldsymbol{x}, v_\|, t)$ such that $f = f_0 + \Omega_e \psi df_0/dv_\| + F$, by means of which Eqs. (1) reduce to

$$\partial_t F + [\phi + v_\| \psi, F] = 0, \qquad \Delta\psi - d_e^{-2}\psi = -e \int v_\| F dv_\|, \qquad \Delta\phi = \frac{\Omega_i}{n_0} \int F dv_\|. \quad (2)$$

Stationary reconnection. In order to study the reconnection process in its nonlinear stage, let us consider a simple X point configuration with ongoing reconnection. The stationary flux and stream functions are given, in polar coordinates, by

$$\psi = j_0 I_2(r/d_e)\cos(2\theta), \qquad \phi = \omega_0 r^2 \sin(2\theta), \quad (3)$$

where j_0 and ω_0 are constants, I_2 is the modified Bessel function of the second order, $x/y = \tan\theta$, and $r = (x^2 + y^2)^{1/2}$. This is a particular solution of the fluid model mentioned above with $\Delta\phi = 0$, $\Delta\psi = d_e^{-2}\psi$, and it is shown in Fig. 3(*a*). We then impose the inflow field lines to have different, constant temperatures: those coming from left equal to $T_0 + \delta T$, while those coming from the right equal to $T_0 - \delta T$. The first question we want to answer is: which is the form of the perturbation, if any, generated by such temperature difference on the reconnected field lines flowing away from the X point, *i.e.* inside the island? The stationary drift kinetic equation has solutions $F = F(\phi + v_\| \psi)$ which may change functional dependence on any of the stream lines $\phi + v_\| \psi = \text{const}$. Note that these particle trajectories, and the separatricies $\phi + v_\| \psi = 0$, are different for different values of $v_\|$.

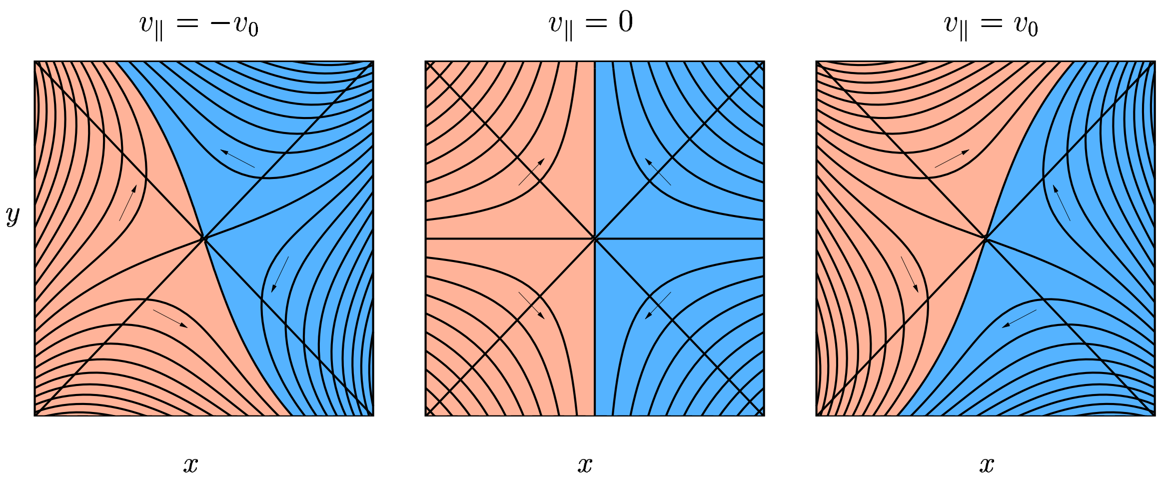

Figure 1: particle trajectories for three values of $v_\|$

In Fig. 1, particle trajectories for three values of $v_\|$ are shown. The particle trajectories coincide with the ψ isolines of Eq. (3) for $v_\| = -\infty$. With increasing $v_\|$, they shift untill they coincide with the isolines of ϕ for $v_\| = 0$, and again to the ψ isolines but with the other separatrix for $v_\| = +\infty$. It is shown that particles coming either from the hot or from the cold side never cross the separatrix $\phi + v_\| \psi = 0$. Since the position of this separatrix depends on $v_\|$, the distribution function in turn depends on the position: it is a superposition of the hot $(T_0 + \delta T)$ electron distribution function F_+ for $v_\parallel < v_s$ and the cold $(T_0 - \delta T)$ function F_- for $v_\parallel > v_s$, where $v_s \equiv -\phi/\psi$ is a function of position. As a consequence, the moments of F (density, current, and temperature) depend on position in the upper and lower quadrants. More specifically, they are functions of ϕ/ψ only:

$$\int_{-\infty}^{+\infty} dv_\parallel F v_\parallel^n = \int_{-\infty}^{v_s} dv_\parallel F_+ v_\parallel^n + \int_{v_s}^{+\infty} dv_\parallel F_- v_\parallel^n. \tag{4}$$

