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Current Drive via Autoresonance and Intermittent Trapping Mechanisms - A Numerical Study.

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A numerical study of the relativistic motion of a representative distribution of 10^4 electrons in a configuration consisting of two circularly polarized electromagnetic waves counterpropagating along a constant magnetic field is presented. Relatively high averaged parallel velocities have been found for the various sets of parameters considered, indicating the possibility of generating currents in such configurations. Two major features are revealed from this analysis: i) a high velocity peaking in the parallel velocity distribution function, and ii) a tendency of a plateau formation in this distribution. The current associated with a high velocity peaked distribution will be of low collisionality and is thus expected to need less power to sustain it.

The generation of continuous currents in thermonuclear plasmas is presently one of the major goals of the fusion research project around the world. A number of schemes have been proposed for driving currents in plasmas and much progress both theoretically and numerically have been made during the last twenty years, but a complete satisfactory and reliable scheme is still searched for. In a series of papers $[1-5]$, the possibility has lately been forwarded of utilizing the autoresonance (AR) acceleration mechanism proposed by Davydovskii [6] and Kolomenskii and Lebedev [7] for driving currents. This current drive scheme is based on a configuration consisting of two circularly polarized electromagnetic waves counterpropagating along a constant magnetic field. The introduction of these two waves in the system was done in an attempt to overcome two basic difficulties encountered in utilizing the AR mechanism for practical plasmas, namely: a) the necessity of having exact appropriate initial conditions, and b) the necessity of having the indices of refraction of the waves equal to 1. It was found that, due to the nonlinear interaction of the two waves, a traveling ponderomotive well was generated allowing for trapping the electrons. This trapping process may condition the particles for going into AR interactions which results in a high parallel velocity of the electrons. In addition, the trapped particles moving with the velocity of the ponderomotive well constitute by themselves a stream of charged particles in a well defined direction, thus being a current in the plasma. For details of the operation of these mechanisms, we refer the reader to references [4] and [5].

As is well known, associated with the two waves interaction is the possibility for a stochastic behavior. This feature is reflected in the intermittent nature of the two just mentioned mechanisms for driving currents, and can clearly be seen as abrupt changes of the velocity of the particles for very short periods of time. When considering a single particle analysis, this intermittency might in principle diminish considerably the effectiveness of the current drive scheme suggested. In ref. [5], it was suspected that this will not be the case when a distribution of particles is considered. In that case, the intermittent nature of the motion of a single particle is masked in average by the regular motion of the other particles. To test this conjecture, we have to resort to a numerical analysis with a distribution of particles which might be a reasonable representative of relativistic plasmas. We have thus considered a Jüttner - Synge [9] distribution of electrons.

$$f(v) = nm^3(2\pi mT)^{-3/2}(\frac{\pi}{2z})^{1/2} \frac{(1-(v/c)^2)^{-5/2}\ e^{-z(1-(v/c)^2)^{-1/2}}}{K_2(z)}, \quad (1)$$

where f is the velocity distribution of the electrons, $z = mc^2/T$, (T being the temperature) and $K_2(z)$ is a normalization constant. This distribution takes explicitly into account the relativistic nature of the motion. Starting from this distribution for the initial velocities, we have followed the individual trajectories of the particles evaluated from the set of the equations of motion. This set of equations was derived from an Hamiltonian description of the motion in terms of action - angle variables P_ϕ, P_ψ, ϕ, ψ, and explicitly read:

$$\begin{aligned} \dot{\phi} &= \frac{1}{\gamma}\Big[+\overline{k}_1(\overline{k}_1P_\phi - \overline{k}_2P_\psi) + 1 \\ &\quad +(\overline{A}_1\sin\phi + \overline{A}_2\sin\psi)R^{-1}\Big] - \overline{\omega}_1, \quad (2) \\ \dot{\psi} &= \frac{1}{\gamma}\Big[-\overline{k}_2(\overline{k}_1P_\phi - \overline{k}_2P_\psi) + 1 \\ &\quad +(\overline{A}_1\sin\phi + \overline{A}_2\sin\psi)R^{-1}\Big] - \overline{\omega}_2, \quad (3) \end{aligned}$$