As an illustration, we chose Maxwellian distributions for the hot and cold particles, with thermal velocities $v_0 \pm \delta v$ and densities $n_0 \pm \delta n$, respectively. To the first approximation in the perturbations amplitude we have

$$F_\pm = \pm f_0 \left(\frac{\delta n}{n_0} - \frac{\delta v}{v_0} + 2 \frac{v_\parallel^2}{v_0^2} \frac{\delta v}{v_0} \right), \tag{5}$$

where $f_0 = (n_0/\sqrt{\pi} v_0) \exp(-v_\parallel^2/v_0^2)$, and $\delta v = \delta T / 2 m_e v_0$. The moments of $F_\pm$ are obtained by integration over $v_\parallel$ and are function of $\tilde{\phi} = \phi / v_0 \psi$:

$$n = n_0 \left[\frac{2}{\sqrt{\pi}} \frac{\delta v}{v_0} \tilde{\phi} e^{-\tilde{\phi}^2} - \frac{\delta n}{n_0} \mathrm{erf}(\tilde{\phi}) \right], \qquad j = \frac{e n_0 v_0}{\sqrt{\pi}} \left[\frac{\delta n}{n_0} + 2 \frac{\delta v}{v_0} (\frac{1}{2} + \tilde{\phi}^2) \right] e^{-\tilde{\phi}^2}$$

$$T = m_e v_0^2 \left[\frac{\delta n}{n_0} \left(\frac{1}{\sqrt{\pi}} \tilde{\phi} e^{-\tilde{\phi}^2} \right) + \frac{\delta v}{v_0} \left(\frac{2}{\sqrt{\pi}} (\frac{1}{2} + \tilde{\phi}^2) \tilde{\phi} e^{-\tilde{\phi}^2} - \mathrm{erf}(\tilde{\phi}) \right) \right] \tag{6}$$

and are showed in Fig.2 for the $\delta n / n_0 = 0$ case.

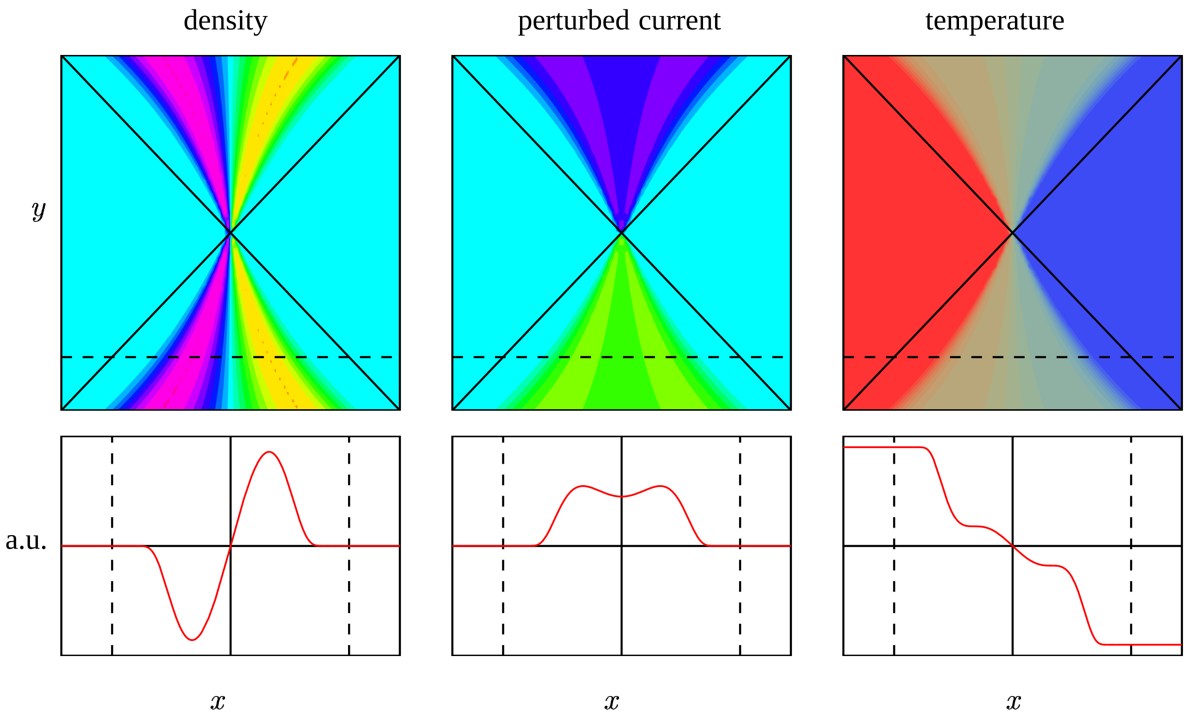

Figure 2: density, perturbed current density, and temperature. First row: contour plots; second row: profiles along the dashed line in the contour plots.

These are the sources in the Poisson and Ampère laws. A second question then arises: which are the effects of such perturbations on the electromagnetic potentials? A simple solution to the selfconsistent problem is found in the proximity of the X point, where the term proportional to d_e^{-2} in Ampère's law can be neglected. In this case both Poisson and Ampère laws reduce to the form $y''(\theta) + 4y(\theta) = g(\theta)$, for which general solutions are known. Note that $g(\theta)$, here representing the sources, vanishes for $3\pi < 4\theta < 5\pi$ and $-\pi < 4\theta < \pi$, *i.e.*, outside the island. The solution is efficiently expressed in term of rotation angles of the separatrices

$$\theta_\phi = \frac{\Omega_i}{8\omega_0}\int_{\pi/4}^{3\pi/4} d\theta \cos(2\theta) n(\theta); \qquad \theta_\psi = \frac{1}{8j_0}\int_{\pi/4}^{3\pi/4} d\theta \cos(2\theta) j(\theta), \tag{7}$$

for the electrostatic potential ϕ and the magnetic flux ψ, respectively. The angles of rotation are clockwise for $x > 0$ and anti-clockwise for $x < 0$. They measure the tilting of the separatrices of the fields ϕ and ψ of Eq. (3), respectively, due to the imposed temperature difference of the incoming field lines. Contour plots of the adjusted fields are shown in Fig. 3(*b*). The parameter dependence can be clarified by the limits

$$\begin{aligned} \frac{\theta_\phi}{\theta_\psi} &= \frac{\delta n/n_0 + \delta v/v_0}{\delta n/n_0 + 2\delta v/v_0}\left(\frac{v_A j_0}{\omega_0}\right)^2, &\quad \text{for } \frac{v_0 j_0}{\omega_0} \ll 1; \\ &= \frac{\delta n/n_0}{\delta n/n_0 + \delta v/v_0}\left(\frac{v_A^2 j_0}{v_0\omega_0}\right), &\quad \text{for } \frac{v_0 j_0}{\omega_0} \gg 1, \end{aligned}$$

where v_A is the Alfvén velocity.

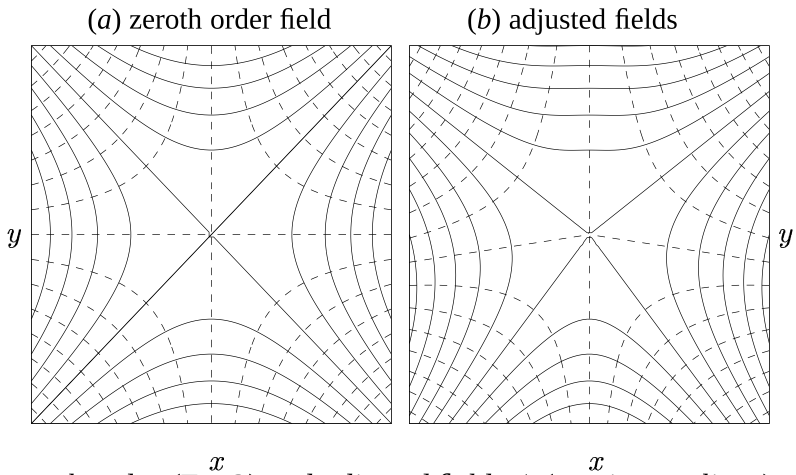

Figure 3: zeroth order (Eq. 3) and adjusted fields ψ (continuous lines) and ϕ (dashes).

Discussion. In a simple X point geometry it has been shown how a left–right asymmetric temperature $T(x, y)$ leads to a breaking of the up–down symmetry of the electromagnetic potentials. We point out that the functional dependence of the moments of the distribution function on ϕ/ψ is a general result, not restricted to the specific functions chosen in Eq. (3). An open question is whether the stationary reconnection process described above is stable to kinetic instabilities in the presence of a temperature difference.

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