$$\dot{P}_\phi = -\frac{\overline{A}_1}{\gamma}\Big[R\cos\phi - \overline{A}_2\sin(\phi-\psi)\Big], \quad (4)$$

$$\dot{P}_\psi = -\frac{\overline{A}_2}{\gamma}\Big[R\cos\psi + \overline{A}_1\sin(\phi-\psi)\Big], \quad (5)$$

where:

$$\begin{aligned} R &= [2(P_\phi + P_\psi)]^{(1/2)}, \quad (6) \\ \gamma &= 1 + [(\overline{k}_1P_\phi - \overline{k}_2P_\psi)^2 \\ &\quad +(R+\rho)^2 + \sigma^2]^{1/2}, \quad (7) \end{aligned}$$

with $\rho = \overline{A}_1\sin(\phi) + \overline{A}_2\sin(\psi)$ and $\sigma = \overline{A}_1\cos(\phi) + \overline{A}_2\cos(\psi)$. $\overline{A}_1$ and $\overline{A}_2$ are the normalized components of the vector potential:

$$\begin{aligned} \overrightarrow{A} = B_0x\overrightarrow{e}_y + \overline{A}_1\Big\{&[\sin(\overline{k}_1\overline{z} - \overline{\omega}_1\overline{t})]\overrightarrow{e}_x \\ &+[\cos(\overline{k}_1\overline{z} - \overline{\omega}_1\overline{t})]\overrightarrow{e}_y\Big\} \\ +\overline{A}_2\Big\{&[-\sin(\overline{k}_2\overline{z} + \overline{\omega}_2\overline{t})]\overrightarrow{e}_x \\ &+[\cos\overline{k}_2\overline{z} + \overline{\omega}_2\overline{t})]\overrightarrow{e}_y]\Big\}, \quad (8) \end{aligned}$$

We have normalized the Hamiltonian to mc^2 and the other normalized quantities are: $\overline{A}_{1,2} = eA_{1,2}/mc^2$, $\overline{t} = \Omega t$, $\overline{\omega}_{1,2} = \omega_{1,2}/\Omega$, $\overline{k}_{1,2} = ck_{1,2}/\Omega$, $\overline{z} = \Omega z/c$, Ω being the gyrofrequency eB_0/mc. The normalized wavenumbers $\overline{k}_{1,2}$ have been evaluated according to the usual dispersion relation:

$$\overline{k}_{1,2} = \sqrt{\overline{\omega}^2_{1,2} - \frac{e_0\overline{\omega}_{1,2}}{(\overline{\omega}_{1,2}-1)}}, \quad (9)$$

where $e_0 \equiv \omega^2_{pe}/\Omega^2$.

In solving numerically these equations, we have made, for convenience, a change of variables from action - angle to velocities and coordinates, which are also more appropriate when dealing with a Jüttner distribution. We have assumed a uniform distribution of the particles in coordinates space x and z in a box of size $\Delta x = \Delta y = \Delta z = 1$. The numerical scheme is a fourth order Runge - Kutta with a variable time step. The equations (2-5) admit the following invariant quantity [2]:

$$I = \gamma - \overline{\omega}_1P_\phi - \overline{\omega}_2P_\psi. \quad (10)$$

The time step in the integration scheme has been chosen small enough to guarantee that $(I^{\text{NUM}} - I^0)/I^0$ is always smaller than 10^{-4} for all the electrons, where I^{NUM} is the numerically computed value of I, and I^0 its initial value.

In order to examine the possible realization of these current driving mechanisms based either on multiple AR acceleration process or intermittent trapping, we follow a representative distribution of 10^4 particles during 3.10^4 normalized time units and evaluate the averaged parallel velocity of the particles at every time unit. Such a simulation required about 10^6 time steps for each particle.

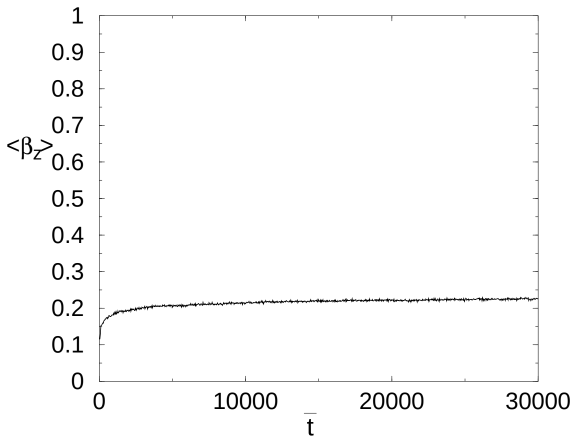

Figure 1(a)

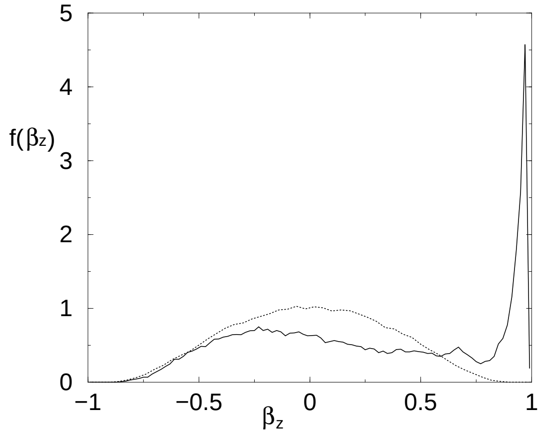

Figure 1(b)

Figure 1: (a) the evolution of the averaged normalized velocity $< \beta_z >=< v_z > /c$ *with respect to the normalized time* $\bar{t}$. *The values of the parameters are* $e_0 = 0.1$, $\overline{\omega}_1 = 3.0$, $\overline{\omega}_2 = 0.7$, $\overline{A}_1 = 0.45$ *and* $\overline{A}_2 = 0.06$.
(b) The parallel velocity distribution of the electrons for the same set of parameters, at $\bar{t} = 30.000$ *(solid line) and at* $\bar{t} = 0$ *(dotted line).*

In Fig. 1(a), we see that, starting from zero, the average parallel velocity increases considerably and tends to saturate to a value around $0.22c$. A significant feature of this generated current is readily seen when inspecting the distribution of parallel velocities of the electrons after the interaction has taken place, as presented in Fig. 1(b). Here, one notices the pronounced peak in the high velocity range of the distribution, making this distribution strongly asymmetric. This fact is of great practical significance due to the low collisionality associated with these fast electrons, thus reducing the power requirement to sustain a given current against collisions.

We tend to attribute this saturation effect to the fact that, after a long enough time, all the particles which could have participated in the AR interaction via the ponderomotive well, took already this course. For details, we refer the reader to Ref. $[5]$.

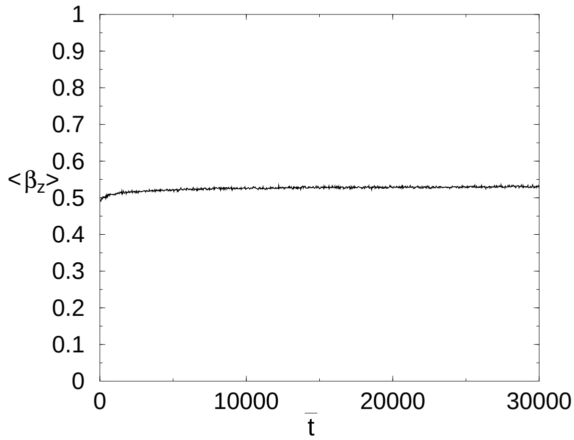

Figure 2(a)

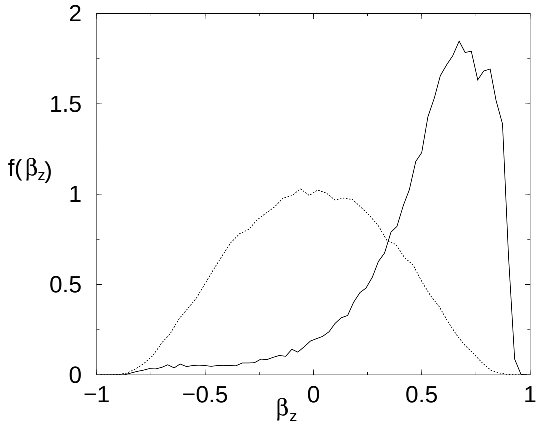

Figure 2(b)

The case studied and presented in Fig. 1 corresponds to a configuration which exhibit primarily the operationality of an AR mechanism. However, the possibility of driving currents via the intermittent trapping mechanism has been also treated in this study and results are presented in Fig. 2. Figures 2(a) an 2(b) again correspond to Fig. 1(a) and 1(b). Let us notice that here, even higher saturated averaged parallel velocity ($< \beta_z >\simeq 0.5$) are reached. One should notice also in Fig 2(b) the plateau structure of the β_z distribution function which is the signature of the generation of a current as was recognized many years ago in the original papers of Fisch [11].

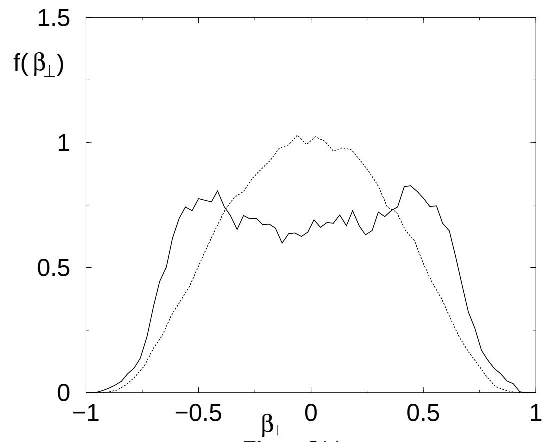

Figure 2(c)

Figure 2: Frames (a) and (b) are the same as in Fig.1 with the parameters $e_0 = 0.06$, $\overline{\omega}_1 = 0.7$, $\overline{\omega}_2 = 0.03$, $\overline{A}_1 = 0.5$ and $\overline{A}_2 = 0.25$.
(c) The perpendicular velocity distribution of the electrons at $\bar{t} = 30.000$ (solid line) and at $\bar{t} = 0$ (dotted line). The parameters are the same as in (a) and (b).

From the analysis and our discussions, it is expected that only the parallel velocity distribution will be considerably distorted from the initial Jüttner symmetric distribution. While, in the perpendicular direction, one should not expect such an equivalent distortion. Inspecting Fig 3(c), one realizes that this is indeed the case. Comparing the initial $\beta_\perp$ distribution (solid line) with the final one (dotted line), one notices that there is not a significant change in the symmetry characteristics of the distributions.

In summary, the results which have been presented indicate in a transparent manner that, indeed, the generation of currents is possible. Two features revealed from this analysis should be emphasized: a) the high velocity peaking in the parallel velocity distribution function, and b) the tendency of the formation of a plateau in this distribution. The peaking in the distribution leads to the generation of a current of fast electrons. Such a current will last longer due to the low collisionality of these electrons;

As to the plateau formation in the parallel velocity distribution, this might indicate the possibility of a global quasilinear process in the system. If it turns out to be true, it will allow for an almost standard theoretical analysis of the evaluation of the needed power for sustaining the current [12].

Finally, for further validation of these current drive schemes, more elaborated numerical studies will have to be performed, taking into account more realistic aspects of the plasma. In this context, the effects due to collisions in the system are expected to be of great interest.

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